## **Thermodynamically Consistent Atomic Radii of Main Group Elements**

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## **Abstract**

Atomic radius is a fundamentally important quantity shaping multiple properties of atoms, molecules, and materials. Due to its fundamental importance, precise quantification and interpretation of atomic radii has been the subject of numerous studies for more than a century. It has resulted in a broad variety of estimations of atomic radii which so far have not been amenable to extensive verification by experiment. The present study follows our recent work demonstrating that electron iso-density surfaces, contoured at a certain cut-off density, accurately represent molecular surfaces determined via thermodynamic phase change data. Here we provide thermodynamically consistent estimates of atomic radii for the main group elements.

Atomic radii are fundamentally important quantities governing many properties of atoms such as electronegativity [1, 2], hardness [3], polarizability [4, 5], ionization potential [6, 7], and electrophilicity index [8, 9]. Precise quantification of atomic radii is crucial in numerous applications in physics and chemistry, including but not limited to theoretical studies of solvent effects via continuum solvation models [10-13], (supra-)molecular interactions [14-16], materials properties such as electrical conductivity [17], melting points [18], porosity [19] and magnetic behavior [20], strength of chemical bonds [21], non-covalent interactions [22-26], design of van der Waals (vdW) materials [27-30], and many other applications. Due to its fundamental importance, quantification of atomic radii has been a subject of numerous studies for more than a century, starting from the pioneering work of Meyer [31], followed by a wide variety of experimental [32-37] and theoretical approaches [38-47] developed in the past decades.

A review of the literature reveals a broad spread, and even some inconsistencies, among available estimations of atomic radii. For experimental methods, atomic radii are commonly determined via interatomic distances measured in the condensed phase. The radii estimated this way are therefore susceptible to be largely influenced by interatomic interactions, resulting in broad range of radii estimations heuristically classified as covalent, ionic, metallic, or vdW radii [48, 49]. Radii estimation from interatomic distances also commonly relies on the assumption of the additivity of atomic radii and approximating the most probable value of each radius from a broad distribution of interatomic distances between a specific element and probe atom(s), which introduces some arbitrariness and uncertainty. The other sources of inaccuracy in conventional experimental methods are due to neglecting the dependence of atomic radii on the chemical environment [50- 52], electron distribution anisotropy and bond polarity [33, 53], and crystallographic direction [54]. In contrast, theoretical methods do typically not suffer from such issues. Nevertheless, the uncertainty in experimental reference data has hindered a fair comparison and extensive verification of the theoretical approaches. Among available theoretical methods, the Bader method which considers the atomic radii as the distance to iso-density surfaces for a certain cut-off density is probably the most widely-accepted approach [55-58]. However, extensive and broad application of this method has long been hindered by the ambiguity in defining the most appropriate value of the cut-off density due to lack of experimental verification [59], as discussed above.

In our recent studies [59, 60], we shed light on the above-mentioned issues by exploiting a recent method allowing precise experimental evaluation of the surfaces of molecules via thermodynamic phase change data [11] and comparing them with iso-density surfaces evaluated for different cutoff densities. We demonstrated that for a large dataset of studied compounds, adopting a cut-off density of 0.0016 a.u., which is roughly at the mid-point of the values suggested by Bader (0.002 a.u.) [61] and Boyd (0.001 a.u.) [44], yields iso-density surfaces that almost perfectly match with the experimental estimates of molecular surfaces. As pointed out by Ashcroft and co-workers, outstanding advantages of this approach stems from linking atomic radii to electron density as an (in principle) experimentally observable quantity, and also because Pauli, exchange, or same-spin repulsions are mainly determined by electron density at the outermost regions of atoms [55]. Additionally, this approach also benefits from the possibility of highly precise estimation of electron density via modern quantum chemical methods, allowing to uniquely define atomic surfaces, taking into account atomic surface asymmetry and anisotropy, and being experimentally verified for a large dataset of compounds [59]. Therefore, in the present study we exploit this approach to provide an estimation of atomic radii for the main group elements. Because the employed value of the cut-off density (0.0016 a.u.) is obtained with reference to experimental thermodynamic phase change data, the atomic radii estimated by this approach are coined thermodynamically consistent.

To estimate atomic radii via electron iso-density surfaces, we computed electron densities of elements in the isolated state (isolated atoms) at coupled cluster CCSD(T) level of theory, which is conventionally considered to be the gold standard in quantum chemistry. We used the large Def2-QZVPPD quadruple-zeta basis set, which was previously shown by us to be highly accurate in reproduction of experimental estimations of molecular resurfaces [59]. Additionally, to study the importance of relativistic corrections which might be important for heavy elements, we also used the ANO-RCC basis set and the Douglas–Kroll–Hess second-order scalar relativistic Hamiltonian [62] which have been commonly used to evaluation of atomic radii [42, 55]. In comparison to single reference computations, the much more computationally demanding multireference methods are more robust, especially for open shell elements where multiple spin configurations might contribute to the expectation values of atomic properties. Nevertheless, for evaluation of electron density and radii approximation, less demanding single reference

computations, even at DFT level of theory, can closely reproduce the results obtained via multireference computations, as demonstrated by Rahm et al [55]. Therefore, in this study, we considered the Complete Active Space Self-Consistent Field (CASSCF) multireference computations only for the first 36 elements, to investigate the accuracy of various applied methods. To approximate atomic radii from electron density distributions, we spherically averaged the electron density for shells with 0.0001 Å radii intervals and took the mid-point of the shell with closest value to the required cut-off density as the atomic radius. All electronic structure computations were carried out in Orca 5.0.3. [63] and post-processed by Multiwfn software [64]. For all studied levels of theory, details of the estimated atomic radii based on different cut-off densities from 0.001 a.u. to 0.002 a.u. with 0.0001 a.u. intervals are provided as supplementary information. Our results show that the radii estimated with the Def2-QZVPPD basis set closely agree with those obtained with the ANO-RCC basis set and based on relativistic correction as well as via CASSCF, with average unsigned deviation of only 0.020 Å and 0.015 Å, respectively. Accordingly, for sake of consistency among elements and also with our previously reported results [59, 60], we report the atomic radii based on computations with the Def2-QZVPPD basis set. According to the results reported in Table 1, our estimated atomic radii are on average 6.8% smaller than those reported by Rahm et al., which are based on a cut-off density of 0.001 a.u. Furthermore, the atomic radii estimated from electron density distribution of isolated atoms are in good agreement with the values reported by us for H, C, N, O, and F elements in our previous study obtained from a different approach. In that work, we found the atomic radii of isolated elements via optimization and in a way that the radii of atoms in molecules evaluated through free and effective volumes reproduce most accurately the overall molecular surfaces [60]. Accordingly, for the effective volumes calculated via iterative Hirshfeld partitioning, we obtained atomic radii of 1.47, 1.87, 1.65, 1.49, and 2.06 Å for H, C, N, O, and F elements, respectively, which are in good agreement with results reported here.







For the challenging case of hydrogen atom for which large discrepancies of predicting atomic radius has been reported in several studies [33, 67, 68], our previously reported and current estimations seem quite consistent and are also in better agreement with those parameterized in the COSMO-RS solvation model and reported by Bondi and Alvarez, in comparison to other methods (Table 1). Accordingly, we propose a radius of 1.41 Å for hydrogen.

In summary, we provide quantum chemistry-based estimates of atomic radii for main group elements. The radii are computed as the distance to the nucleus of spherically averaged iso-density surfaces contoured at 0.0016 a.u. The radii obtained from this approach are grounded on experimental phase change thermodynamic data related to molecular surfaces, and are thus thermodynamically consistent.

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