# Structure and Excitation Spectra of Third-Row Transition Metal Hexafluorides Based on Multi-Reference Exact Two-Component Theory

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The structures and some vertical excitation energies of third-row transition metal hexafluorides  $(MF_6, M = \text{Re}, \text{Os}, \text{Ir}, \text{Pt}, \text{Au}, \text{Hg})$  were calculated using the multi-reference configuration interaction (MRCI) theory based on exact two-component (X2C) Hamiltonian. The spin-orbit coupling (SOC) was variationally included at the Hartree-Fock level, enabling us to analyze the SOC at the orbital level. The excitation spectra were assigned based on the double group, a relativistic group theory applicable to states with the SOC. This study provides a fundamental understanding of the ligand field splitting, including the SOC, that is useful for the photochemistry and spin chemistry involving heavy elements.

# I. INTRODUCTION

 $MX_6$ -type complexes, where M is a central metal element and X is a ligand, serve as excellent models for introducing ligand field splitting – an essential factor in predicting the electronic and spin states of transition metal complexes. This subject is frequently discussed in inorganic chemistry books  $[1-3]$  $[1-3]$ . While some  $MX_6$  complexes exhibit the expected  $O_h$  symmetry based on their molecular formula, the Jahn-Teller (JT) effect [\[4,](#page-7-2) [5\]](#page-7-3) can induce distortions that lead to another molecular structure such as  $D_{4h}$  symmetry. The smallest and most extensively reported  $MX_6$  complexes would be  $MF_6$ . This complex garners attention not only in introductory chemistry texts but also due to its environmental impact of  $SF_6$  [\[6,](#page-7-4) [7\]](#page-7-5), notable oxidation properties when M is a third-row transition metal  $[8-10]$  $[8-10]$  and UF<sub>6</sub> in nuclear fuel reprocessing [\[11,](#page-7-8) [12\]](#page-7-9). A comprehensive overview of the history and physical and chemical properties of  $MF_6$ is provided in ref [13.](#page-7-10)

Traditional ligand field theory, developed for light elements, does not adequately capture the intricate electronic behaviors exhibited by heavy transition metals. The relativistic effect is significant when describing the chemistry of heavy elements [\[14](#page-7-11)[–19\]](#page-7-12). The relativistic effect can be classified into the following two effects: i) scalar relativistic effect, where the s and p orbitals are contracted while the d and f orbitals are expanded, and ii) spin-orbit coupling (SOC), which is the interaction of the electron spin and the magnetic field induced by a charge in relative motion, leading to the splitting of the degenerate orbitals with  $l \geq 1$ , where l is the quantum number of the electronic angular momentum. The former effect can be sufficiently taken into account by introducing a pseudo-potential [\[20–](#page-7-13)[23\]](#page-8-0), where core electrons are replaced with a fitted potential for valence molecular properties. The latter effect can become small in

molecules with low symmetry even if heavy elements are included because of the orbital angular momentum quenching [\[1,](#page-7-0) [3,](#page-7-1) [24\]](#page-8-1), and thus this effect can often be ignored. However, the SOC can become significant and has to be carefully considered in complexes with high symmetry, which is the target of our study.

Another important aspect is including static correlation effects to treat the degeneracy in the  $MF<sub>6</sub>$  molecules. Most previous studies of  $\text{MF}_6$  [\[25](#page-8-2)[–38\]](#page-8-3) were based on single-reference methods such as the density functional theory (DFT) or coupled cluster theory. These methods are sufficient for closed-shell systems but may encounter problems when there is significant multi-reference effect In the context of the vibrational coupling with the JT effect, multi-reference methods with the SOC were employed by Balasubramanian's group for metal clusters [\[39–](#page-8-4)[42\]](#page-8-5), by Mondal for the first-row transition-metal trihalides [43,](#page-8-6) [44,](#page-8-7) and by Koseki's group for third-row transition-metal tetrahydrides [\[45,](#page-8-8) [46\]](#page-8-9). However, such an approach has not been applied for  $\text{MF}_6$  molecules.

The static JT distortion itself is well known in the structures of  $\text{MF}_6$  molecules with non-zero spin. However, the structure of the third-row transition metal hexafluorides may be stable in the presence of JT distortion. It is currently recognized that  $\text{Re}\mathbf{F}_6$  and  $\text{Os}\mathbf{F}_6$  show the dynamic JT effect [\[47](#page-8-10)[–49\]](#page-8-11), and the structures of  $\text{ReF}_6$ ,  $OSF_6$ , Ir $F_6$ , and Pt $F_6$  are very close to  $O_h$  based on electron-diffraction experiments in the gas phase [\[50–](#page-8-12)[52\]](#page-8-13), single-crystal X-ray diffractometry [\[53\]](#page-8-14), and extended Xray absorption fine structure (EXAFS) [\[54\]](#page-8-15). Even if it exists, the JT distortion may be smaller than experimental errors. In theory, the most sophisticated calculation for the third-row transition metal hexafluorides was carried out at the coupled cluster singles and doubles with perturbed triples (CCSD(T)) level with the SOC at the DFT level [\[33\]](#page-8-16). Their calculated structures agree with the experiment well, but they compared the averaged bond lengths of the  $D_{4h}$  structures and experimental structures in  $O_h$  symmetry. It was reported that the SO effect stabilizes the  $O_h$  structures of PtF<sub>6</sub> [\[55,](#page-8-17) [56\]](#page-9-0) and

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 $\mathrm{IrF_6}^-$  [\[57\]](#page-9-1) based on four-component DFT and  $\mathrm{CCSD(T)}$ methods, respectively. However, a systematic study for third-row transition metal hexafluorides based on relativistic multi-reference theory has not been reported.

In this study, the effect of the SOC in third-row transition metal hexafluorides is extensively investigated. The multi-reference configuration-interaction (MRCI) method based on the exact two-component Hamiltonian is employed to calculate the structures and vertical excited energies. The calculated electronic excited states, including the SOC, are assigned based on the double group, which is a relativistic group symmetry, taking the SOC into account. The schematic diagrams based on the one-electron orbital Figure [1](#page-2-0) are also proposed.

#### II. THEORY

Assigning the symmetry of  $MF_6$  in the relativistic framework requires the double group, which is a relativistic symmetry group. Although the double group is well known [\[16,](#page-7-14) [24,](#page-8-1) [58–](#page-9-2)[60\]](#page-9-3), we briefly describe it to clarify the notation in this work.

An operator for a rotation by a finite angle  $\phi$  about the quantization axis can be expressed as follows [\[61,](#page-9-4) [62\]](#page-9-5):

$$
\hat{R}_z(\phi) = \exp\left(\frac{-i\hat{J}_z\phi}{\hbar}\right),\tag{1}
$$
\n
$$
\hat{R}_z(\phi)|J, M_J\rangle = \exp(-iM_J\phi)|J, M_J\rangle,
$$

where  $i$  is the imaginary unit,  $J$  is an angular momentum (whether orbital, spin, or total angular momentum) and  $M_J$  is the projection of the angular momentum along the axis z. The representation matrix of this rotation with respect to the basis  $M_J = -J, -J+1, \ldots, J$  can be expressed by

$$
\mathbf{D}^{J}\left(R_{\phi}\right) = \begin{pmatrix} \exp[iJ\phi] & 0 & \dots & 0 \\ 0 & \exp[i(J-1)\phi] & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \exp[-iJ\phi] \end{pmatrix} \tag{2}
$$

A trace of this matrix is

<span id="page-1-0"></span>
$$
\chi^{J}(\phi) = e^{-iJ\phi} \sum_{l=0}^{2J} (e^{i\phi})^{l}
$$

$$
= \frac{\sin[(J+1/2)\phi]}{\sin(\phi/2)}.
$$
(3)

This is the character of the representation  $\mathbf{D}^{J}(R_{\phi})$  with respect to the rotation  $\phi$ . For example, in the case of respect to the rotation  $\phi$ . For example, in the case of  $R_{\phi} = C_4$  ( $\phi = \pi/2$ ) and  $J = 1/2$ ,  $\chi^J(\phi) = \sqrt{2}$  is obtained.

The periodicity of eq [3](#page-1-0) can be obtained for the cases where  $J$  is an integer or a half-integer:

<span id="page-1-1"></span>
$$
\chi^{J}(\phi \pm 2\pi) = +\chi^{J}(\phi) ; \text{ (an integer } J), \tag{4}
$$

$$
\chi^{J}(\phi \pm 2\pi) = -\chi^{J}(\phi); \text{ (a half-integer } J), \tag{5}
$$

$$
\chi^{J}(\phi \pm 4\pi) = +\chi^{J}(\phi);
$$
 (an integer/a half-integer J), (6)

In the non-relativistic (NR) case, the molecular symmetry is described by the spatial symmetry and the electronic orbital angular momentum  $(M_J = M_L)$ . In thfis case, the quantum number  $(J = L)$  is an integer, and rotation by  $2\pi$  returns the system to the original state (eq. [4\)](#page-1-1). That is, rotation by  $2\pi$  is equivalent to the identity operator  $\hat{E}$ . This is the case of a normal point group. Meanwhile, in the relativistic case, the projection of the total angular momentum  $M_J = M_L + M_S$  is a good quantum number, and J becomes a half-integer when the system contains an odd number of electrons. Here, rotation by  $2\pi$  reverses the sign (eq [5\)](#page-1-1). In both cases,  $\hat{R}_{\phi=4\pi} = \hat{E}$ is satisfied.

The symmetry operations in the double group are obtained from the direct product of the point group and the group  $\left\{\hat{E}, \hat{Q}\right\}$ , where  $\hat{Q} = \hat{R}_{\phi=2\pi}$ . Several columns are added to the character table to describe additional irreducible representations that correspond to the oddnumber electron systems.  $\dot{Q}$  does not disturb the point group (cf. eq [4\)](#page-1-1) and only affects the additional irreducible representations (cf. eq [5\)](#page-1-1).

The great orthogonality theorem also holds in the double group. Hence, the direct product of irreducible representations and the reduction of the reducible representation can be performed as the point group. For example, the splitting of the one-electron molecular orbitals belonging to the irreducible representations  $e_q$  and  $t_{2q}$  in  $O_h$  symmetry due to the SOC in the double group can be expressed as follows:

$$
e_g \otimes e_{1/2,g} = u_{3/2,g},\tag{7}
$$

$$
t_{2g} \otimes e_{1/2,g} = u_{3/2,g} \oplus e_{5/2,g}.
$$
 (8)

Here  $e_{1/2,g}$  is the representation of the electron spin in the double group.

In the electronic structure theory, the SO splitting of the states is found when the total spin is nonzero. In the case of group theory, however, whether  $J$  is an integer or a half-integer is important (cf., eqs [4](#page-1-1) and [5\)](#page-1-1). Even when the SOC contributes, if  $J$  is an integer (that is, an even number of electrons in the system), the representation of the system is the same as the point group of the NR case. The double group is applied only when the number of electrons is odd.

# III. COMPUTATIONAL DETAIL

All calculations of this study were carried out by using a development version of the DIRAC program [\[63,](#page-9-6) [64\]](#page-9-7); the precise version and build information are shown in the output files, see ref [\[65\]](#page-9-8). Our calculations are based on



<span id="page-2-0"></span>shown. The energy differences  $(\text{cm}^{-1})$  and symmetry labels (Mulliken's symbol) are obtained at the X2C/AOC-HF/dyall.v3z *b*<sub>2*g*</sub> corresponds to  $d_{xy}$ -type orbital, where the z-axis is the axial axis of MF<sub>6</sub>. Schematic molecular structures are also notation,  $b_{2g}$  corresponds to  $d_{xy}$ -type orbital, where the z-axis is the axial axis optimized geometries shown in Table [V.](#page-5-0) The orbital energy differences are obtained with  $n$  ( $1 \le n \le 4$ ) electrons in six  $t_{2g}$ -like FIG. 1. Schematic orbital energy diagram and electronic configurations for  $t_{2g}$ -type orbitals at the SF and SO levels. In our shown. The energy differences (cm ) and symmetry fabels (Multiken's symbor) are obtained at the X2C/MOC-IIF/dyall.v3z<br>level. Schematic electronic configurations are described based on the results at the X2C/MRCI/dyall.v3z orbitals and with 128 electrons in the closed orbitals. The option ".OPENFAC=1.0" is specified in the DIRAC calculations to satisfy Koopmans's theorem.

the spin-free (SF) and spin-orbit (SO) relativistic Hamiltonians. For the one-electron part of the Hamiltonian, the exact two-component Hamiltonian (X2C) [\[66\]](#page-9-9) is employed for both SF and SO relativistic calculations. For the two-electron term, the non-relativistic Coulomb operator is employed for the SF calculation, and the Gaunt term is added in the atomic mean-field approximation (AMFI) [\[67\]](#page-9-10) scheme for the SO calculation [\[68\]](#page-9-11). The nuclear potential is described by the Gauss-type finitesize nuclear model [\[69\]](#page-9-12). For all elements in this study, dyall.v3z basis sets [\[70–](#page-9-13)[72\]](#page-9-14) are employed in the uncontracted form.

The multi-reference configuration interaction (MRCI) theory was employed to compute the wavefunction for geometry optimization and vertical excitation energies. For the MRCI calculation, we employed the LUCITA mod-

ule [\[73–](#page-9-15)[75\]](#page-9-16) for the SF Hamiltonian and the KRCI module [\[76–](#page-9-17)[78\]](#page-9-18) for the SO Hamiltonian. In these modules, one can flexibly specify the number of holes in the CI space, and this is called the general-active-space configurationinteraction (GASCI) method. The active space employed in this study is shown in Table [I.](#page-3-0) Here, the completeactive-space CI (CASCI) calculation is employed in the space constructed by the  $t_{2g}$  orbitals in the SF or by the  $u_{3/2,g}$  and  $e_{5/2,g}$  orbitals in the SO levels (GAS2), and the two-electron excitation is allowed from the GAS2 space. One-electron excitations from GAS1 and GAS2 (a total of two electrons) are also allowed. The truncation of the virtual orbitals shown in Table [II](#page-3-1) is determined so that the two orbitals are energetically well-separated. The same CI parameters are employed for both SO and SF calculations. The reference orbitals of the MRCI cal-

<span id="page-3-0"></span>TABLE I. Generalized active space (GAS) models for CI wave functions of the  $MF_6$  molecules. The parameters m and n that determine the occupation constraints of the subspaces are described in Table [II](#page-3-1)

			$accumulated \# of electrons$
	$\#$ of orbitals	min	max
GAS3	m	$24+n$	$24+n$
GAS <sub>2</sub>	3	$22+n$	$24+n$
GAS1	12	23	24
frozen	(52)		

<span id="page-3-1"></span>TABLE II. Parameter values for CI calculations of the  $MF_6$ molecules. n refers to the number of correlated electrons of GAS2 space  $(t_{2g}$  orbitals in the SF and  $u_{3/2,g}$  and  $e_{5/2,g}$  orbitals in the  $SO$ ).  $m$  refers to the number of virtual orbitals included in the GAS3 space. "small" is employed for the optimization, and "large" is employed for the excited state and JT distortion energy calculations



culation are generated with the average-of-configuration Hartree-Fock (AOC-HF) method [\[79\]](#page-9-19) or the HF method. The detailed parameters are listed in Table [III.](#page-3-2)

We also employed the CCSD(T) method [\[80\]](#page-9-20) for the optimization of HgF<sub>6</sub>, assuming  $O_h$  symmetry. The 32 electrons from the highest occupied molecular orbital (HOMO) are correlated, and the virtual space is truncated at 5 Hartree.

Unless otherwise explicitly mentioned, we optimized the two axial and four equatorial M-F bonds of  $MF_6$ molecules, assuming they belong to the  $O_h$  or  $D_{4h}$  symmetry group. The SURFIT program [\[81\]](#page-9-21) was used to obtain the equilibrium structure in a two-dimensional potential energy surface for several molecules. Numerous one-point calculations were performed for the other molecules by changing the bond length by a step size

<span id="page-3-2"></span>TABLE III. Parameter values to obtain the reference orbitals for the CI calculations of the  $\text{MF}_6$  molecules provided in Table [II](#page-3-1) at each relativistic Hamiltonian (REL). Only the central metal M is shown in the table. "closed" refers to the number of electrons occupied in the closed shell orbitals.  $m/n$  shown in "open" means that  $m$  electrons are distributed to  $n$  orbitals in the AOC-HF method. The HF method was employed when the open-shell electron was zero

	REL CI shell Re Os Ir Pt Au Hg			
	SO small closed 128 128 128 132 132 134			
	open $1/4$ $2/4$ $3/4$ 0 $1/2$ 0			
	large closed 128 128 128 -			
	open $1/6$ $2/6$ $3/6$ $4/6$ -			
SF.	small closed 128 128 128 128 132 134			
	open $1/6$ 2/6 3/6 4/6 1/2			$\Omega$
	large closed 128 128 128 -			
	open $1/6$ 2/6 3/6 4/6 -			

TABLE IV. Optimized bond lengths of HgF<sub>6</sub>  $(r)$  in Å obtained using different methods. The dyall.v3z basis sets were employed.

<span id="page-3-3"></span>

method $CCSD(T)$		MRCI		PBE0[36]		
X2C	SO SF		SO SF	SO <sup>a</sup>	- SF	
		1.920 1.927 1.889 1.892 1.885 1.887				
<sup>a</sup> The two-electron spin-orbit effect is simulated by a						
		screened-nuclear-spin-orbit approach.				

of 0.001 Å. The details are explained in the Supporting Information. The  $O<sub>h</sub>$  structure was obtained using the twofit program, which is a utility of the DIRAC code.

The projection analysis [\[82\]](#page-9-22) using the intrinsic atomic orbital [\[83\]](#page-9-23) was carried out for the  $PtF_6$  molecule. At the SF level, two electrons are distributed in four orbitals, and 130 electrons occupy the closed shell in the AOC-HF calculation. In the projection analysis step, 0.01 and 1.99 electrons are distributed in the virtual  $e'_{g}$ and  $e_g$  orbitals and the occupied  $b_{2g}$  orbitals to obtain the gross population of the virtual  $e'_{g}$  orbitals. At the SO level, 132 electrons occupy the closed shell in the HF calculation. In the projection analysis step, 0.01 (3.99) electrons are distributed in the virtual  $u'_{3/2,g}$  and  $e_{5/2,g}$ orbitals (the occupied  $u_{3/2,g}$  orbitals) (cf., Figure [2\)](#page-7-15). In the atomic and molecular calculations, an option of the DIRAC code, .OPENFAC=1.0, was employed.

#### IV. RESULTS AND DISCUSSION

### 1. Structures

To estimate the dynamic correlation effect in the employed GAS scheme in the MRCI method, the bond lengths of  $HgF_6$  obtained by the MRCI are compared with the  $CCSD(T)$  values (cf. Table [IV\)](#page-3-3). The symmetry of the HgF<sub>6</sub> molecule is fixed to  $O_h$ . From the comparison between the MRCI and CCSD(T) values, the MRCI method underestimates the dynamic correlation on the bond length by 0.031-0.035 Å. From the viewpoint of dynamic correlation, the MRCI provides similar accuracy to that of the PBE0 functional of DFT [\[36\]](#page-8-18). Although this deviation is similar in magnitude to the JT distortion (cf. Tables [V](#page-5-0) and [VII\)](#page-5-1), it would not matter because the multi-reference effect would be more significant for determining the symmetry of the open-shell  $\text{MF}_6$  systems. The MRCI and CCSD(T) methods provide the SO effects in a similar order of magnitude. This means that the coupling between the SO effect and the dynamic correlation would not affect the conclusion of our study.

The SO effect is not a correction of the SF calculation but can change the molecular point group symmetries at equilibrium geometry. Our results at the MRCI level are listed in Table [V.](#page-5-0) The most sophisticated literature values at the SF-CCSD(T) and SO-BLYP levels are listed in Table [VI.](#page-5-2) The geometries were obtained with several experimental methods [\[50,](#page-8-12) [51,](#page-8-19) [53,](#page-8-14) [54\]](#page-8-15), but here we show

only one set of experimental values obtained by electron diffraction in the gas phase [\[52\]](#page-8-13). The space symmetry of Ir $F_6$  is  $O_h$  at both the SF and SO levels, but the orbital is splitting at the SO level. The space symmetry of  $PtF_6$ is changed at the SF  $(D_{4h})$  and SO  $(O_h)$  levels, where the electronic structure becomes closed-shell. From the comparison between the SF-MRCI and SO-MRCI values, the SOC reduces the JT distortions of  $\text{Re}F_6$  and  $\text{Os}F_6$ . This trend is not reported at the DFT level [\[33\]](#page-8-16). From the orbital energy levels shown in Figure [1,](#page-2-0) the SO splittings of ReF<sub>6</sub>, OsF<sub>6</sub>, and PtF<sub>6</sub> are larger than the splitting due to JT distortion at the SF level. Our bond lengths at the SF-MRCI level are smaller than those at the SF- $CCSD(T)$  level shown in Table [IV,](#page-3-3) while is consistent with the conclusion obtained from Table [IV.](#page-3-3) The JT distortion reduced by the SO effect is also observed in the stabilization energy (cf. Table [VII\)](#page-5-1).

The electronic structure of  $\text{AuF}_6$  may be different from the others.  $AuF_6$  has ungerade-type orbitals with higher energies than  $t_{2g}$ -type orbitals, as shown in Figure S1 of the Supporting information.  $AuF_6$  should not be explained like the other third-row transition metal fluorides. More careful investigation of the reference orbitals and active spaces is required, and thus, we do not study  $AuF_6$  in the subsequent sections. The bond length of  $AuF_6$  shown in Table [V](#page-5-0) is different from the literature value, but we also found a local minimum of  $\text{AuF}_6$  that is closer to the literature value (1.913 Å  $(\times 2)$ ) and 1.898  $\AA(\times 4)$  at the SF-X2C/MRCI/dyall.v3z level), which is 6354 cm<sup>−</sup><sup>1</sup> less stable than the one shown in Table [V.](#page-5-0)

Similar schematic figures have been previously reported for group 10 hexafluorides [\[55,](#page-8-17) [56\]](#page-9-0) and group 9 monoanionic hexafluorides [\[57\]](#page-9-1) namely that  $d_4$ -type configurations predict octahedral symmetries, but they did not explicitly mention the triplet spin-state at the SF level. Furthermore, the influence of the SOC on the electron affinity (EA) discussed in ref [33](#page-8-16) can also be rationalized with reference to Figure [1.](#page-2-0) By adding the SO correction, these authors demonstrated that the EA of  $\text{OsF}_6$ becomes smaller than that of  $IrF_6$ . From Figure [1,](#page-2-0) the added electron predominantly occupies the  $e_{5/2,q}$  spinor of  $\text{IrF}_6$  in the SO level, which is energetically higher compared to the one at the SF level, where it occupies the triply degenerate  $t_{2g}$  orbitals.

#### A. Excitation spectra

Tables [VIII](#page-6-0) and [IX](#page-6-1) list the vertical excitation energy at the SF and SO levels. We could not simply correlate the states calculated at the SO level with the states without the SOC (SF calculations) except for the  $\text{ReF}_6$ case, where  ${}^2E_g \rightarrow E_{1/2,q}, E_{3/2,q}$  is observed. Even if the degeneracy of states is the same, the electronic structure can differ. This is exemplified by the ground state of IrF<sub>6</sub>, which is the <sup>4</sup> $A_{2g}$  and  $U_{3/2,g}$  states at the SF and SO levels, respectively.

The molecular symmetry of the double group can be

assigned as done in the point group at the NR level, using the direct product of the representation and Bethe's descending symmetry method [\[84,](#page-9-24) [85\]](#page-9-25). The total symmetry of  $\text{Re}F_6$  with one valence electron corresponds to the occupied spinor. In the bosonic case (even number of electrons), whether  $D_{4h}$  (OsF<sub>6</sub>) or  $O_h$  (PtF<sub>6</sub>), the doubly degenerate states are  $E_q$ , and the closed-shell states are  $A_1$ . Regarding the non-degenerate states with an openshell configuration for  $\text{OsF}_6$ , according to the direct product of the representation of orbitals in the double group (e.g., refs [24,](#page-8-1) [86\)](#page-9-26)

<span id="page-4-0"></span>
$$
e_{1/2,g} \otimes e_{3/2,g} = b_{1g} \oplus b_{2g} \oplus e_g
$$
  
\n
$$
e_{3/2,g} \otimes e_{3/2,g} = a_{1g} \oplus a_{2g} \oplus e_g
$$
  
\n(9)

 $b_{1g}$  and  $b_{2g}$  corresponds to  $d_{x^2-y^2}$  and " $d_{xy}$ " type basis sets, respectively. In addition, the Mulliken population analysis [\[87\]](#page-9-27) at the one-component basis set level using the DIRAC code shows that " $d_{xy}$ ",  $d_{yz}$ , and  $d_{zx}$  orbitals contribute to  $e_{1/2}$ ,  $e_{3/2}$ , and  $e'_{3/2}$  spinors.  $a_{1g}$  does not contribute to an open-shell state. Based on these conditions, the states with  $e_{1/2}(1)e_{3/2}(1)$  and  $e_{3/2}(1)e_{3/2}(1)$ type configuration correspond to  $B_{1q}$  and  $A_{2q}$ , respectively. To distinguish the triply degenerate states of  $O_h$ ,  $T_1$ , and  $T_2$ , we employed Bethe's method of descending symmetry [\[84,](#page-9-24) [85\]](#page-9-25). By going down to a lower symmetry,  $D_{4h}$ , the triple degeneracy splits as follows [\[86\]](#page-9-26):

$$
T_{1g} \to A_{2g} \oplus E_g
$$
  
\n
$$
T_{2g} \to B_{2g} \oplus E_g
$$
\n(10)

In other words, the  $A_{2q}$  ( $B_{2q}$ ) states at the  $D_{4h}$  structure, which was calculated in slightly distorting the structure from  $O_h$  to  $D_{4h}$ , corresponds to the  $T_{1g}$   $(T_{2g})$  states. The  $A_{2g}$  and  $B_{2g}$  states in  $D_{4h}$  symmetry can be distinguished based on the occupation number of the spinor, as shown in eq [9.](#page-4-0) For  $IrF_6$ , a Fermionic system (odd number of electrons) in  $O<sub>h</sub>$  symmetry, the quadruply degenerate states are always in  $U_{3/2}$  symmetry. The doubly degenerate states  $(E_{1/2}$  and  $E_{5/2})$  can be distinguished from a doublet-like electronic configuration (one open-shell orbital, the others are closed-shell) in the CI wavefunction. The electronic configuration  $[u_{3/2}(4), e_{1/2,6s}(1)]$ contributes to the  $E_{1/2}$  state, where Ir's 6s orbital contributes to  $e_{1/2,6s}$ . In the  $E_{5/2}$  state, a Kramers pair (two spinors) of four spinors in  $u_{3/2}$  are occupied, and another electron occupies the  $e_{5/2}$  spinor. The assignment at the SF level, which has been extensively investigated and thus will not be described here, was carried out based on the orbital occupation number under the  $C_{2v}$  symmetry using Bethe's method and the phase of the Slater determinants of the CI wavefunctions.

Although the experimental UV/vis spectra were reported [\[88,](#page-9-28) [89\]](#page-9-29), vibrational calculations using more sophisticated electronic structure theory would be essential to compare with the experimental spectra. Further efforts are also needed to assign the experimental spectra. They are beyond the scope of the present study and will be pursued in future studies.

<span id="page-5-0"></span>TABLE V. Bond lengths in Å, state and symmetry for metal hexafluorides at the most stable states calculated at X2C/MRCI/dyall.v3z level

	SO-MRCI		SF-MRCI
			$\text{ReF}_6   D_{4h}$ 1.785 (×2), 1.814 (×4) $ ^2B_{2g}/D_{4h}$ 1.773 (×2), 1.820 (×4)
			$\text{OsF}_6 _{D_{4h}}$ 1.819 (×2), 1.790 (×4) $ {}^3B_{2g}/D_{4h}$ 1.828 (×2), 1.785 (×4)
IrF <sub>6</sub> $O_h$ 1.799		$4A_{2q}/O_h$ 1.798	
	$PtF_6   O_h 1.806$		$\left  \frac{3B_{2g}}{D_{4h}} \right $ 1.774 (×2), 1.814 (×4)
AuF <sub>6</sub> $O_h$ 1.839			$\left  \frac{2B_{2g}}{D_{4h}} \right $ 1.849 (×2), 1.796 (×4)

TABLE VI. Summary of the most sophisticated literature values for bond lengths  $(\hat{A})$ , state, and symmetry for metal hexafluorides and the experimental values in the gas phase

<span id="page-5-2"></span>

SO-ZORA-BLYP [33]	$SF-CCSD(T)$ [33]	$\exp. \; [52]$
$\overline{\text{ReF}_6 \mid D_{4h} }$ 1.899 (×2), 1.878 (×4)	$\left[{}^{2}B_{2g}/D_{4h} \right]$ 1.802 (×2), 1.844 (×4) 1.829(2)	
	$OsF_6 _{D_{4h}}$ 1.877 (×2), 1.892 (×4) $ {}^3A_{1g}/D_{4h}$ 1.856 (×2), 1.816 (×4) 1.828(2)	
$IrF_6   O_h 1.894$	$^{4}A_{1q}/O_h$ 1.832	1.839(2)
	$\Pr{F_6   D_{4h} 1.915 (\times 2), 1.909 (\times 4)^a  ^3 A_{1g}/D_{4h} 1.823 (\times 2), 1.856 (\times 4)   1.852(2)}$	
AuF <sub>6</sub> $O_h$ 1.936	$\left  \frac{2B_{2q}}{D_{4h}} \right $ 1.897 (×2), 1.878 (×4)	

<sup>a</sup> ref [56](#page-9-0) reports  $O_h$  symmetry with 1.867 Å using the B3LYP functional based on the Dirac-Coulomb-Gaunt Hamiltonian.

TABLE VII. JT distortion energy  $(E_{\text{JT}}$  in cm<sup>-1</sup>), which is the difference between the energy obtained at the optimized geometry shown in Table [V](#page-5-0) and optimized bond length (r in Å) in  $O_h$  symmetry

<span id="page-5-1"></span>

	$E_{\rm JT}$	r		
		SF SO SF SO		
		ReF <sub>6</sub> 415 123 1.804 1.804		
		$OsF_6$ 364 71 1.799 1.800		
$PtF_6$ 369 - 1.801			$\sim$	

#### B. Population analysis

Finally, the SO effect of the  $MF_6$  is analyzed from the viewpoint of the atomic SO splitting. Figure [2](#page-7-15) shows a schematic diagram of the ligand field splitting of the atomic  $d$  orbitals at the SF and SO levels as well as the gross population of the  $\mathrm{PtF}_6$  . This figure is similar to Figures 19.4 and 19.5 of ref [58,](#page-9-2) but we added the gross population of  $PtF_6$  obtained by the projection analysis at the X2C/HF/dyall.v3z level of theory in this study. The projection analysis is a method to obtain the contribution from the atomic orbital to the molecular orbital (MO) based on the two- or four-component Hamiltonian that can work with the one-component basis sets, as demonstrated in refs. [\[82,](#page-9-22) [90–](#page-10-0)[93\]](#page-10-1).

From the gross population,  $5d_{5/2}$  ( $5d_{3/2}$ ) contributes to the  $u'_{3/2,g}(u_{3/2,g})$  spinors more significantly. This means that the SO splitting at the atomic orbital level remains even under the ligand field, and the SO splitting changes the degeneracy behavior of the MOs. According to the Mulliken population analysis using the one-component basis sets, the  $d_{xy}$ ,  $d_{yz}$ , and  $d_{xz}$  orbitals are distributed to the  $e_{5/2,g}$  and  $u_{3/2,g}$  orbitals, and  $d_{x^2}$ ,  $d_{y^2}$ , and  $d_{z^2}$ contribute to  $u'_{3/2,g}$  orbitals. Therefore,  $u'_{3/2,g}$  orbitals would have similar properties to those of  $e'_{g}$  orbitals, al-

though the assignments are different at the SF and SO levels. In fact, the total gross populations of d orbitals of  $u'_{3/2,g}$  and  $e'_{g}$  have almost the same magnitude.

# V. CONCLUSION

The geometry and excitation spectra of third-row transition metal hexafluorides are discussed based on the SFand SO-X2C MRCI theory. The electronic states involved in the excitation spectra are assigned based on the double group. Including the SOC variationally in the X2C Hamiltonian allows for analysis of SO splitting at the MO level. Our schematic MO diagram can effectively demonstrate how the SOC influences the structures of the metal hexafluorides. From the comparison between the SF and SO results, the SOC dramatically changes a well-known picture of the ligand field without the SOC: triply degenerate  $t_{2g}$  orbitals are splitting to one  $e_{5/2,g}$  and doubly degenerate  $u_{3/2,g}$  orbitals. The SO effect should not be obtained as a correction using a single-reference method such as DFT in molecules with multi-reference characters. The SO splitting destroys the degeneracy of the MOs at the SF level and quenches the JT distortion. The excited states at the SO level cannot simply correlate to those at the SF level due to the strong SOC. A strong SO effect is also observed in the population analysis.

The observation in this study should be applicable to photochemistry [\[94\]](#page-10-2) and spin chemistry [\[95\]](#page-10-3) of heavy element systems. Further expansion of this study to the vibrational properties, including the dynamic JT effect, is also interesting. In the context of theoretical chemistry, these kinds of heavy metal complexes with high symmetry would be an attractive target for relativistic correlated theory aimed at strong correlation, such as density matrix renormalization group (DMRG) theory

<span id="page-6-0"></span>TABLE VIII. Symmetry representations (sym), degeneracies (deg) for the energy levels, leading CI configurations, calculated vertical excitation energies in  $cm^{-1}$ , for  $ReF_6$  and  $OsF_6$  at the geometries shown in Table [V.](#page-5-0) The orbital labels are shown in Figure [1.](#page-2-0) The integer in the parentheses is the occupation number. For example, e.g.,  $e_{3/2,g}(2)$  refers to the  $e_{3/2,g}$  orbitals occupied by two electrons

	$\overline{\text{SF}}$					$\overline{SO}$		
	sym		deg configuration energy		sym	$\deg$	configuration	energy
$\overline{\rm Re} \rm F_6$	$\overline{^2B}_{2g}$	$\overline{2}$	$b_{2g}(1)$	$\boldsymbol{0}$	$E_{3/2,g}$	2	$e_{3/2,g}(1)$	$\theta$
	${}^2E_g$	4	$e_g(1)$	1312	$\mathcal{E}_{1/2,g}$	$\overline{2}$	$e_{1/2,g}(1)$	566
					$E_{3/2,\underline{g}}$	$\overline{2}$	$e'_{3/2,g}(1)$	5304
$\overline{\mathrm{Os}}\mathrm{F}_{6}$	$\sqrt[3]{B_{2g}}$	3	$e_q(2)$	$\theta$	$A_{1g}$	1	$e_{1/2,q}(2), e_{3/2,q}(2)$	$\overline{0}$
	${}^3E_g$	6	$e_g(1)b_{2g}(1)$	1100	E <sub>g</sub>	$\overline{2}$	$e_{1/2,g}(1)e_{3/2,g}(1)$	241
	$B_{2g}$	1	$e_q(2)$	8490	$B_{2g}$	1	$e_{1/2,g}(1)e_{3/2,g}(1)$	444
	$A_{1g}$	1	$e_q(2)$	8677	$B_{2g}$	1	$e_{1/2,g}(1)e_{3/2,g}(1)$	587
	${}^1E_g$	$\overline{2}$	$e_g(1)b_{2g}(1)$	9643	E <sub>g</sub>	$\boldsymbol{2}$	$e_{1/2,g}(1)e'_{3/2,g}(1)$	4944
	$A_{1g}$	1	$e_g(2), b_{2g}(2)$	9946	$A_{2g}$	1	$e_{3/2,g}(1)e'_{3/2,g}(1)$	5290
	$A_{1g}$	1	$e_g(2), b_{2g}(2)$	20873	$A_{1g}$	1	$e_{1/2,q}(2), e_{3/2,q}(2)$	5666
							$e'_{3/2,g}(2)$	
					$B_{2a}$	1	$e_{1/2,g}(1)e'_{3/2,g}(1)$	11548
					$B_{2q}$	1	$e_{1/2,g}(1)e'_{3/2,g}(1)$	11860
					$E_q$	2	$e_{3/2,g}(1)e_{3/2,g}'(1)$	12278
					$A_{2g}$	1	$e_{3/2,g}(1)e_{3/2,g}'(1)$	12676
					$A_{1g}$	1	$e'_{3/2,g}(2)$	24299

<span id="page-6-1"></span>TABLE IX. Symmetry representations (sym), degeneracies (deg) for the energy levels, leading CI configurations, calculated vertical excitation energies in  $cm^{-1}$ , for ReF<sub>6</sub> and OsF<sub>6</sub> at the geometries shown in Table [V.](#page-5-0) The orbital labels are shown in Figure [1.](#page-2-0) The integer in the parentheses is the occupation number. For example, e.g.,  $u_{3/2,g}(3)$  refers to the  $u_{3/2,g}$  orbitals occupied by three electrons



<sup>a</sup> The excitation from the GAS1 to GAS2 spaces also contributes.

 $<sup>b</sup>$  The excitation from the GAS2 to GAS3 spaces also contributes.</sup>

[\[96,](#page-10-4) [97\]](#page-10-5), driven similarity renormalization group (DSRG) theory [\[98\]](#page-10-6), and DMRG-tailored coupled cluster theory [\[99\]](#page-10-7). Feature calculations using these theories may provide results closer to the experimental ones. This study also provides fundamental knowledge about the chemistry of heavy metal complexes. The results would serve as useful examples for a lecture on inorganic chemistry.

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<span id="page-7-15"></span>FIG. 2. Schematic diagram of the ligand field splitting of the atomic d orbital at the SF and SO levels. The gross population of the Pt  $5d$  orbitals in PtF<sub>6</sub> obtained by the projection analysis are also shown.

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• MF6 suppl.pdf: Details of the optimization and the schematic orbital energy diagrams, including the AuF<sup>6</sup> molecule

# VI. SUPPLEMENTAL MATERIALS

The following Supporting Information is available free of charge.

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