Abstract

Water transport through minuscule pores is widespread in the natural world and holds significant implications in various technological applications [1, 2, 3, 4]. Radha et al. [5] has observed a significant increase in the water flow within graphene-based capillaries that are only a few nanometers or Angstrom-sized thick. By applying the Hagen-Poiseuille theory with confined water properties under continuum modelling, along with molecular dynamic simulations, Neek-Amal et al. [6] modelled these capillaries with rigid wall channels and attributed this enhancement to the high density and viscosity of water inside these nano capillaries. As Graphene sheets are flexible [7], we represent these graphene-based nanochannels with a deformable channel-wall model by using the small displacement structural mechanics and perturbation theory presented by Gervais et al. [8], and Christov et al. [9], respectively. We assume the lubrication assumption in the shallow nanochannels, and using the microstructure of confined water along with slip at the capillary boundaries and disjoining pressure [6], we derive the model for deformable nanochannels. The newly derived model also facilitate the flow dynamics of Newtonian fluids under different conditions as its limiting cases, which has been previously reported in the literature [8, 9, 6, 10, 11, 12]. Using the model, we study the effect of flexibility of graphene sheet on the flow rate. We also investigate how the applied pulsating pressure influences the behavior of the water flow rate within these flexible nano capillaries, as applying pulsating pressure fields or vibrations is a classical method for enhancing flow-rate of complex fluids through porous mediums such as channels and tube capillaries [13, 14, 15]. We compare the prediction of flow rate from both including the flexibility of the channel wall, and the application of pulsating pressure with the experimental observations by Radha et al. [5] and predictions from the molecular dynamic simulation by Neek-Amal et al. [6] which were well fitted by their rigid-wall model. We find that both the flexibility of the graphene sheet and the pulsating pressure fields to these flexible channels intensify the rapid flow rate through nano/Angstrom-size graphene capillaries.

Keywords: Flexible nanochannels, nano/Angstrom-size capillaries, pulsating pressure flow, high-speed flow

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Under Review
Introduction

Water transport through nanocapillaries, such as in biological systems and nanoscale materials, is ubiquitous in the natural world and has many applications in technology [16, 17, 18, 19]. In biological systems, capillary action in nano capillaries enables efficient water transport in plants, allowing them to absorb water from the soil and distribute it throughout their tissues [20, 21]. It also facilitates the movement of water in the vascular systems of humans and animals, such as in blood vessels and the microcapillaries of the human body [22]. From a technological standpoint, flexible nanocapillary flow has been harnessed for various applications. Some notable examples include: flexible nanochannels are utilized in lab-on-a-chip devices for precise manipulation and analysis of small volumes of fluids [23, 24]. These flexible nanochannels enable miniaturized systems for tasks such as chemical and biological sensing, DNA sequencing, and drug delivery [25, 26, 27]. Flexible graphene nanochannels are employed in water purification, desalination, filtration and separation of biomolecules processes due to their ability to selectively allow certain molecules or particles to pass through while blocking others [28, 29]. Due to the high surface area and confined flow of water, the flexible nanochannels are incorporated into heat exchange systems for efficient cooling [30, 31, 32, 33]. Inkjet printers use pressure pulses to eject the jet through nano-micro channels to control the flow of ink onto paper or other surfaces. The precise manipulation of tiny droplets enables high-resolution printing [34, 35, 36].

Radha et al. [5] has observed fast water flow within graphene-based nanocapillaries. Neek-Amal et al. [6] used a combination of the Hagen-Poiseuille theory, with confined water properties under continuum modeling, and the molecular dynamic simulations to study the capillaries. They represented the capillaries as rigid wall channels and presented a model for the flow rate. They attributed the fast water flow to the high density and viscosity of water molecules confined within these extremely small nano capillaries. Although the Graphene sheets are flexible [7], the flexibility of a shallow channel plays a crucial role in influencing both the effective pressure drop across the channel and the resulting flow profile [8, 9]. This is primarily because the flow rate is highly sensitive to the cross-section length scale, exhibiting a fourth power dependence [9]. Consequently, even minor deformations in the channel’s geometry can lead to significant alterations in the pressure drop and flow behavior within the system.

Figure 1: Schematic diagram of the upper-half part of the shallow channel of length $L$ and the cross-section of width $W$, and height $H$. 

Radha et al. [5] has observed fast water flow within graphene-based nanocapillaries. Neek-Amal et al. [6] used a combination of the Hagen-Poiseuille theory, with confined water properties under continuum modeling, and the molecular dynamic simulations to study the capillaries. They represented the capillaries as rigid wall channels and presented a model for the flow rate. They attributed the fast water flow to the high density and viscosity of water molecules confined within these extremely small nano capillaries. Although the Graphene sheets are flexible [7], the flexibility of a shallow channel plays a crucial role in influencing both the effective pressure drop across the channel and the resulting flow profile [8, 9]. This is primarily because the flow rate is highly sensitive to the cross-section length scale, exhibiting a fourth power dependence [9]. Consequently, even minor deformations in the channel’s geometry can lead to significant alterations in the pressure drop and flow behavior within the system.
Gervais et al. [8] provided a satisfactory model of the deformation-induced change in the flow rate by empirically connecting a Hookean elastic response with the lubrication approximation for Stokes flow. Nonetheless, their model involves a fitting parameter that needs to be determined through experimentation for every channel shape. Christov et al. [9] relate the fitting parameter in Gervais et al. [8] using a perturbation technique for the flow. Therefore, using the small displacement structural mechanics and perturbation theory presented by Gervais et al. [8], and Christov et al. [9], respectively, in this paper we represent these graphene-based nanochannels with a deformable channel-wall model.

Under the lubrication assumption in the shallow nanochannels (specifically, the ratio of the channel’s height to its width and the ratio of the channel’s height to its length are both assumed small), and using the microstructure of confined water along with slip at the capillary boundaries and disjoining pressure [6], we study the effect of flexibility of graphene sheet to the flow rate. Additionally, we examine the influence of applied pulsating pressure on the water flow rate within these flexible nano capillaries. The application of pulsating pressure fields or vibrations is a well-known technique for enhancing the flow rate of complex fluids through porous mediums, including channels and tube capillaries [13, 14, 34, 35, 36], however, for a Newtonian fluid (such as water), there is no effect of the pulsating pressure on the flow-rate in a rigid-wall channel/tube [37]. We conduct a comparison between the predicted flow rates considering both the flexibility of the channel wall and the application of pulsating pressure with the experimental findings reported by Radha et al. [5], as well as from the molecular dynamic simulations by Neek-Amal et al. [6] using their well-fitted rigid-wall model.

We consider a shallow rectangular channel with a length $L$, width $W$, and height $H$, satisfying the condition $H \ll W \ll L$ as shown in figure 1. The upper wall of the channel is a flexible graphene sheet sealed at the edges of the vertical channel wall which can undergo deformation. The time-dependent flow $Q(t)$ because of the applied pulsating pressure field occurs along the $x$-direction. As a result of the normal stresses exerted by the flow on the walls, the soft top wall of the channel deforms in the positive $z$-direction, away from the $x−y$ plane. This deformation defines the steady shape of the channel’s top wall as $z = H(x, y) = H_0 + \delta(x, y, t)$, where $\delta(x, y, t)$ is the deformation along the vertical direction as depicted in figure 1. A pulsatile pressure field $p(t)$ is applied at the reservoir at $x = 0$ and the exit pressure is assumed to be zero for reference purposes. Currently, we do not assume any particular magnitude for the displacement, but we anticipate that as the pressure is pulsatile, therefore $\delta$ can be greater and less than zero, and the magnitude $|\delta| \ll W$ in our problem.

2 Governing equations

2.1 Cauchy equations

The Cauchy’s equation and the continuity equation for an incompressible fluid are given by

$$\rho \left( \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right) = -\nabla p - \rho \mathbf{g} + \nabla \cdot \mathbf{\tau},$$

$$\nabla \cdot \mathbf{v} = 0,$$  

(1a,b)
where \( \mathbf{v} = [u \ v \ w] \) is the fluid velocity, \( p \) is the fluid pressure, \( \rho \) is the fluid density, \( \mathbf{g} \) is the gravitational body force, and \( \tau \) is the total deviatoric stress tensor.

### 2.1.1 Boundary conditions

Boundary conditions play an important role in determining the solution. We assume that the fluid cannot penetrate the channel wall, therefore on the boundary \( \Gamma \) it gives

\[
\mathbf{v} \cdot \mathbf{n}_{\text{wall}} = 0,
\]

where \( \mathbf{n}_{\text{wall}} \) is the unit outward normal vector on the wall. Generally, the no-slip boundary condition at the fluid-solid interface is a fundamental notion in fluid mechanics. However, fluid flow at the nano and Angstrom scale require a certain degree of tangential velocity (Navier slip) in order to match experimental observations \cite{38, 39, 40, 41}. This leads to

\[
\theta \mathbf{v} \cdot \mathbf{m}_{\text{wall}} + (1 - \theta) \tau \mathbf{n}_{\text{wall}} \cdot \mathbf{m}_{\text{wall}} = 0,
\]

where \( \mathbf{m}_{\text{wall}} \) is the tangential unit vector along the channel wall. Also, the arbitrary parameter \( \theta \) meets \( 0 \leq \theta \leq 1 \). Here, \( \theta = 0 \) and \( 1 \) corresponds to pure-slip and no-slip boundary conditions. The symmetry boundary condition at the centreline of the channel \( z = 0 \) demands the velocity normal to the centreline and the Cauchy traction vector \( \mathbf{t} \) tangential to the centreline is zero. These two conditions can be expressed as

\[
\mathbf{v} \cdot \mathbf{n}_{\text{centreline}} = 0,
\]

and

\[
\mathbf{t} \cdot \mathbf{m}_{\text{centreline}} = 0,
\]

respectively, where \( \mathbf{n}_{\text{centreline}} \) and \( \mathbf{m}_{\text{centreline}} \) are the unit normal and unit tangent vector to the symmetry boundary, respectively. The traction on the boundary, which is equivalent to a Neumann boundary condition, is expressed as

\[
\mathbf{t} = (-p \mathbf{I} + \tau) \mathbf{n}_{\text{centreline}}.
\]

### 3 The model: Involving microstructure of confined water

#### 3.1 Structural mechanics: small displacement mechanics

Gervais et al. \cite{8} performed the scaling analysis and showed that if the top wall is thick and the deformations are shallow, then the internal strains along vertical (\( \delta/W \) along \( z \) direction) and lateral (\( \Delta W/H \)) directions are proportional to \( p/E \), where \( p \) is the pressure and \( E \) is the elastic modulus. For \( H/W \ll 1 \), the strains could be rearranged to \( \delta/H = c p W/E H \), where \( \delta \) is the change in height due to shallow deformations and \( c \) is an unknown constant. Therefore Gervais et al. \cite{8} approximated, the width-averaged height of the channel along the length as

\[
H(x) = H_o \left( 1 + \alpha \frac{p(x) W}{E H_o} \right),
\]

where \( p(x) \) is a function of \( x \).
where $p(x)$ is the pressure at any longitudinal direction $x$ and $0 < \alpha < 2/3$. Young’s modulus of a graphene membrane is $E = 1$ TPa \cite{7, 12}. $H_0$ is the initial height of the channel when $\delta = 0$. However, the $\alpha$ is a fitting parameter that varies with the different geometry of the channel and needs to calculate explicitly from the experiments. To overcome this issue, Christov et al. \cite{9} performed the perturbation analysis using the isotropic quasi-static plate bending and the Stokes equations and figured out that for rectangular cross-section $\alpha = (1/60)(W/T)^3(1 - \nu^2)$, where $T$ is the thickness of the upper horizontal wall and $\mu$ is the Poisson’s ratio of the material (for incompressible material $\nu = 0.5$ \cite{13}).

### 3.2 2D planar model

We consider fully developed, time-periodic, incompressible laminar flow in a rectangular channel of height $H$ and width $W$ as shown in the schematic diagram (figure 1). The channel is assumed to be sufficiently long and wide in comparison to the height (i.e. $H/W \ll 1$, and $H/L \ll 1$) to use a two-dimensional planar model \cite{9, 44, 45}. The external forcing vibrations are applied using a pulsatory pressure gradient in the water reservoirs similar to the pistons placed at infinity from the channel wall \cite{37}. We also exclude any hydrodynamic instability caused in the transience flow field due to pulsatory pressure. We further assume a very small expansion or contraction due to deformability in comparison to the height of the channel, $\delta/H \ll 1$, which is caused by the pressure difference between the fluid and the atmospheric conditions in the deformable channel.

We assume the cartesian velocity components $u$ and $w$ along longitudinal and vertical directions $x$ and $z$, respectively. The $z$ coordinate is measured from the channel’s mid-plane. Therefore using the lubrication assumptions in the shallow cross-section of the channel, we retain the leading order terms as described in \cite{9}. Using the impermeable solid-wall boundary condition, we get $w(z = -H(x)/2) = w(z = H(x)/2) = 0$. In the leading order terms, using the impermeable solid-wall boundary condition, the normal velocity vanishes everywhere, i.e.

$$w(z, t) = 0.$$  \hspace{1cm} (8)

Also, we neglect gravitational forces over capillary forces. Under these assumptions for $H/W \ll 1$, and $H/L \ll 1$, the Cauchy’s equation \cite{1} can be written as

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{\eta}{\rho} \frac{\partial^2 u}{\partial z^2},$$  \hspace{1cm} (9)

$$0 = -\frac{\partial p}{\partial z},$$

where $\eta$ and $\rho$ are the viscosity and density of confined water, respectively. Using the Navier slip, the tangential solid-wall boundary condition gives

$$u(z = -H/2, t) = u(z = H/2, t) = \lambda \left| \frac{\partial u}{\partial z} \right|,$$  \hspace{1cm} (10)

where $\lambda = \eta(1 - \theta)/\theta \geq 0$, is the slip length \cite{16}.
3.3 Fourier Representation

In the standard lubrication assumption, we assume the pressure is only dependent on the longitudinal direction. We consider the width-averaged pressure gradient to be

$$-\frac{\partial p}{\partial x} = -\frac{dp}{dx} = p_o + \sum_{n=1}^{\infty} \left( p_{s,n} \sin(\omega nt) + p_{c,n} \cos(\omega nt) \right),$$

where $p_o \geq 0$ is the steady pressure gradient. Also, $p_{s,n}$, and $p_{c,n}$ represent the sine and cosine amplitudes of the pressure gradient of $n^{th}$ harmonics of the oscillatory pressure field. In deformable channels under small displacement approximations, all coefficients $p_o$, $p_{s,n}$, and $p_{c,n}$ might weakly depend on ‘$x$’. In compact form, the above function can be written as

$$-\frac{\partial p}{\partial x} = -\frac{dp}{dx} = p_o + \sum_{n=1}^{\infty} p_n \exp(i\omega nt),$$

where $p_n = p_{c,n} - ip_{s,n}$, and only the real part of the pressure gradient represent the solution. Similarly the velocity can be represented as

$$u = u_o + \sum_{n=1}^{\infty} \left( u_{s,n} \sin(\omega nt) + u_{c,n} \cos(\omega nt) \right),$$

$$= u_o + \sum_{n=1}^{\infty} u_n \exp(i\omega nt),$$

where $u_o$, and $u_n$ are the width-averaged velocities. Also, the $u_n = u_{c,n} - iu_{s,n}$ and only the real part of the flow profile represent the solution.

3.4 Analytical solution

Substituting equations (12), and (13) in the model equation (9) along $x$ direction, we get

$$\sum_{n=1}^{\infty} \left( i\omega n u_n - \frac{\eta d^2 u_n}{\rho d^2 z^2} - \frac{p_n}{\rho} \right) \exp(i\omega nt) - \frac{p_o}{\rho} - \frac{\eta d^2 u_o}{\rho d^2 z^2} = 0.$$

(14)

To satisfy equation (14), the steady and the time-dependent terms for each harmonic requires

$$\frac{p_o}{\rho} + \frac{\eta d^2 u_o}{\rho d^2 z^2} = 0,$$

(15)

and

$$i\omega n u_n - \frac{\eta d^2 u_n}{\rho d^2 z^2} - \frac{p_n}{\rho} = 0,$$

(16)

respectively. After solving equations (15), and (16), we get

$$u_o = -\frac{p_o}{2\eta} z^2 + c_1 z + c_2,$$

and

$$u_n = -\frac{ip_n}{\rho \omega n} + c_{3,n} \sinh \left( z \sqrt{\frac{i\rho \omega n}{\eta}} \right) + c_{4,n} \cosh \left( z \sqrt{\frac{i\rho \omega n}{\eta}} \right),$$

(17)
respectively. From equation [17], the velocity gradients are given by
\[
\frac{\partial u_0}{\partial z} = -\frac{p_o}{\eta} z + c_1, \quad \text{and} \quad \frac{\partial u_n}{\partial z} = c_{3,n} \sqrt{\frac{i \rho \omega n}{\eta}} \cosh \left( z \sqrt{\frac{i \rho \omega n}{\eta}} \right) + c_{4,n} \sqrt{\frac{i \rho \omega n}{\eta}} \sinh \left( z \sqrt{\frac{i \rho \omega n}{\eta}} \right),
\]
respectively. Using the symmetry boundary condition at \( z = 0 \) for all time \( t \), i.e. \( \left( \frac{\partial u}{\partial z} \right)_{z=0,t} = 0 \) in equation [18], we get
\[
c_1 = c_{3,n} = 0.
\]
Similarly, using the tangential solid-wall boundary condition with Navier slip for all time \( t \) from equation [10], we obtain
\[
-\frac{i p_n}{\rho \omega n} + c_{4,n} \cosh \left( \frac{H^2}{2} \sqrt{\frac{i \rho \omega n}{\eta}} \right) = \lambda \left| c_{4,n} \sqrt{\frac{i \rho \omega n}{\eta}} \sinh \left( \frac{H^2}{2} \sqrt{\frac{i \rho \omega n}{\eta}} \right) \right|,
\]
which yields
\[
c_2 = \frac{p_o}{2\eta} H^2 + \lambda \left| -\frac{p_o H}{\eta} \right|;
\]
and
\[
c_{4,n} = -\frac{i p_n}{\rho \omega n} \cosh \left( \frac{H^2}{2} \sqrt{\frac{i \rho \omega n}{\eta}} \right) - \frac{\lambda}{\text{sgn}(c_{4,n})} \left| \sqrt{\frac{i \rho \omega n}{\eta}} \sinh \left( \frac{H^2}{2} \sqrt{\frac{i \rho \omega n}{\eta}} \right) \right|,
\]
respectively. Here \( \text{sgn}(c_{4,n}) \) is the ‘sign function of argument \( (c_{4,n}) \)’. As we know for a steady part, the pressure is decreasing along the channel length, therefore we assumed \( p_o \geq 0 \). For a meaningful solution we require \( \text{sgn}(c_{4,n}) = 1 \). Hence substituting equations [17], [19], [21], and [22] in equation [13], the generalised velocity profile under pulsating field can be written as
\[
u = \frac{p_o}{2\eta} \left( \frac{H^2}{4} - z^2 \right) + \frac{p_o H}{2\eta} + \sum_{n=1}^{\infty} \frac{i p_n}{\rho \omega n} \left[ \cosh \left( z \sqrt{\frac{i \rho \omega n}{\eta}} \right) - \lambda \sqrt{\frac{i \rho \omega n}{\eta}} \sinh \left( \frac{H^2}{2} \sqrt{\frac{i \rho \omega n}{\eta}} \right) \right] \exp(i \omega n t).
\]
We define \( k \) as a non-dimensional frequency or kinetic Reynolds number as
\[
k = \frac{\omega H^2 \rho}{4\eta}.
\]
Equation (23) can be further written as,

\[
  u = \frac{p_o}{2\eta} \left( \frac{H^2}{4} - z^2 \right) + \lambda \frac{p_o H}{2\eta} - \sum_{n=1}^{\infty} \frac{H^2 p_n e^{i\omega nt}}{4\eta} \left[ \frac{\cosh \left( \frac{(2z/H)\sqrt{ikn}}{\cosh(\sqrt{ikn})} \right)}{\cosh(\sqrt{ikn}) - \frac{2\lambda}{H} \sqrt{ikn} \sinh(\sqrt{ikn})} - 1 \right].
\]

(25)

In equation (25), the real part of the pressure gradient \( p_n e^{i\omega nt} \) is the valid physical condition. Therefore the ‘Term A’ mentioned using underbrace need to be also real for valid physical flow profile. Using properties of confined water in nano and angstrom-scale capillaries, where \( H \) is \( O(10^{-9}) \) m, \( \rho \sim O(10^3) \) kg/m\(^3\), and \( \eta \sim O(10^{-1} - 10^{-3}) \) Pa s (see section (3.5) on the microstructure of confined water), the kinetic Reynolds number \( k \) is approximately \( O(10^{-12}) \omega \approx 0 \). Assuming \( \beta = \sqrt{ikn} \), therefore, even for very large values of applied \( \omega \), and \( n^{th} \) harmonic of applied pulsating pressure, the \( \beta \) tends to zero. Hence in equation (25), the ‘Term A’ can be written to

\[
  \lim_{\beta \to 0} \frac{1}{\beta^2} \left[ \frac{\cosh \left( \frac{(2z/H)\beta}{\cosh(\beta)} \right)}{\cosh(\beta) - \frac{2\lambda}{H} \beta \sinh(\beta)} - 1 \right] = \lim_{\beta \to 0} \frac{2 \sinh^2(z\beta/H) - 2 \sinh^2(\beta/2)}{\beta^2 \left( \cosh(\beta) - \frac{2\lambda}{H} \beta \sinh(\beta) \right)}
\]

(26)

\[
  \begin{align*}
  &+ \lim_{\beta \to 0} \frac{2\lambda}{H} \beta \sinh(\beta) \\
  &\quad \div \beta^2 \left( \cosh(\beta) - \frac{2\lambda}{H} \beta \sinh(\beta) \right),
\end{align*}
\]

Term B

Term C

where \( \beta = \sqrt{ikn} \). We find that irrespective of the sign of \( \beta \sinh(\beta) \), the limit of Term B will not be affected and is real. Also for a physically valid flow profile, Term C has to be real. The real part of the limit of Term C depends on the sign of the real part of \( \beta \sinh(\beta) \). Also with in the experimental range, the \( 0.5H\sqrt{\omega n/\eta} \approx O(10^{-6} \sqrt{\omega n}) \ll 10 \), which upon substitution we get the real part of \( \sqrt{ikn} \sinh(\sqrt{ikn}) = \beta \sinh(\beta) < 0 \). Hence for the real solution, equation (26) can be simplified to

\[
  2 \left( \frac{z^2}{H^2} - \frac{1}{4} \right) - \lim_{\beta \to 0} \frac{2\lambda}{H} \beta \sinh(\beta) \div \beta^2 \left( \cosh(\beta) + \frac{2\lambda}{H} \beta \sinh(\beta) \right) = 2 \left( \frac{z^2}{H^2} - \frac{1}{4} \right) - \frac{2\lambda}{H}.
\]

(27)

Substituting equation (27) back in equation (25), we get

\[
  u = \frac{p_o}{2\eta} \left( \frac{H^2}{4} - z^2 \right) + \lambda \frac{p_o H}{2\eta} + \sum_{n=1}^{\infty} \frac{p_n e^{i\omega nt}}{2\eta} \left[ \left( \frac{H^2}{4} - z^2 \right) + \lambda H \right]
\]

(28)

\[
  = \left[ \frac{p_o}{2\eta} + \sum_{n=1}^{\infty} \frac{p_n}{2\eta} e^{i\omega nt} \right] \left[ \left( \frac{H^2}{4} - z^2 \right) + \lambda H \right].
\]
The volume flow rate in a deformable nanochannel at any time \( t \) is given by

\[
Q(t) = 2W \int_0^{H(x)/2} u \, dz = \frac{W}{12\eta} \left[ p_o + \sum_{n=1}^{\infty} p_n e^{(i\omega nt)} \right] \left[ H(x)^3 + 6\lambda H(x)^2 \right].
\]  

(29)

As we know from equation (7), that

\[
H(x) = H_o \left( 1 + \alpha \frac{p(x)W}{EH_o} \right).
\]  

(30)

Using equation (12), and substituting equation (30) in equation (29), we get

\[
Q(t) = -WH_o^3 \left[ \frac{dp}{dx} \right] \left[ \left( 1 + \alpha \frac{p(x)W}{EH_o} \right)^3 + \frac{6\lambda}{H_o} \left( 1 + \alpha \frac{p(x)W}{EH_o} \right)^2 \right].
\]  

(31)

We assume \( p(x = 0) \) to be the relative oscillatory inlet pressure with respect to the pressure at the outlet of the channel, i.e., \( p(x = L) = 0 \). In nano and Angstrom scale channels, the viscosity \( \eta(H) \) and density \( \rho(H) \) is a function of the height of the channel [6]. Under small displacements of the wall, we assume the viscosity and density to be \( \eta(H) \sim \eta(H_o) \), and \( \rho(H) \sim \rho(H_o) \), respectively.

We integrate equation (31) along the channel length \( L \) for a given applied oscillatory pressure field at the inlet of the channel, which yields

\[
\int_x^L Q(t) \, dx = -\frac{W}{12\eta} \frac{H_o^3}{E} \left[ \frac{dp}{dx} \right] \left[ \left( 1 + \alpha \frac{p(x)W}{EH_o} \right)^3 + \frac{6\lambda}{H_o} \left( 1 + \alpha \frac{p(x)W}{EH_o} \right)^2 \right] dp,
\]  

(32)

\[
(L - x)Q(t) = \frac{H_o^4 E}{12\eta \alpha} \left( \frac{1}{4} \left[ \left( 1 + \alpha \frac{p(x)W}{EH_o} \right)^4 - 1 \right] + \frac{2\lambda}{H_o} \left[ \left( 1 + \alpha \frac{p(x)W}{EH_o} \right)^3 - 1 \right] \right).
\]  

(33)

As the flow rate \( Q(t) \) is not a function of longitudinal directional. Therefore substituting \( x = 0 \) in equation (33) gives the volume flow rate in the channel as

\[
Q(t) = \frac{H_o^4 E}{48\eta L \alpha} \left( 1 + \alpha \frac{\Delta p W}{EH_o} \right)^4 - 1 + \frac{8\lambda}{H_o} \left[ \left( 1 + \alpha \frac{\Delta p W}{EH_o} \right)^3 - 1 \right],
\]  

(34)

where \( p(x = 0) = \Delta p \) contains both the the steady and the oscillatory pressure field at \( x = 0 \). Therefore the mass flow rate can be written as

\[
\dot{m} = \rho Q(t).
\]  

(35)

In order to calculate the mean volumetric flow in a time-periodic quasi-steady state over a complete oscillatory pressure field cycle in the channel, we can integrate it as

\[
\bar{Q} = \frac{\omega}{2\pi} \int_0^{2\pi/\omega} Q(t) \, dt.
\]  

(36)

Similarly the mean mass flow rate across one channel of dimensions \( L \times H \times W \) is given by

\[
\bar{\dot{m}} = \rho \bar{Q}.
\]  

(37)
Radha et al. [5], and Neek-Amal et al. [6], observed that the density $\rho$ and viscosity $\eta$ of the confined water inside a nanochannel is significantly large (for $H_o \lesssim 12\,\text{Å}$) and varies with the size of the nanocapillary. In hydrophobic nanocapillaries, the presence of excluded volume near the confining walls reduces the available volume for a given mass of water, leading to increased density. Additionally, the confining walls induce a structuring effect that enhances the viscosity of water [5, 47, 48, 6]. Therefore, for a given channel height, Neek-Amal et al. [6] assumed a uniform density in the channel under a continuum approach and proposed a model for density and viscosity of the confined water as a function of the height of the channel, which are

$$\rho(H_o) = \rho_o f(H_o),$$

and

$$\eta(H_o) = \eta_o g(H_o),$$

respectively, where $\rho_o \approx 1\,\text{g/cm}^3$ and $\eta_o \approx 0.89\,\text{mPa s}$ are the bulk density and viscosity of water at room temperature, respectively. Also, the functions $f(H_o)$, and $g(H_o)$ are given by

$$f(H_o) = 1 + ae^{-H_o/\delta_1}, \quad \text{and} \quad g(H_o) = 1 + be^{-H_o/\delta_2},$$

where $\delta_1$ and $\delta_2$ are constants.
respectively, where the values of $a = 10.9$, $b = 6.23 \times 10^4$, $\delta_1 = 2.2$ Å, and $\delta_2 = 1.19$ Å are calculated by fitting the functions to the molecular dynamic simulations as shown in figure 2 [6]. Also, these functions $f(H_o)$ and $g(H_o)$ approaches unity for $H_o \gg \delta_1, \delta_2$.

Figure 3: Variation of the entropic pressure $\Delta p_e$ (k bar) and capillary pressure $\Delta p_c$ (bar) (in the inset) with width-averaged channel height. The data is taken from [3, 5]. Reproduced/Adapted from [M. Neek-Amal, A. Lohrasebi, M. Mousaei, F. Shayeganfar, B. Radha, F. M. Peeters; Fast water flow through graphene nanocapillaries: A continuum model approach involving the microscopic structure of confined water. Appl. Phys. Lett. 20 August 2018; 113 (8): 083101. https://doi.org/10.1063/1.5037992; Figure 3], with the permission of AIP Publishing.

In nano and angstrom-scale capillaries, the hydrostatic pressure is likely to have little effect on the dynamics of flow which is many orders of magnitude smaller than the capillary pressure $\Delta p_c$, and disjoining pressure $\Delta p_d$. The capillary pressure is due to interfacial tension, which is given by $\Delta p_c = 2\sigma \cos(\phi)/H$, where $\sigma \approx 70$ mN/m is the surface tension and $\phi$ is the contact angle between water and surface (graphene/graphite). In different studies, the contact angle varies between 55° to 127° [49, 50, 6]. Further, the disjoining pressure is due to the enhanced ordering of water structure in nanocapillaries. It is given by $\Delta p_d = -\frac{1}{A} \left( \frac{\partial G}{\partial H} \right)_{T,V,A} = \Delta p_{vdW} + \Delta p_e$, which consists of van der Waals (vdW) pressure $\Delta p_{vdW} = \frac{A_H}{6\pi H^3}$ and entropic pressure $\Delta p_e = \frac{RT}{V_m} \ln(f(H_o))$ components, where $A_H$ is the Hamaker constant for water-graphite interaction.

The values for the parameters are taken from the experiments and the fitted confined water properties from [3, 6], which are $RT = 2494.2$ J/mol, which are $V_m = 18$ cm$^3$/mol, $W = 1300$ Å, $L = 10^4$ Å, $\lambda = 600$ Å, $\rho_o = 1$ g/cm$^3$, $\eta_o = 0.89$ mPa s, $A_H = 35zJ$, $N = 200 \times 30$, $\gamma = 2\sigma \cos(89.96^o) = 0.1$ mN/m, where $\sigma = 70$ mN/m is the surface tension of water. We also know the experimental upper capping wall thickness $T = 100$ nm (given in supplementary information in [5]). Using these experimental parameters, we show the variation of the entropic pressure $\Delta p_e$ (k bar) and capillary pressure $\Delta p_c$ (bar) (in the inset) with width-averaged channel height in

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4 Results and discussion

The mass flow-rate $\dot{m}$ without oscillatory pressure field (i.e., $p_n = 0$) in the rigid-wall $N$ nanochannels of dimensions $L \times H \times W$ using equations (7), (29), and (35) can be calculated as

$$\dot{m}_{\text{rigid}} = \rho N Q = \frac{\rho NW}{12\eta L} \Delta p \left[ H_o^3 + 6\lambda H_o^2 \right],$$

(41)

where $Q$ is the volume flow-rate in a single nanochannel. Also, here $\Delta p = p(x = 0)$ is the inlet pressure. Therefore using the microstructure properties of confined water, we can write

$$\dot{m}_{\text{rigid}} = \rho N Q = \frac{\rho_o NW}{12\eta_o L} \frac{f(H_o)}{g(H_o)} \Delta p \left[ H_o^3 + 6\lambda H_o^2 \right],$$

(42)

where $\Delta p = \Delta p_{vdW} + \Delta p_e + \Delta p_c = \frac{A_H}{6\pi H_o^3} + \frac{RT}{V_m} \ln(f(H_o)) + \frac{\gamma}{H}$ as presented by [6].

Using equations (34) and (35), the mass flow rate for the flexible wall $N$ graphene channels using the microstructure properties of confined water and under an oscillatory pressure field can be written as

$$\dot{m}_{\text{flexible}} = \frac{\rho_o NW}{12\eta_o L} \frac{f(H_o)}{g(H_o)} \Delta p \left[ H_o^3 \left( 1 + \frac{3}{2} \left( \alpha \frac{\Delta p W}{E H_o} \right) + \left( \alpha \frac{\Delta p W}{E H_o} \right)^2 + \frac{1}{4} \left( \alpha \frac{\Delta p W}{E H_o} \right)^3 \right) \right. \right.$$ 

$$\left. + \frac{6\lambda}{H_o} \left( 1 + \left( \alpha \frac{\Delta p W}{E H_o} \right) + \frac{1}{3} \left( \alpha \frac{\Delta p W}{E H_o} \right)^2 \right) \right],$$

(43)

where $\Delta p = p(x = 0)$ is the applied pressure at the inlet of the channel, which contain both the steady and oscillatory pressure field, i.e. $p_o \neq 0$, and $p_n \neq 0$. Using equation (43), we can easily identify the following limits:

I. The mass flow rate in the rigid wall $N$ nanochannels under pulsating pressure field, i.e. $\alpha = 0$: Using equation (43), the mass flow rate for the rigid channel under an oscillatory pressure field can be written as

$$\dot{m}_{\text{rigid}} = \frac{\rho_o NW}{12\eta_o L} \frac{f(H_o)}{g(H_o)} \Delta p \left[ H_o^3 \left( 1 + \frac{6\lambda}{H_o} \right) \right],$$

(44)

where $\Delta p = p(x = 0)$ is the applied pressure at the inlet of the channel, which contain both the steady and oscillatory pressure field, i.e. $p_o \neq 0$, and $p_n \neq 0$.

II. The mass flow rate in the flexible wall $N$ nanochannels without pulsating pressure field, without slip, and without confined water properties, i.e. $\lambda = 0$, $p_n = 0$, $f(H_o) = g(H_o) = 1$:

$$\dot{m}_{\text{flexible}} = \frac{\rho_o NW}{12\eta_o L} \Delta p H_o^3 \left[ 1 + \frac{3}{2} \left( \alpha \frac{\Delta p W}{E H_o} \right) + \left( \alpha \frac{\Delta p W}{E H_o} \right)^2 + \frac{1}{4} \left( \alpha \frac{\Delta p W}{E H_o} \right)^3 \right].$$

(45)
Equation (45) is the same as the analytical model expression under lubrication assumptions derived by Gervais et al. [8].

**III. The mass flow rate in the rigid wall $N$ nanochannels without pulsating pressure field i.e. $\alpha = 0, p_n = 0$: Under these limits, we obtain equation (42) as presented by Neek-Amal et al. [6].**

**IV. The mass flow rate in the rigid wall $N$ nanochannels without pulsating pressure field, without slip and without confined water properties, i.e. $\alpha = 0, \lambda = 0, p_n = 0, f(H_0) = g(H_0) = 1$: Under these limits, we obtain**

$$\dot{m}_{\text{rigid}} = \frac{\rho_o NW}{12\eta_o L} \Delta p H_o^3,$$

which is a classical result of Hagen-Poiseuille flow in channels [10, 11, 12].

Further using equations (43), and (44), we see that the difference between the mass flow rate from the flexible and rigid nanochannel can be written as

$$\dot{m}_{\text{flexible}} - \dot{m}_{\text{rigid}} = \frac{\rho_o NW f(H_0)}{12\eta_o L} g(H_0) \Delta p H_o^3 \left[ \left( \frac{3}{2} \left( \frac{\Delta p W}{E H_o} \right) + \left( \frac{\Delta p W}{E H_o} \right)^2 + \frac{1}{4} \left( \frac{\Delta p W}{E H_o} \right)^3 \right) + \frac{6\lambda}{H_o} \left( \frac{\Delta p W}{E H_o} + \frac{1}{3} \left( \frac{\Delta p W}{E H_o} \right)^2 \right) \right].$$

In equation (47), we observe that the additional non-linear terms $\left( \alpha \Delta p W/E H_o \right)$ because of flexibility increases the mass flow rate. Also, in the following section we will find that the mean mass flow rate in a complete pure oscillatory cycle of pressure field increases the mass flow rate from the channel, whereas in the case of a rigid channel, the increment mass flow rate due to half oscillatory cycle of pressure gets nullified from the other half cycle of pressure-field. In the following section, we will discuss the effect of flexibility and pulsating pressure field on the flow rate of these nanochannels and compare the flow rate observed in the experiments by Radha et al. [5], and the predictions from molecular simulations by Neek-Amal et al. [6] using the well-fitted rigid-wall model described by Neek-Amal et al. [6].

**4.1 Effect of thickness of the top capping graphene sheet wall on mass flow rate**

Using the experimental parameters, pressure [5], and the fitted confined water properties [6], we show the mass flow rate as a function of the height of the nano/Angstrom-sized channels in figure 4(a). In figure 4(b), we show the percentage change in flow rate when the flexibility of the graphene upper capping wall is considered. In Both figures 4(a,b), the black line shows the mass flow rate in the experimental graphene channels (where we choose $\alpha = 0$, i.e. $T = \infty$ because $\alpha \propto (W/T)^3$) to include the experimental and Molecular Dynamic simulation’s predictions which were described and shown well-fitted using the rigid channel model [5, 6].

The black arrow indicates the increasing values of upper capping wall thickness $T$, which varies from 35nm (green color) to 100nm (magenta color) in intervals of 5nm. We find that the
Figure 4: Figure (a) shows the mass flow rate as a function of the height of the nano/Angstrom-sized flexible channels on the log-log scale. In (b), we show the percentage change in flow rate when the flexibility of the graphene upper capping wall is considered on a semilog plot. The black and red arrows indicate the increasing and corresponding decreasing values of upper capping wall thickness $T$ and $\alpha$, respectively. In Both (a,b), we compare the mass flow rate in the experiments by Radha et al. [5] and from the Molecular Dynamic simulation’s by Neek-Amal et al. [6] using their well-fitted rigid-wall model (shown with the black line) to the flow rates when flexibility is considered.

The flow-rate prediction for the magenta line (when $T = 100$ nm) matches well with the experimental predictions for the rigid channel as shown by a black line ($T = \infty$) for all channel heights $H_o > 4$. This indicates that the pressure fields due to disjoining pressure and capillary interfacial tension were not enough to deform the upper capping of the graphene channel in experiments. Although for $H_o < 4$, we see the deviation between the magenta and black line, where upto 5-7% increment of flow rate is predicted as shown in figure 4(b). This deviation is within the errorbar of the experimental values [5]. This shows that the derived flexible channel wall model could be more accurate to model flexible graphene nanochannel in comparison to the rigid wall model by Neek-Amal et al. [6]. Assuming graphene as the incompressible material ($\nu = 0.5$), we calculated the corresponding increasing values of $\alpha$ with decreasing $T$ from the perturbation theory, where $\alpha = (1/60)(W/T)^3(1-\nu^2)$ as shown by the red arrow, which varies between $(0.027 \leq \alpha \leq 0.64)$.

We also observe, as the thickness of the upper capping wall decreases, the mass flow rate inside the channel increases. For $T = 35$ nm, the percentage change in flow rate due to the flexibility of the wall increased by more than 80% as shown by the green line. We noticed, for channel heights $H_o > 10$, the change in flow rate due to the flexibility of the wall is negligible. It is because the pressure field due to disjoining pressure and capillary interfacial tension decreases with channel height (as shown in figure 3) and becomes quite low enough to deform the upper wall of the channel.

By applying an additional constant pressure at the reservoir (i.e. at $x = 0$) which is ten times of disjoining pressure and capillary pressure at channel height $H_o = 1$, we show the mass flow.
rate prediction in figure 5(a,b). Here, the black and red arrows, shows the varying values of $\mathcal{T}$, and $\alpha$ where the varying interval remain the same as in 4(a,b).

In 5(a), we find that the flow rate increases with channel height unlike in figure 4(a). This is because we applied the additional pressure supply to all channel heights whereas in Radha et al. [5], the disjoining and interfacial pressure inside the channel were decreasing drastically with the channel height (shown in figure 3). We find that the flow rate in the rigid channel scales as $\dot{m}_{\text{rigid}} \sim \Delta p$, whereas for the flexible channels, for negligible wall-displacement perturbation, the flow rate scales as $\dot{m}_{\text{flexible}} \sim \Delta p$ for $\left(\alpha \Delta p W / E H_o\right) \ll 1$, and for large perturbation $\dot{m}_{\text{flexible}} \sim \Delta p^4$. We also find that, for a given thickness $\mathcal{T}$, the percentage change in flow rate in the smaller height of the channel is much larger than the larger height of the channels. For the larger height of the channel, where $\left(\alpha \Delta p W / E H_o\right) \ll 1$, the change in flow rate scales as $(\dot{m}_{\text{flexible}} - \dot{m}_{\text{rigid}})/\dot{m}_{\text{rigid}} = \Delta \dot{m}/\dot{m} \sim (\Delta p / H_o)$, whereas for the smaller height channel, the wall perturbation due to deformity is large and the change in flow rate scales as $\sim (\Delta p / H_o)^3$. Hence as the channel height decreases for the given reservoir pressure, $\Delta \dot{m}/\dot{m}$ increases $H_o^{-1}$ followed by $H_o^{-3}$ after a threshold with the channel height as shown in 5(b).

### 4.2 Effect of varying pressure on the mass flow rate

We take a channel height $H_o = 10$, and the confined water properties at the $H_o = 10$ in this section. We keep the other experimental parameters as it is and show the mass flow rate as a function of varying reservoir pressure in figure 6(a). In figure 6(b), we show the percentage

Figure 5: Figure (a) shows the mass flow rate as a function of the height of the nano/Angstrom-sized flexible channels when additional constant pressure $\Delta p = 10\Delta p_{H_o=1}$ is applied on a log-log plot. In (b), we show the percentage change in flow rate with respect to the rigid wall of the channel on a semilog plot. The black and red arrows indicate the increasing and corresponding decreasing values of upper capping wall thickness $\mathcal{T}$ and $\alpha$, respectively.
change in flow rate when the flexibility of the graphene upper capping wall is considered as a function of varying reservoir pressure.

In Both figures 6(a, b), the data shown from the green line to the purple line indicate \( \alpha \) is increasing (shown with red arrow) from 0 to 0.1, respectively, and corresponding wall thickness \( T \) is decreasing (shown with black arrow). For all data values, the nonlinear term due to flexibility is between \( 0 \leq \left( \alpha \Delta pW/EH_o \right) \leq 0.9 \), where the maximum value 0.9 occur for the purple data for \( \alpha = 0.1 \), and \( \Delta p/\Delta p_{H_0=1} = 175 \). We find that for the rigid channel, the \( \dot{m}_{\text{rigid}} \sim \Delta p \) as shown with green data in figure 6(a), whereas for the flexible channels (shown between red to purple colors), the flow rate scales as \( \dot{m}_{\text{flexible}} \sim \Delta p^2 \) for \( \left( \alpha \Delta pW/EH_o \right) \sim O(1) \), and \( \dot{m}_{\text{flexible}} \sim \Delta p^4 \) for \( \left( \alpha \Delta pW/EH_o \right) \sim O(1) \). We find that the non-linear increment of mass flow rate in figure 6(a) is due to \( \left( \alpha \Delta pW/EH_o \right) \sim O(10^{-1}) \).

On the other hand in 6(b), we find that the percentage change in mass flow rate for the rigid channel, and the flexible channels are \( \Delta \dot{m}_{\text{rigid}}/\dot{m}_{\text{rigid}} \sim \Delta p^0 \), \( \Delta \dot{m}_{\text{flexible}}/\dot{m}_{\text{rigid}} \sim \Delta p \) for \( \left( \alpha \Delta pW/EH_o \right) \ll 1 \) and \( \Delta \dot{m}_{\text{flexible}}/\dot{m}_{\text{rigid}} \sim \Delta p^3 \) for \( \left( \alpha \Delta pW/EH_o \right) \sim O(1) \), respectively. We also find that as the flexibility parameter \( \alpha \) increases for the given reservoir pressure, the mass flow increases in the channel. We notice for \( \alpha = 0.02 \) and \( \alpha = 0.1 \), at \( \Delta p/\Delta p_{H_0=1} = 175 \), the percentage change in mass flow rate \( (\Delta \dot{m}_{\text{flexible}} - \dot{m}_{\text{rigid}})/\dot{m}_{\text{rigid}} \) is 20% (shown in red circle), and 120% (shown in purple left-side triangle), respectively. This states that, due to the flexibility of graphene sheets, the channel deformations effect becomes significantly important and substantially increases the mass flow rate.
Figure 7: We show the mass flow rate in one oscillatory cycle for rigid ($\alpha = 0$) (a), and for the flexible channel ($\alpha = 0.1$) (b) as a function of non dimensionalise time. The red arrow indicates the increasing value of the pressure.

### 4.3 Effect of pulsating/oscillatory pressure in the reservoir on the mass flow rate

We take a channel height $H = 10$, and the confined water properties at the $H = 10$ in this section. We use $\alpha = 0$, for rigid channels, and $\alpha = 0.1$ for flexible channels. We give a pulsating reservoir pressure $\Delta p = p_1 L \sin(\omega t)$, where $p_1 L$ (amplitude of the applied oscillatory pressure field at reservoir, i.e. at $x = 0$) is varying between 0 to $p_1 L = 325\Delta p_{H_o=1}$ indicated by the data lines from black to pastel green. We keep the other experimental parameters as it is and show the mass flow rate in one oscillatory cycle for rigid ($\alpha = 0$), and flexible channel ($\alpha = 0.1$) as a function of non dimensionalise time in figure 7(a) and 7(b), respectively. Also, please note that we choose the $\Delta p$ has only one sinusoidal pressure field here to show the pulsation effect simply, but our derivation and model (i.e. equation (43)) for $\Delta p$ is not restricted to only one oscillatory field, in fact it can have any number of oscillatory pressure field with different amplitude and frequencies.

We find that for the first half cycle (when the $\sin(\omega t) \geq 0$), the flow rate increases with pressure in both the rigid and flexible channels. We find that the ratio of maximum flow rate (when $p_1 L = 325\Delta p_{H_o=1}$ and $\omega t = \pi/2$), is $\dot{m}_{\text{flexible}} / \dot{m}_{\text{rigid}} = 3.42$. This means that the pulsating/oscillatory pressure in the reservoir intensifies the mass flow rate in a flexible channel. We also noticed that, in the next half cycle (when the $\sin(\omega t) \leq 0$), the flow rate is in the opposite direction for both channels. In the case of a rigid wall channel, the amount of flow reversal is the same as the amount of flow in the first half cycle, this gives the net averaged flow over a complete cycle to be zero for all pure oscillatory pressure fields. On the other hand in the case of a flexible channel, the amount of flow reversal is negligibly small in the next half cycle (when the $\sin(\omega t) \leq 0$) as compared to the amount of flow in the first half cycle. This states that pulsating flow field shows a net positive averaged mass flow rate in a flexible channel.
4.4 Effect of pulsating pressure and flexibility of graphene sheet on the mass flow rate

We take a channel height $H_o = 10$, and the confined water properties at the $H_o = 10$ in this section. We use $\alpha = 0$, for rigid channels, and $\alpha = 0.1$ for flexible channels. We give a pulsating reservoir pressure $\Delta p = p_o L + p_1 L \sin(\omega t)$, where $(p_o L$ and $p_1 L$ are the amplitudes of the applied steady and oscillatory pressure field at reservoir, i.e. at $x = 0$). The $p_o L$ is varying between 0 to $p_o L = 175 \Delta p_{H_o=1}$, and $p_1 = 2p_o$, respectively. We keep the other experimental parameters as it is and show the averaged mass flow rate in one oscillatory cycle for the flexible channel ($\alpha = 0.1$) without oscillatory pressure field (i.e. $p_1 = 0$) in green color, and with oscillatory pressure field (i.e. $p_1 = 2p_o$) in red color in figure 8 as a function of varying pressure, respectively. We also show the averaged mass flow rate for the rigid channel ($\alpha = 0$) with the black line in the same figure 8.

We find that the flexibility of the wall increases the averaged mass flow rate for all varying pressure fields in case of without oscillatory pressure field in comparison to the rigid wall channel as shown with green and black data lines. We find that for $p_o L = 162.5 \Delta p_{H_o=1}$, the averaged mass flow rate in rigid channel wall is $1.7x10^{-3}$ g/cm$^3$, whereas for flexible channel ($\alpha = 0.1$) in the absence of the oscillatory pressure field, it is $3.6x10^{-3}$ g/cm$^3$. Therefore, slight deformability in the channel increases the averaged mass flow rate by 2.12 times, which is significant. We further find that, as we introduced the oscillation field in the pressure in addition to the constant pressure field, the averaged mass flow rate remains unchanged in the rigid wall channel as shown with a black line, whereas the oscillatory field intensifies the flow rate further in the flexible channel (shown in red). We find that for $p_o L = 162.5 \Delta p_{H_o=1}$, the averaged mass flow rate in rigid channel wall with oscillations is $1.7x10^{-3}$ g/cm$^3$, whereas, for flexible channel ($\alpha = 0.1$) with oscillatory pressure field, it is $8.8x10^{-3}$ g/cm$^3$, which is 5.2 times higher in flow rate with...
respect to rigid channel and 2.44 times higher when there is no oscillatory pressure field in flexible channel.

Additionally, we also find that, as the magnitude of the pulsating pressure increases in a flexible channel, the time-averaged flow rate also increases with it. The reason is that in the case of a rigid channel the mass flow rate is linearly proportional to $\Delta p$, therefore when we time-averaged the flow rate, the integration of the oscillatory part of the pressure field becomes zero in the full cycle. On the otherhand, in the case of the flexible channel, the flow rate (from equation (43)) consist of non-linear terms of $\Delta p$, due to the non-linearity the time-averaging of the oscillatory part of the pressure field in a complete cycle is non-zero and increases with the amplitude of the oscillatory pressure field (shown in red).

5 Conclusion

In this paper, we derived a model for the mass flow rate in the rigid and deformable nanochannels for the nanoconfined water transport by using the small displacement structural mechanics and perturbation theory presented by Gervais et al. [8], and Christov et al. [9], respectively under the lubrication approximation. For the validation purpose, we show that the newly derived model also facilitate the flow dynamics of Newtonian fluids under different conditions as its limiting cases, which has been previously reported in the literature [8, 9, 6, 10, 11, 12].

In our study, we focus on investigating the impact of two key factors: the flexibility of the graphene sheet and the application of a pulsating pressure field on the flow rate. We find that as the flexibility $\alpha$ increases or corresponding thickness $T$ or elastic modulus $E$ of the wall decreases, the flow rate inside the channel increases. We find that the flow rate in flexible channels scales as $\dot{m}_{\text{flexible}} \sim \alpha^0$ for $(\alpha \Delta p W/EH_o) \ll 1$, $\dot{m}_{\text{flexible}} \sim \alpha$ for $(\alpha \Delta p W/EH_o) \sim O(10^{-1})$ and $\dot{m}_{\text{flexible}} \sim \alpha^3$ for $(\alpha \Delta p W/EH_o) \sim O(1)$. We also find that, for a given thickness $T$, the percentage change in flow rate in the smaller height of the channel is much larger than the larger height of the channels. As the channel height decreases for the given reservoir pressure and thickness, the $\Delta \dot{m}/\dot{m}$ increases with $H_o^{-1}$ followed by $H_o^{-3}$ after a threshold with the channel height as shown in [5, b).

Additionally, to gain insights into the mechanisms and dynamics of fluid flow in nanocapillaries, and to understand how they can be manipulated or controlled for various applications, we analyze the effects of applying a pulsating pressure field, which involves periodically varying the pressure exerted on the reservoir. We find that due to the oscillatory pressure field, there is no change in the averaged mass flow rate in the rigid channel. On the otherhand, the flow rate increases in the flexible channels with the increasing magnitude of the oscillatory pressure field. Also, in flexible channels depending on the magnitude of the pressure field either steady or oscillatory or both, the averaged mass flow rate dependence varies from $\Delta p$ to $\Delta p^4$ as the pressure field increases.

We find that although the confined water properties increase the density upto 1.6 times in the channel and the slip also plays a major role in increasing the mass flow rate in the nanocapillaries, the flexibility of these channels and oscillatory pressure field at the reservoir intensify the flow rate through these channels several times.
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