

# Lessons learned from multi-objective automatic optimizations of classical three-site rigid water models using microscopic and macroscopic target experimental observables

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## Abstract

<sup>2</sup>  
<sup>3</sup> The development of accurate water models is of primary importance for molecular  
<sup>4</sup> simulations. Despite their intrinsic approximations, three-site rigid water models are  
<sup>5</sup> still ubiquitously used to simulate a variety of molecular systems. Automatic opti-  
<sup>6</sup> mization approaches have been recently used to iteratively optimize three-site water

7 models to fit macroscopic (average) thermodynamic properties, providing “state-of-the-  
8 art” three-site models that still present some deviations from the liquid water properties.  
9 Here we show results obtained by automatically optimizing three-site rigid water models  
10 to fit a combination of microscopic and macroscopic experimental observables. We use  
11 *Swarm-CG*, a multi-objective particle-swarm-optimization algorithm, for training the  
12 models to reproduce the experimental radial distribution functions of liquid water at  
13 various temperatures (rich in microscopic-level information on, *e.g.*, the local orienta-  
14 tion and interactions of the water molecules). We systematically analyze the agreement  
15 of these models with experimental observables and the effect of adding macroscopic in-  
16 formation into the training-set. Our results demonstrate how adding microscopic-rich  
17 information in the training of water models allows achieving state-of-art accuracy in an  
18 efficient way. Limitations in the approach and in the approximated description of water  
19 in these three-site models are also discussed, providing a demonstrative case useful for  
20 the optimization of approximated molecular models in general.

## 21 Introduction

22 The development and optimization of classical molecular models is typically challenging and  
23 time-consuming.<sup>1,2</sup> Despite notable progresses in developing efficient methods and optimiza-  
24 tion approaches,<sup>3-8</sup> accurately predicting experimental observables and ensuring transfer-  
25 ability across varying thermodynamics conditions remains in most cases a significant chal-  
26 lenge.<sup>9-11</sup> A considerable example is the case of water, for which current state-of-art models  
27 struggle in matching all the relevant cases of interest at the same time,<sup>12</sup> *e.g.*, bulk proper-  
28 ties,<sup>13</sup> free energy of hydration of compounds,<sup>14</sup> stabilization of lipid membranes,<sup>15</sup> interac-  
29 tion with proteins,<sup>16</sup> *etc.*

30 Although intrinsically approximated, classical three-site rigid water models are widely used  
31 in molecular dynamics (MD) simulations.<sup>13</sup> One key requirement is that such simplified  
32 models can capture fairly well the properties of water even relying on a reduced number of

33 parameters. In such a representation, the interaction potential is centered on three sites,  
34 each of which corresponds to one of the atoms in the water molecule (O, H, H). Early ver-  
35 sions of these models, including, *e.g.*, TIP3P<sup>17</sup> and SPC,<sup>18</sup> were originally parameterized to  
36 accurately reproduce basic thermodynamic properties, *e.g.*, density and enthalpy of vapor-  
37 ization under standard conditions. Despite their age, these models continue to be extensively  
38 utilized in classical MD simulations, and most general-purpose forcefields are parametrized  
39 on them.<sup>19–21</sup> With the increase in computing power, it has become possible to perform  
40 high-throughput parameterization, often in an automatic fashion,<sup>22,23</sup> by considering a large  
41 set of experimental observables under different conditions as the reference data to fit.

42 Over the past decade, two notable general-purpose three-site water models that have been  
43 obtained through iterative optimization, TIP3P-FB<sup>24</sup> and OPC3,<sup>25</sup> led to a substantial im-  
44 provement of the state-of-the-art. Such models were refined to accurately reproduce a set  
45 of thermodynamic properties including density, heat of vaporization, coefficient of thermal  
46 expansion, isothermal compressibility, isobaric heat capacity, and static dielectric constant.

47 In particular, TIP3P-FB has been optimized to accurately reproduce these observables over  
48 a wide range of thermodynamic conditions, spanning a total of 40 training points at different  
49 temperatures and pressures. Such a parallel/multi-objective parametrization has a positive  
50 effect on the transferability of the optimized model,<sup>26</sup> *e.g.*, across different conditions. In  
51 contrast, OPC3 was optimized to match such observables under standard conditions (298  
52 K and 1 bar), while simultaneously imposing a constraint on the geometry of the water  
53 molecule. Specifically, a fixed Hydrogen-Oxygen-Hydrogen angle value is imposed to ensure  
54 that the resulting linear quadrupole moment is equal to zero. This constraint is applied  
55 because the quadrupole moment is known to have minimal significance in the context of the  
56 model’s overall performance and accuracy.<sup>27</sup> While both models have demonstrated similar  
57 accuracy in reproducing thermodynamic properties, they do exhibit some distinct character-  
58 istics. TIP3P-FB is characterized by a larger geometry, with a distance of 0.101 nm between  
59 the Oxygen and Hydrogen sites ( $d_{OH}$ ) and 0.164 nm between the Hydrogen sites ( $d_{HH}$ ).

60 Furthermore, the Oxygen site in TIP3P-FB carries a partial charge of -0.848 e. In contrast,  
61 OPC3 exhibits a smaller geometry, with  $d_{OH}$  and  $d_{HH}$  equal to 0.098 nm and 0.160 nm  
62 respectively. Additionally, the Oxygen site in OPC3 has a charge of -0.895 e. The variability  
63 observed between the optimized models may be attributed to an intrinsic limitation aris-  
64 ing from the simplified description of the system. Furthermore, as both models are trained  
65 solely on average parameters derived from a top-down approach, it becomes intriguing to ex-  
66 plore the potential advantages of integrating additional data on microscopic target features  
67 through a bottom-up approach.

68 In recent works, we introduced *Swarm-CG*,<sup>7,9</sup> a versatile optimization software that is able  
69 to integrate bottom-up and top-down references in a multi-objective and multi-directional  
70 optimization framework for coarse-grained models. Building upon *Swarm-CG*'s capabili-  
71 ties, we propose a novel strategy for optimizing three-site water models by incorporating  
72 experimental data on the microscopic structure of water, particularly the radial distribu-  
73 tion functions (RDF) of its atoms. Specifically, we utilize the Oxygen-Oxygen RDF ( $g_{OO}$ ),  
74 Oxygen-Hydrogen RDF ( $g_{OH}$ ), and Hydrogen-Hydrogen RDF ( $g_{HH}$ ) as the primary refer-  
75 ences for deriving our model. While our main objective is not to develop the most accurate  
76 three-site rigid model, we aim to explore the capabilities of *Swarm-CG* and assess the room  
77 for improvement in what can be considered *de facto* a coarse-grained description of water.  
78 The results we obtain are significant for two main reasons. Firstly, we demonstrate that by  
79 selecting optimization targets spanning different scales (micro + macro), such as the RDFs,  
80 density, and dielectric constant, it is possible to obtain an optimized water model with  
81 comparable accuracy to state-of-the-art models like TIP3P-FB and OPC3, while maintain-  
82 ing computational efficiency and robustness. Secondly, our findings allow us to investigate  
83 the chemical and physical origins that control the accuracy limits (indeterminacy) of model  
84 optimization. We investigate how these limits are intrinsic and connected to the physical  
85 constraints of the model itself. The insights gained from this study hold significance not  
86 only for optimizing the specific system presented in this paper but also for any approxi-

87 mated model that relies on higher-accuracy data or incorporates top-down constraints based  
88 on experimental evidence.

## 89 **Methods**

90 The optimization work conducted herein builds on a multi-reference particle swarm op-  
91 timization software that we developed recently: *Swarm-CG*.<sup>7,9</sup> In particular, *Swarm-CG*  
92 has been developed to optimize bonded and non-bonded parameters in molecular mod-  
93 els to fit experimental results (top-down references) and the behaviour seen in all-atom  
94 MD trajectories (bottom-up references). *Swarm-CG* has been successfully tested to opti-  
95 mize a variety of molecular systems (*e.g.*, lipid models<sup>26</sup>). In this paper, *Swarm-CG* has  
96 been adapted for this specific case study (a dedicated variant can be found at: <https://github.com/GMPavanLab/wateropti>). The five parameters of a general three-site rigid  
97 water model that are iteratively tuned (illustrated in Figure 1a) are: (i) the intramolecular  
98 distance between the Oxygen and the Hydrogen sites,  $d_{OH}$ , (ii) the intramolecular distance  
99 between the two Hydrogen sites,  $d_{HH}$ , (iii) the absolute charge of the Oxygen site,  $q$ , and the  
100 two functional parameters of the Lennard-Jones potential which is centered on the Oxygen  
101 site, namely (iv) sigma  $\sigma$  and (v) epsilon  $\epsilon$ . We conducted our optimizations initializing  
102 swarms composed of 15 particles in the first and third subsections of the results, and 26 par-  
103 ticles in the second subsection. In each optimization procedure, a series of classical molecular  
104 dynamics simulations are performed and their discrepancy from the target properties is eval-  
105 uated according to a scoring function (described below). Finally, the obtained optimized  
106 models are simulated at various temperature across the liquid regime, and observables of  
107 interest are computed.

### 109 **Scoring function**

110 To quantify the discrepancy between the RDFs obtained from the simulations of the mod-  
111 els vs. the experimental ones from liquid water (at various temperatures), we introduced a

112 scoring function based on the Earth mover’s distance (EMD) or Wasserstein distance.<sup>28</sup> The  
113 Wasserstein distance is a measure of the dissimilarity between two probability distributions,  
114 based on the concept of optimal transport.<sup>29</sup> It represents the minimum cost of transforming  
115 one distribution into the other, where the cost is proportional to the distance between pairs  
116 of points. In our case, we used the Wasserstein distance to compare the simulated RDFs  
117 with the experimental RDFs, with the distance matrix representing the differences between  
118 the radial distance of the bins of the distributions. In our work, we modified the standard  
119 computation of the Wasserstein distance by using the square of the distance matrix instead  
120 of the distance itself. Such a modification allowed to better account (weights more) the dif-  
121 ference between the  $g(r)$  at larger distance, which is important for capturing the long-range  
122 behavior of the water-water interactions and to avoid overfitting on short-range interactions.  
123 Preliminary tests demonstrated that this provided the best setup to compare  $g(r)$  curves as  
124 a whole in the most robust way. This modification also allows mitigating potential problems  
125 emerging from the fact that classical three-site water models usually have difficulty repro-  
126 ducing the first peak of the RDFs (due to the fact that quantum effects are not included in  
127 the description of the system).<sup>30</sup>  
128 In particular, the scoring function used in the optimization presented the first subsection of  
129 the results is:

$$S = EMD_{g_{OO}} + EMD_{g_{OH}} + EMD_{g_{HH}} , \quad (1)$$

130 where  $S$  represents the score and  $EMD_{g_{OO}}$ ,  $EMD_{g_{OH}}$ , and  $EMD_{g_{HH}}$  represent the Earth  
131 mover’s distance measurements of the three RDFs considered, namely the Oxygen-Oxygen,  
132 Oxygen-Hydrogen, and Hydrogen-Hydrogen, respectively. In this way, the scoring function  
133 does not capture discrepancies only in terms of distances and spatial displacement of the  
134 water molecules respect to each other, but also in terms of their natural orientation. This  
135 provides us with a scoring function which is rich in microscopic structural information on  
136 the system.

137 The optimizations presented in the second and third subsection of the results involved not

138 only fitting of microscopic features, but also the density and static dielectric constant (macro-  
139 scopic observables). The adopted score is expressed as:

$$S = w_{EMD}(EMD_{g_{OO}} + EMD_{g_{OH}} + EMD_{g_{HH}}) + w_{\rho} |\rho_{sim} - \rho_{exp}| + w_{\varepsilon} |\varepsilon_{sim} - \varepsilon_{exp}| , \quad (2)$$

140 where the first term represents the difference between the simulated and experimental RDFs  
141 for each type of particle-particle correlation. The second and third terms take into account  
142 the difference between the simulated and experimental values of density  $\rho$  and static dielectric  
143 constant  $\varepsilon$ , respectively. Each term in the score function has a weight assigned to it, which  
144 determines its relative importance in the optimization process. The weights were chosen as  
145  $w_{EMD} = 0.5$ ,  $w_{\rho} = 0.3$ , and  $w_{\varepsilon} = 0.2$ . Preliminary tests demonstrated that these weights  
146 ensured a balanced representation in the optimization process, allowing us to prioritize and  
147 place emphasis on fitting the RDFs over other macroscopic features of the systems. A  
148 comparison of experimental RDF with simulated  $g(r)$  examples scored according to our  
149 metrics is present in Figure S1 of the Supporting Information.

## 150 Results and Discussion

151 This part is organized as follows. The first subsection presents the results of the model opti-  
152 mized to reproduce the experimental RDFs ( $g_{OO}$ ,  $g_{OH}$ , and  $g_{HH}$ ) under standard conditions  
153 of 298 K and 1 bar. This approach focuses primarily on a pure bottom-up methodology,  
154 where the optimization is driven by the microscopic features of the water model. In the sec-  
155 ond subsection, we extend our analysis by optimizing the model to reproduce not only the  
156 RDFs but also experimental density and static dielectric constant. Furthermore, the system  
157 is trained at two additional temperatures, specifically 280 K and 343 K. This comprehensive  
158 optimization approach aims to capture a broader range of experimental observables, combin-  
159 ing both bottom-up and top-down references. Finally, the last subsection provides a detailed  
160 investigation into the indeterminacy of the optimization problem within the context of the

161 three-site representation.

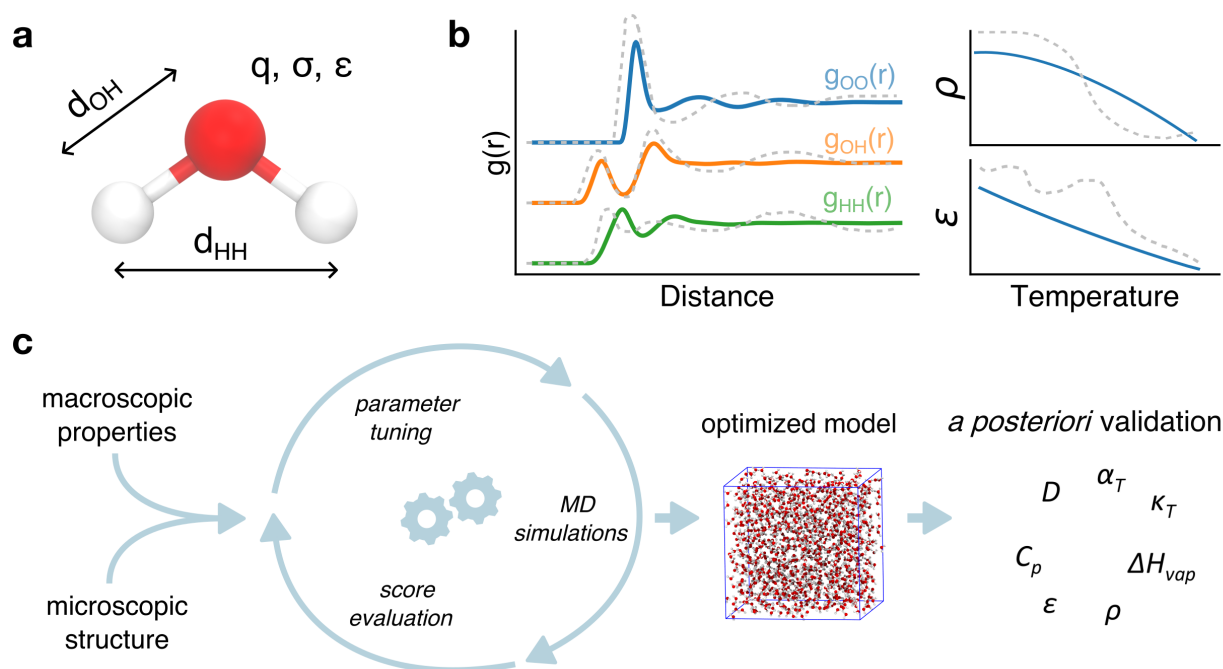


Figure 1: Overview of the study. **a** Representation of a water molecule and schematic of the five parameters that define the three-site model. **b** Experimental data used in this work: radial distribution functions  $g(r)$  (bottom-up reference), liquid water density ( $\rho$ ) as a function of temperature and static dielectric constant ( $\epsilon$ ) as a function of temperature (top-down references). **c** Workflow diagram illustrating the process of the study. Reference experimental data serve as the targets guiding the optimization process. Swarm-CG runs iterative MD simulations, adjusting the parameters of the water molecule to reach the best match with the reference experimental data. The resulting optimized model is then evaluated and validated *a posteriori* against a set of experimental observables at different temperatures not in the training set.

## 162 Multi-objective optimization based on microscopic system features

163 In a first optimization test, we trained the optimized water model according to a purely  
164 bottom-up approach to reproduce the experimental RDFs ( $g_{OO}$ ,  $g_{OH}$  and  $g_{HH}$ ) of water at  
165 the standard conditions of 298 K and 1 bar. At every iteration, *Swarm-CG* tests new param-  
166 eters in the attempt to minimize the discrepancy between the  $g_{OO}$ ,  $g_{OH}$  and  $g_{HH}$  obtained  
167 from the model and the experimental ones in standard conditions. The obtained results are  
168 presented in Figure 2. A comparison with other popular three-site water models of the same



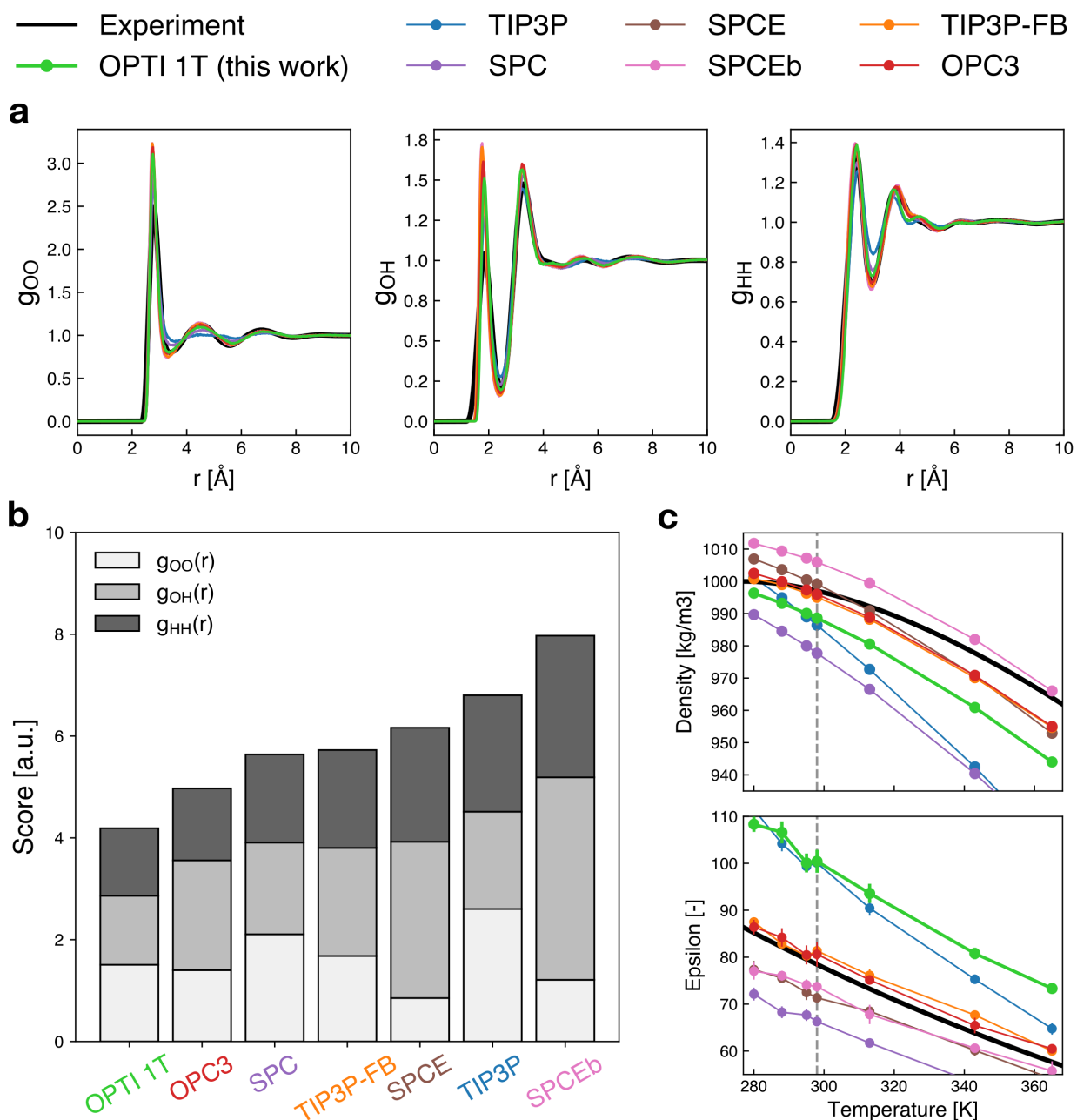


Figure 2: Results obtained from the first optimization, where the model OPTI 1T has been trained to reproduce experimental RDFs at 298 K and 1 bar. **a** RDFs reproduction and comparison with other three-site water models: complete data (without superposition of curves) are provided in the Supplementary Information (Figure S3). **b** Ranking of RDFs reproduction accuracy based on our score. **c** *A posteriori* validation of the model with respect to density and static dielectric constant. Dashed vertical gray lines indicate the temperature at which the model was trained.

169 type (OPC3,<sup>31</sup> TIP3P-FB,<sup>24</sup> SPC,<sup>18</sup> SPCE,<sup>32</sup> SPCEb<sup>33</sup> and TIP3P<sup>17</sup>) is also provided. A  
170 summary of the parameters for these models can be found in Table S1 of the Supplementary  
171 Information.

172 According to the score that we formulated to quantify the deviation of the simulated RDFs  
173 from the experimental reference (equation (1)), our model exhibited the highest level of ac-  
174 curacy in replicating the experimental RDFs (Figure 2b). Despite the fact that a model  
175 optimized as such is found the best one, this is not surprising since our model was optimized  
176 to reproduce experimental RDFs. Nonetheless it can be noticed that also in our case the  
177 Oxygen-Hydrogen RDF is overlocalized which appears as unavoidable in such models, where  
178 the nuclear quantum effects are not explicitly included. Similarly, in the attempt to fit at  
179 best the 2<sup>nd</sup> and 3<sup>rd</sup> solvation shell in the Oxygen-Oxygen RDF (identified by the 2<sup>nd</sup> and  
180 3<sup>rd</sup>  $g_{OO}$  peaks) produces an unavoidable overlocalization of the 1<sup>st</sup> peak (an enlarged plot of  
181 the RDFs around the solvation shells is provided for clarity in Figure S2 of the Supporting  
182 Information).

183 It is worth noting that, even at the training temperature, the value of the dielectric constant  
184 predicted by our best model deviated from the target by a substantial amount. Such a lack  
185 of accuracy can be attributed to the fact that training the water model on RDFs alone does  
186 not provide sufficient information on the interactions between atoms. As a result, quantities  
187 such as the static dielectric constant, which depends on dipole fluctuations and is sensitive  
188 to the charges on the water model, are not reproduced accurately enough if the model is  
189 not trained to do so. This also means that, although the RDFs are well reproduced, this is  
190 not a sufficient condition for macroscopic properties to emerge spontaneously in the system.  
191 These considerations motivated us to incorporate also additional top-down experimental tar-  
192 gets into the scoring function. The results of this integration are illustrated in the following  
193 section.

194 In terms of computational time, the refinement of 5 parameters of the water model at single  
195 temperature condition required 4 days (wall-clock time) to reach convergence (500 swarm

196 iterations) using 26 particles in the swarm and using 36 CPU cores, each simulation running  
197 on 9 CPU cores equipped with a GPU.

198

## 199 Multi-objective multi-temperature optimization based on microscopic 200 and macroscopic observables

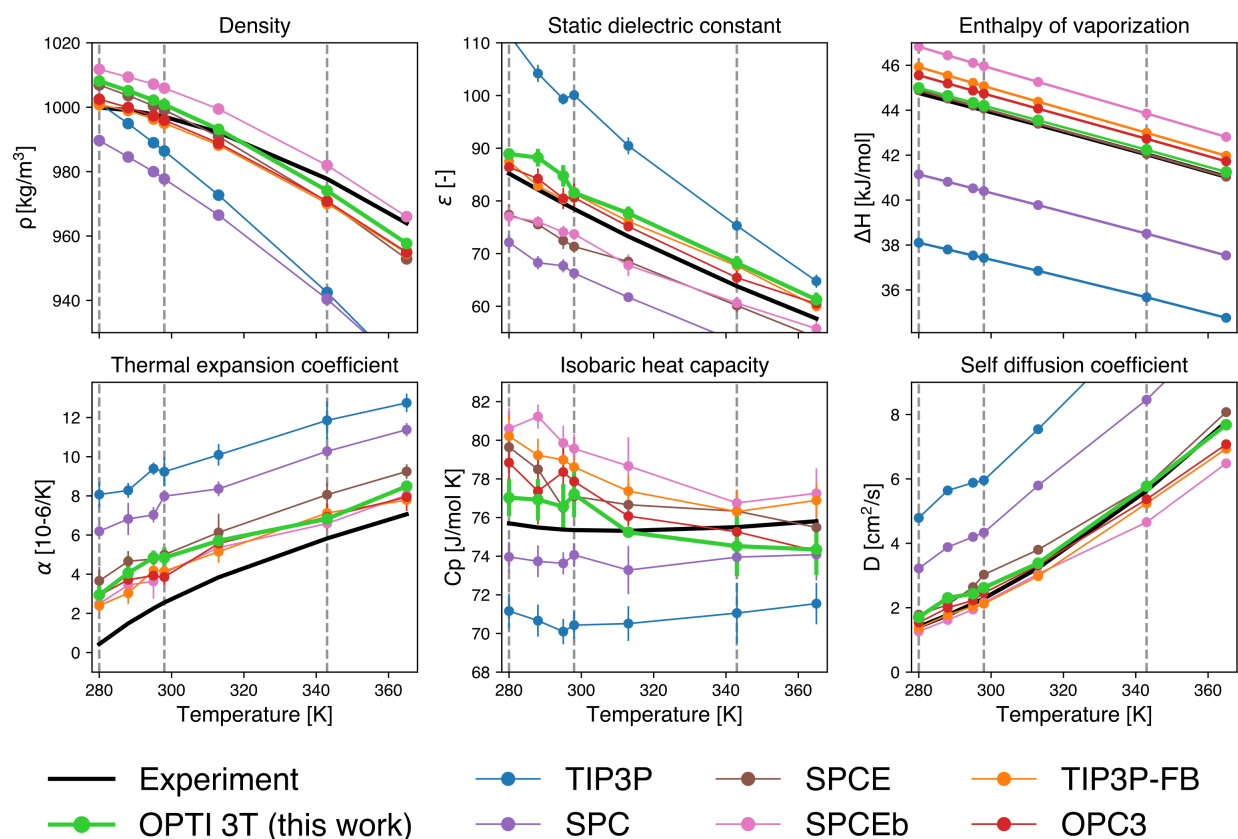


Figure 3: Validation of the model OPTI 3T, trained at the temperatures of 280 K, 298 K, and 343 K. Reproduction of RDFs, isothermal compressibility, surface tension, and quantification of the deviation of simulated observables with respect to experimental data are provided in the Supporting Information (Figures S3-S6)

201 In a second test, we trained our water model using a hybrid approach, incorporating both  
202 top-down and bottom-up references, to attain an accurate reproduction of the experimental  
203 radial distribution functions (RDFs), density, and static dielectric constant at three distinct

204 temperatures: 280K, 298K, and 343K. This is thus a multi-temperature multi-objective op-  
205 timization combining top-down (microscopic) and bottom-up (macroscopic) target observ-  
206 ables. Figure 3 illustrates the model’s performance in reproducing observables of interest  
207 in the liquid regime, namely density, static dielectric constant, enthalpy of vaporization,  
208 thermal expansion coefficient, isobaric heat capacity and self-diffusion coefficient. The plots  
209 containing RDFs, the values of isothermal compressibility and surface tension are present in  
210 the Supporting Information (Figures S3, S4 and S5). The overall accuracy of the model that  
211 we obtained can be compared with the most advanced state-of-the-art data-driven trained  
212 models, such as TIP3P-FB<sup>24</sup> and OPC3.<sup>31</sup> In particular, it is worth noting the agreement  
213 of our model (OPTI-3T, in green) with the experimental enthalpy of vaporization and self-  
214 diffusion coefficient at all explored temperatures. The enthalpy of vaporization reflects the  
215 strength of interactions between water molecules in the liquid state, representing the energy  
216 required to transition a molecule from the liquid to the vapor phase. On the other hand, the  
217 self-diffusion coefficient characterizes the dynamics of individual molecule diffusion within  
218 the liquid, indicating the ease of movement in a medium comprised of other water molecules.  
219 Notably, our optimized model demonstrates remarkable agreement with experimental results  
220 for these two parameters, despite not being explicitly targeted during training. This agree-  
221 ment underscores the significance of training the model on the radial distribution function  
222 and these two additional macroscopic targets, as they provide essential information for ac-  
223 curately reproducing fundamental thermodynamic and kinetic properties at the local level.  
224 A quantification of the accuracy of our OPTI-3T model by means of average deviation from  
225 the various experimental observables is present in Figure S6 of the Supporting Information.  
226 Overall, these results show the striking positive effect of training the water model based on  
227 microscopic information rich observables (*e.g.*, the RDFs), and how microscopic characteris-  
228 tics of the model significantly influence most of its properties.

229 In terms of computational time, the refinement of 5 parameters of the water model at 3 levels  
230 of temperature required 8 days (wall-clock time) to reach convergence (300 swarm iterations)

231 using 15 particles in the swarm and using 36 CPU cores, each simulation running on 9 CPU  
232 cores equipped with a GPU.

233 These results give rise to several important considerations. Firstly, the results obtained  
234 with our method demonstrate that, despite the fact that our model reproduces globally well  
235 the explored thermodynamic properties across the different conditions, the performances of  
236 OPTI-3T are not distant from those of *e.g.*, OPC3 and TIP3P-FB. This demonstrates that,  
237 substantially, there is a limited room for radically improving the performances of three-site  
238 rigid water models. All our results suggest that there is an intrinsic limit in the accuracy  
239 that is achievable with models where the representation of the water molecule is so simpli-  
240 fied. This leads us to fundamental questions. What are the key factors underpinning such  
241 limits? Are these imputable, *e.g.*, to limitations in the optimization method itself, or to  
242 intrinsic limits of the model? In the next section, we will deeper investigate these questions,  
243 obtaining interesting insights.

## 244 **Intrinsic physical limits and indeterminate optimizations of rigid** 245 **three-site water models**

246 Recently, Izadi *et al.*<sup>31</sup> suggested that three-site water models somehow possess inherent ac-  
247 curacy limitations due to their oversimplified nature, which hinders their ability to achieve a  
248 complete and experimentally consistent reproduction of observables across the liquid phase.  
249 Nevertheless, an relevant question that remains unanswered is the precise reason behind this  
250 instrinsic limitation.

251 The performance of an automatic optimization procedure may be significantly influenced by  
252 *a priori* choices concerning the methodology and training variables. As a result, the model's  
253 ability to accurately fit different observables may vary to some extent. In the case of OPC3,  
254 for example, it was considered crucial to impose constraints on the geometry of the molecules  
255 in order to ensure a quadrupole moment of zero.<sup>31</sup> In the case of TIP3P-FB, a predominant  
256 emphasis was placed on a top-down approach, involving the simultaneous fitting to multiple

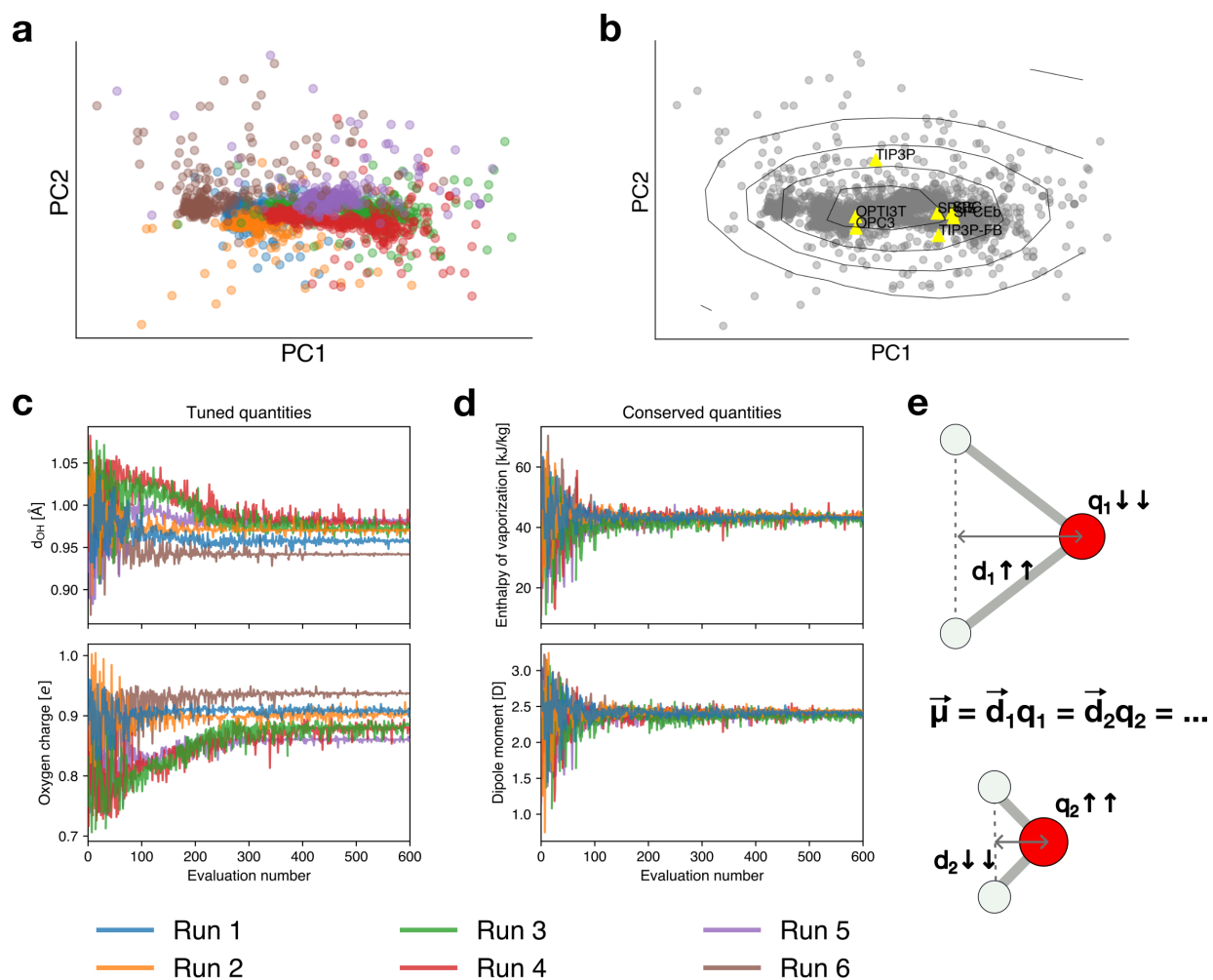


Figure 4: Results obtained by running a series of identical optimizations, initialized in different points of the parameters space. **a** Models obtained as solutions, displayed with principal component analysis. Each color represents a different optimization run. **b** Density isolines. **c** Parameters that are tuned during the optimization -  $d_{HH}$  and Oxygen charge - as a function of the number of iterations. **d** Example of quantities calculated *a posteriori* that are related to the energy of interaction between molecules - *e.g.*, enthalpy of vaporization and dipole moment - as a function of the number of iterations. **e** Schematic representation of the solutions obtained from the series of optimizations: inter-dependence between size of the molecule and partial charges.

257 thermodynamic observables across the liquid regime of water.<sup>24</sup> To investigate the impact of  
258 these initial conditions and gain a comprehensive understanding of the optimization process,  
259 we conducted a series of six optimizations under identical constraints. Specifically, we mini-  
260 mized the discrepancy of radial distribution functions (RDFs), density, and static dielectric  
261 constant, at the standard conditions of 298 K and 1 bar. Since the results obtained with  
262 such optimization cycles vary to some extent, this setup did not produce a single solution  
263 (identical in all six runs) but rather a group of solutions. Noteworthy, the obtained solutions  
264 demonstrate a comparable score (as illustrated in Figure S7 of the Supporting Information).  
265 Moreover, we conducted a Principal Component Analysis (PCA) of all explored solutions  
266 achieved through the use of *Swarm-CG*. Figure 4a shows the high density regions (*i.e.*, the  
267 solutions projected on the first two principal components) that represent the optimal solution  
268 obtained from our optimization cycles. The different colors represent the different runs. The  
269 contour lines illustrated in Figure 4b represent density isolines, which enable us to identify  
270 the regions of higher density points containing a set of optimal water models (according to  
271 our scoring function), which are characterized by slightly different set of parameters. These  
272 data demonstrate how *Swarm-CG* brings the model systematically not to a to a specific so-  
273 lution (*i.e.*, to a specific optimal model), but to a region of the space which contains “equally  
274 optimal, although slightly different solutions”. Interesting questions are, for example, why  
275 the method behaves in this way, and specifically why slightly different solutions are “equally  
276 optimal”.

277 A deeper inspection of the “optimal solutions” provided by *Swarm-CG* revealed interesting  
278 patterns. In particular, it is interesting to observe that the dipole moment of all the models  
279 belonging to this minimum is identical. Namely, despite the fact that their geometry or par-  
280 tial charges can be slightly larger/smaller (Figure 4c) in the various solutions, these change  
281 in such a way that the dipole moment the molecule is conserved. In such a way, the enthalpy  
282 of vaporization is also conserved across the various solutions (Figure 4d).

283 It is worth noting that these two properties (the dipole moment and the enthalpy of vapor-

284 ization) are evaluated *a posteriori* and are not explicitly used to train the models during  
285 the optimization process. Moreover, both such observables are related to the extent of the  
286 intermolecular interactions, as the enthalpy of vaporization is proportional to the potential  
287 energy in the system, and the interaction between dipoles of the molecules plays a key role  
288 in it. Figure 4d presents a visual representation of the variability in charge and geometry  
289 of the water models generated by the different optimization runs. In qualitative terms, an  
290 increase in charge corresponds to a reduction in the size of the molecule, while a decrease  
291 in charge results in an increase in size (Figure 4c,e). Additionally, we observed changes in  
292 the Lennard-Jones interactions with variations in sigma and epsilon values (Figure S8 in the  
293 Supporting Information). The obtained results and considerations illustrate how the collec-  
294 tive properties of these models are largely controlled by the interplay of molecular dipoles  
295 and their interactions with each other.

296 Since in such a simplified three-site models the majority of water-water intermolecular in-  
297 teraction are largely governed by the dipole moment, this introduces an intrinsic level of  
298 indeterminacy in the optimization. Recently, we have observed similar results also in the  
299 framework of the automatic optimization of, *e.g.*, lipid models using *Swarm-CG*, where a  
300 certain level of model accuracy can be achieved, although accompanied by an inherent un-  
301 certainty. While uncertainties can arise from various sources in automatic approaches, such  
302 as the number of objectives, parameter selection, and optimization methods, it is worth not-  
303 ing that the uncertainty we are referring to in this context originates elsewhere. Here, due  
304 to the simplified physical description of the three-site rigid water model, the optimization  
305 problem becomes inherently undetermined as it seeks to find an optimized dipole, which is  
306 a composite variable represented by the product of charge and geometry ( $\mu = qd$ ). This  
307 leads to a degeneracy characterized by different optimal solutions with varying combina-  
308 tions of charge and geometry. To overcome this limitation, a potential improvement could  
309 involve incorporating additional parameters that decouple the geometric and electrostatic  
310 characteristics of the water model during the training process. For example, one approach



311 could be training the model to reproduce a specific geometry obtained with higher accuracy  
312 from quantum mechanical (QM) approaches or by calculating the electrostatic potential.  
313 However, implementing such an approach encounters challenges due to the substantial dif-  
314 ferences between the geometric and electrostatic descriptions at the QM level compared to  
315 the all-atom models, as demonstrated by recent research on OPC3.<sup>25</sup>

316 An interesting outcome of these considerations is that while our approach can achieve opti-  
317 mal solutions at least as good as the state-of-the-art models in a very efficient way, it also  
318 underlines how such all atom water models are *de facto* a coarse grained description of the  
319 real water molecule features. Like other coarse-grained models, they encounter a degeneracy  
320 due to simplified representations of system degrees of freedom, resulting in a certain level of  
321 precision combined with inherent indeterminacy. This indeterminacy implies that different  
322 parametrizations can lead to similar behaviors. Notably, in our case, this degeneracy yields  
323 a set of non-identical solutions that belong to a minimum that is identified by the PCA.

324 These considerations suggest that similar principles used to parameterize other three-site  
325 rigid water models generally encounter similar limitations (*i.e.*, the challenges and con-  
326 straints faced in developing and optimizing water models based on similar principles are  
327 likely to be shared). To reduce the uncertainty in the model outcomes, another possible  
328 solution is to employ extra sites in the model, such as in the four-site models like TIP4P<sup>17</sup> or  
329 TIP4P-ICE.<sup>34</sup> Such an addition permits to expand the degrees of freedom to tune, allowing  
330 for a better fit of some properties , *e.g.*, the curve of density across different temperatures.

331 *De facto*, this underlines how, to improve substantially the performances of three-site water  
332 models, it is necessary to increase the resolution of the model accounting for more degrees  
333 of freedom.

## 334 Conclusions

335 In this work, we explored the effect of combining microscopic-rich and macroscopic-rich in-  
336 formation into a training set of experimental observables used to automatically optimize  
337 classical three-site water models. In particular, as the microscopic target observables, we  
338 use the experimental  $g_{OO}$ ,  $g_{OH}$ , and  $g_{HH}$  radial distribution functions of liquid water at var-  
339 ious temperatures that, altogether, contain information not only of how strongly the water  
340 molecules interact but also on how the molecules are organized in space respect to each  
341 other. A first optimization of the water model under standard conditions (298 K and 1 bar)  
342 using only such a bottom-up (microscopic) reference demonstrated how such microscopic  
343 information alone is insufficient to obtain an experimentally consistent reproduction of all  
344 other screened macroscopic observables for the liquid phase of water, especially for what  
345 pertains to the density and dielectric constant of liquid water.

346  
347 Including in the training-set and in the score, in a second test, the density and static  
348 dielectric constant of liquid water, was then seen to provide considerable improvements. The  
349 obtained model showed a remarkable improvement in reproducing macroscopic properties,  
350 especially with respect to self-diffusion coefficient and enthalpy of vaporization. This also  
351 suggests that these properties – density and dielectric constant – are not strongly dependent  
352 on the  $g(r)$  of water. Overall, we found that our optimized water model (called OPTI-3T  
353 herein) exhibits a comparable level of accuracy as two models, OPC3 and TIP3P-FB, which  
354 were also obtained through automatic optimization approaches and are considered state-of-  
355 the-art models in the realm of three-site rigid water models. Nonetheless, it is worth noting  
356 how, in our case, combining microscopic and macroscopic target properties allows achieving  
357 such a level of accuracy in an efficient way, and with a relatively reduced computational  
358 time (*e.g.*, TIP3P-FB is trained on a large amount of thermodynamic properties at various  
359 temperatures in a computationally intensive process). At the same time, our tests show that  
360 there is little room for further improvement in these models by, *e.g.*, adding more experimen-

361 tal observables in the training-set, etc., which suggested that all these models are somewhat  
362 very similar and possibly nearly consistent with each other, considered the precision that it  
363 is reasonable to expect from them.

364

365 The series of optimizations that we conducted herein under identical constraints (namely,  
366 fitting the experimental RDFs, density and static dielectric constant at standard conditions)  
367 shows that these models are somewhat intrinsically limited in their accuracy. The same  
368 is true in some sense concerning the determinacy of their optimization cycles. The results  
369 shown in Figure 4 show how many of the screened thermodynamic observables are controlled  
370 in these simplified water models by the water dipole ( $\mu$ ), which is a composite variable that  
371 depends on both the charge ( $q$ ) and geometry ( $d$ ) of the water model ( $\mu = qd$ ). This leads to  
372 an inherent indeterminacy in the solutions that are systematically obtained. This means that  
373 different combinations of charge and size can correspond to equally optimal solutions toward  
374 the fitting of the targeted properties. A set of optimal solutions is thus typically obtained in  
375 such automatic optimizations instead of a single specific one. The PCA data of Figure 4a,b  
376 show how all such obtained "optimal" solutions belong, in our case, to the same global high-  
377 density minimum. In Figures 4c,d it is demonstrated how all the slightly different solutions  
378 belonging to such minimum represents share nearly identical molecular dipole and enthalpy  
379 of vaporization. While the broadness of such minima could be interpreted, *e.g.*, as to be  
380 imputable to some kind of statistical error/limit in the particle swarm optimization method  
381 used herein, these results suggest that this is most likely related to an intrinsic indeterminacy  
382 in how the problem is posed. In particular, the degrees of freedom in such "coarse-grained"  
383 atomistic description of the water models are so limited that the optimization process de-  
384 generates, providing equally optimal solutions that are nonetheless different from each other.

385

386 These results are interesting because they demonstrate that, when dealing with the opti-  
387 mization of approximated models, there will be inevitably intrinsic limits due to degeneration

388 of the optimal set of parameters that satisfy the conditions that are posed. In such a case,  
389 further improvements cannot be achieved without introducing additional degrees of freedom  
390 that can decouple in some way such composite variables into the fundamental ones allow-  
391 ing to fine-tune the model. One way could be, *e.g.*, to add some higher (quantum) level  
392 additional constraint that allows to decouple the dependence on the charge ( $q$ ) from that  
393 of geometry ( $d$ ) in the solution. However, the geometry and electrostatics of QM water  
394 molecules are so different from those of these AA models, that tests in this sense proved  
395 inefficient. In the case of the classical water models studied herein, reaching higher precision  
396 thus requires expanding the model's representation by adding additional "classical" degrees  
397 of freedom, for example, allowing for a more flexible and accurate description of the system.  
398 This is exactly the case of the higher precision that can be achieved by, *e.g.*, 4- or 5-site mod-  
399 els.<sup>35,36</sup> Moreover, altering the degrees of freedom can have important effects for example  
400 on subtle dynamical mechanisms associated with water reorientational dynamics as recently  
401 shown.<sup>37</sup> However, these results are also interesting for the development of approximated  
402 molecular models in general. Recently, we have observed similar intrinsic limitations also in  
403 the optimization of, *e.g.*, coarse-grained models of a variety of other molecular systems.<sup>7,9,26</sup>  
404 This observation serves as a valuable lesson for developing models of all kinds, not just in the  
405 context of water simulations. Such inherent limitations and these challenges encountered in  
406 optimizing approximated models demonstrate the importance of considering the complexity  
407 of the system being studied, and the type of information lost with approximated molecular  
408 models. The integration of multiple references, and in particular combining bottom-up and  
409 top-down microscopic/macroscopic-level information in the training-set can improve the effi-  
410 ciency and robustness in the models' optimization. Nonetheless, the results discussed herein  
411 also offer an unambiguous example of how, understanding the physical limits of approxi-  
412 mated models, can provide a precious knowledge for guiding future research towards more  
413 robust and reliable modeling approaches.

## 414 Computational details

### 415 MD Simulations

416 All the simulations have been conducted using GROMACS version 2021.5<sup>38</sup> with the fol-  
417 lowing protocol. The starting systems' configuration is a cubic box containing 1024 water  
418 molecules, arranged in initial random configurations, using Packmol.<sup>39</sup> After a preliminary  
419 energy minimization via steepest descent algorithm (for  $2 \cdot 10^3$  steps), the system is then  
420 equilibrated for 5ns in  $NpT$  ensemble and simulated for 10ns in the same ensemble. Both  
421 these equilibration and production phases are simulated with a 2 fs timestep. We kept tem-  
422 perature constant with velocity rescale thermostat<sup>40</sup> (with a time constant of 0.2 ps), and  
423 pressure constant to 1 bar with cell rescale barostat<sup>41</sup> (with a coupling constant of 1 ps  
424 and compressibility of  $4.5 \cdot 10^{-5}$  bar). A cutoff distance of 1 nm was used for short-range  
425 electrostatic and van der Waals interactions, and the long-range interactions were computed  
426 with the particle-mesh Ewald summation method.<sup>42</sup> Corrections to long-range pressure and  
427 potential energy were considered.<sup>43</sup>

### 428 Observables

429 **Density.** The mass density of water  $\rho$  is calculated as follows:

$$\rho = \frac{N \cdot m_{H_2O}}{\mathcal{N}_A \cdot V_{\text{box}}}, \quad (3)$$

430 where  $N$  is the number of water molecules (1024 in our case),  $m_{H_2O}$  is the mass of water  
431 molecules in u.a.,  $\mathcal{N}_A$  is Avogadro's number, and  $V_{\text{box}}$  is the volume of the simulation box.  
432 Experimental reference data of  $\rho$  are taken from Ref.<sup>44</sup>

433 **Static dielectric constant.** We calculate the static dielectric constant from the fluctu-

434 ations of the total dipole moment  $M$  of the simulation box, *i.e.*, as:

$$\varepsilon = 1 + \frac{\langle M^2 \rangle - \langle M \rangle^2}{3\varepsilon_0 V k_B T}, \quad (4)$$

435 where  $\varepsilon_0$  is the permittivity of the vacuum,  $V$  the volume of the simulation box,  $k_B$  the  
436 Boltzmann constant,  $T$  the temperature of the system, and  $\langle \cdot \rangle$  represents the thermodynamic  
437 average. We calculated this observable using the routine `gmx dipoles` of GROMACS suite.  
438 Experimental reference data of  $\varepsilon$  are taken from Ref.<sup>44</sup>

439 **Radial distribution functions.** We calculated the radial distribution functions of  
440 Oxygen-Oxygen, Oxygen-Hydrogen and Hydrogen-Hydrogen ( $g_{OO}(r)$ ,  $g_{OH}(r)$ ,  $g_{HH}(r)$ ) pairs  
441 with MDAnalysis 2.0.0.<sup>45</sup> We considered a cutoff of 10 Å and 500 equally spaced bins. Ex-  
442 perimental reference data of radial distribution functions are taken from Ref.<sup>46</sup>

443 **Self diffusion coefficient.** The self-diffusion coefficient  $D$  is calculated using Einstein's  
444 relation for a diffusive particle as:

$$D = \lim_{t \rightarrow \infty} \frac{\langle |\vec{r}(t) - \vec{r}(0)|^2 \rangle}{6t}, \quad (5)$$

445 where the quantity in the numerator is the mean square displacement (MSD), averaged over  
446 the trajectories of individual particles. Diffusion coefficients calculated with MD simulation  
447 are often referred to as  $D_{PBC}$ , because they contain systematic errors due to the finite box  
448 size.<sup>47</sup> Following Ref.,<sup>47</sup> it is possible to correct this artifact obtaining the theoretical value  
449 of self-diffusion coefficient of water in an infinite box ( $D_0$ ). To this end, we calculated  $D_{PBC}$   
450 in cubic simulation cells with  $N = 512, 1024, 2048, 4096$  and  $8192$  water molecules. The  
451 protocol used for these simulations is identical to the one described in section , except for a  
452 different production time, *i.e.*, 20 ns ( $N = 512$ ), 15 ns ( $N = 1024, 2048$ ), 10 ns ( $N = 4096,$   
453  $8192$ ). We calculated the various  $D_{PBC}$  values using the `gmx msd` routine of GROMACS,<sup>38</sup>  
454 and  $D_0$  with linear interpolation. Experimental reference data are taken from Ref.<sup>48</sup>

455 **Enthalpy of vaporization.** The enthalpy of vaporization  $\Delta H_{vap}$  of one mole of liquid

456 water in the gas phase can be approximated as:<sup>49</sup>

$$\Delta H_{vap} \approx -U + RT - p_{sat}V - E_{pol} + C , \quad (6)$$

457 where  $U$  and  $V$  are respectively the average potential energy and the volume of one mole of  
458 water molecules at pressure  $p$  and bath temperature  $T$ .  $p_{sat}$  is the value of saturation pressure  
459 at temperature  $T$ . The term  $E_{pol}$  represents the depolarization energy of one mole of water  
460 molecules when it is transferred from the liquid to the gas phase.<sup>32</sup> It can be expressed as:

$$E_{pol} = \frac{(\mu - \mu_{gas})^2}{2\alpha_{gas}} \quad (7)$$

461 where  $\mu$  is the dipole moment of the simulated model,  $\mu_{gas}$  and  $\alpha_{gas}$  are the dipole moment  
462 and average polarizability of a water molecule in the gas phase,<sup>49</sup> respectively. The last  
463 term in equation (6) contains corrections that account for the vibrational effects of water  
464 molecules and non-ideality of the gas phase. These corrections are reported in Ref.<sup>49</sup> for  
465 different temperatures. Experimental reference data are taken from Ref.<sup>49</sup>

466 **Specific heat capacity** . We computed the isobaric heat capacity  $c_p$  using the enthalpy  
467 fluctuation formula, namely:

$$c_p = \frac{\langle H^2 \rangle - \langle H \rangle^2}{Nk_B \langle T \rangle^2} , \quad (8)$$

468 We computed this observable by using the `gmx energy` routine of the GROMACS<sup>38</sup> suite.  
469 The value obtained was then corrected to account for quantum effects that are not considered  
470 in the classically computed heat capacity in eq.(8). Specifically, these corrections include  
471 estimation of intra-molecular vibrational energies (due to the fact that our model is rigid)  
472 and inter-molecular high frequency modes. The values of these correction are reported in  
473 the Ref.<sup>49</sup> of Horn et al. The experimental reference data of  $c_p$  are taken from Ref.<sup>50</sup>

474 **Thermal expansion coefficient**. We calculated the thermal expansion coefficient  $\alpha_T$

475 using the enthalpy-volume fluctuation formula:

$$\alpha_T = \frac{\langle VH \rangle - \langle V \rangle \langle H \rangle}{k_B \langle T \rangle^2 \langle V \rangle}, \quad (9)$$

476 We computed this observable by using the `gmx energy` routine of the GROMACS<sup>38</sup> suite.

477 The experimental reference data of  $\alpha_T$  are taken from Ref.<sup>50</sup>

478 **Isothermal compressibility.** We calculated the thermal expansion coefficient  $\kappa_T$  using  
479 the volume fluctuation formula:

$$\kappa_T = \frac{\langle V \rangle^2 - \langle V^2 \rangle}{k_B \langle T \rangle \langle V \rangle}, \quad (10)$$

480 We computed this observable using the `gmx energy` routine of GROMACS<sup>38</sup> suite. The  
481 experimental reference data of  $\kappa_T$  were taken from Ref.<sup>50</sup>

482 **Surface tension.** The interface between water and void was prepared and simulated  
483 following the good practices outlined in Ref.<sup>51</sup> Firstly, a cubic box containing 1024 water  
484 molecules was equilibrated in the *NPT* ensemble. To represent the void phase, the z-axis of  
485 the simulation box was elongated by a factor of 4. The resulting biphasic system was then  
486 simulated for 50 ns in the *NVT* ensemble. The surface tension of the water-void interface  
487 was calculated using the mechanical or pressure approach,<sup>52</sup> which involves evaluating the  
488 inhomogeneity of the pressure tensor as follows:

$$\gamma(t) = \frac{L_z}{2} \left( P_{zz}(t) - \frac{P_{xx}(t) + P_{yy}(t)}{2} \right), \quad (11)$$

489 where  $L_z$  is the elongation of the z-axis, and  $P_{xx}$ ,  $P_{yy}$ , and  $P_{zz}$  are the diagonal components  
490 of the pressure tensor. To perform the analysis, we used the `gmx energy` routine of the  
491 GROMACS suite.<sup>38</sup> Experimental reference data for the surface tension are obtained from  
492 Ref.<sup>53</sup>



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## 497 Data Availability

498 The input files, relevant analysis script, and the results of the optimizations and simulations  
499 conducted herein are available at <https://github.com/GMPavanLab/wateropti> (temporary  
500 link that will be replaced with a definitive Zenodo archive upon acceptance of the final version  
501 of this paper). Other information needed is available from the corresponding author upon  
502 reasonable request.

## 503 Notes

504 The authors declare no competing financial interests.

## 505 Supporting Information

506 The Supporting Information is available free of charge online. Additional details regarding  
507 the automatic optimizations and molecular simulations conducted (PDF).

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# Lessons learned from multi-objective automatic optimizations of classical three-site rigid water models using microscopic and macroscopic target experimental observables

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Model	Refs.	Sigma [nm]	Epsilon [kJ/mol]	O charge (e)	O-H distance [nm]	H-H distance [nm]
SPC	ref. <sup>[1]</sup>	0.31655700	0.65062900	-0.820000	0.1000000	0.1633000
SPCE	ref. <sup>[2]</sup>	0.31655700	0.65062900	-0.847600	0.1000000	0.1633000
SPCEb	ref. <sup>[3]</sup>	0.31657195	0.64977520	-0.847600	0.1010000	0.1649300
TIP3P	ref. <sup>[4]</sup>	0.31506100	0.63638600	-0.834000	0.0957200	0.1513900
TIP3P-FB	ref. <sup>[5]</sup>	0.31779646	0.65214334	-0.848448	0.1011811	0.1638684
OPC3	ref. <sup>[6]</sup>	0.31742704	0.68369070	-0.895170	0.0978882	0.1598507
<b>OPTI 1T</b>		0.31921898	0.61796336	-0.935014	0.0940835	0.1518014
<b>OPTI 3T</b>		0.31657153	0.69750474	-0.889239	0.0977138	0.1590416

Table S1: Summary of the classical three-site rigid water models compared in this work.

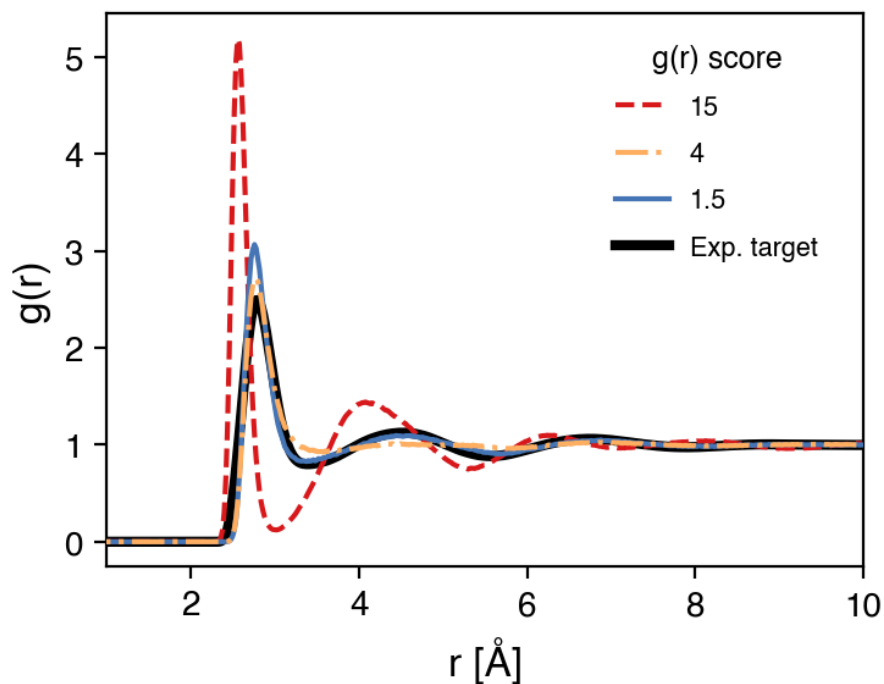


Figure S1: Comparison of experimental RDF and simulated RDFs examples scored according to our metrics. Notably, the orange curve receives a higher score (i.e., worse performance) compared to the blue one, despite capturing the first peak more accurately. The difference in scores arises from the orange curve's poorer reproduction of the second and third peaks. This observation highlights the scoring function preference for favoring long-range reproduction of RDFs in the evaluation of simulated curves.

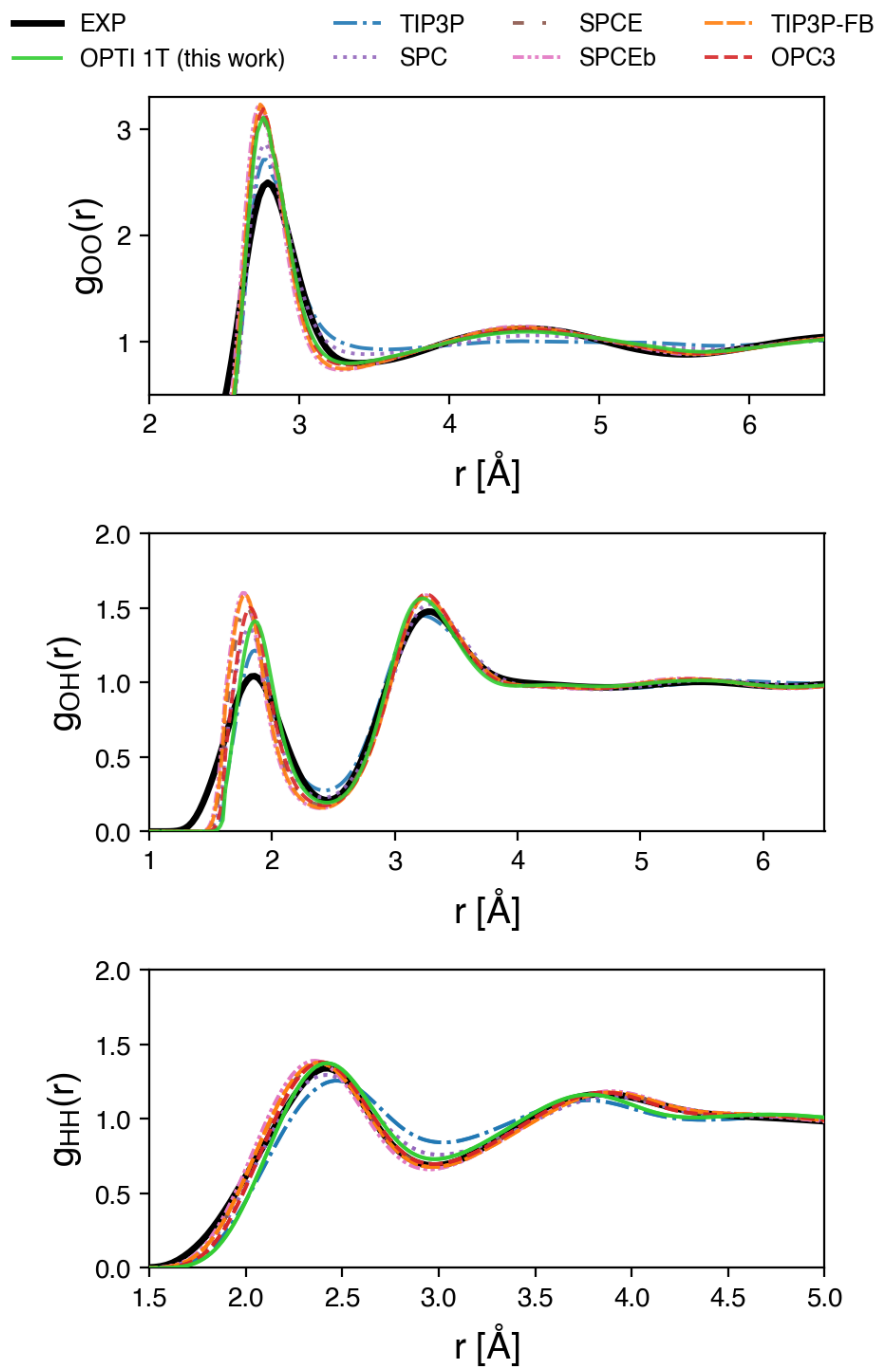


Figure S2: Enlarged plot of the reproduction of RDFs contained in Fig.2a

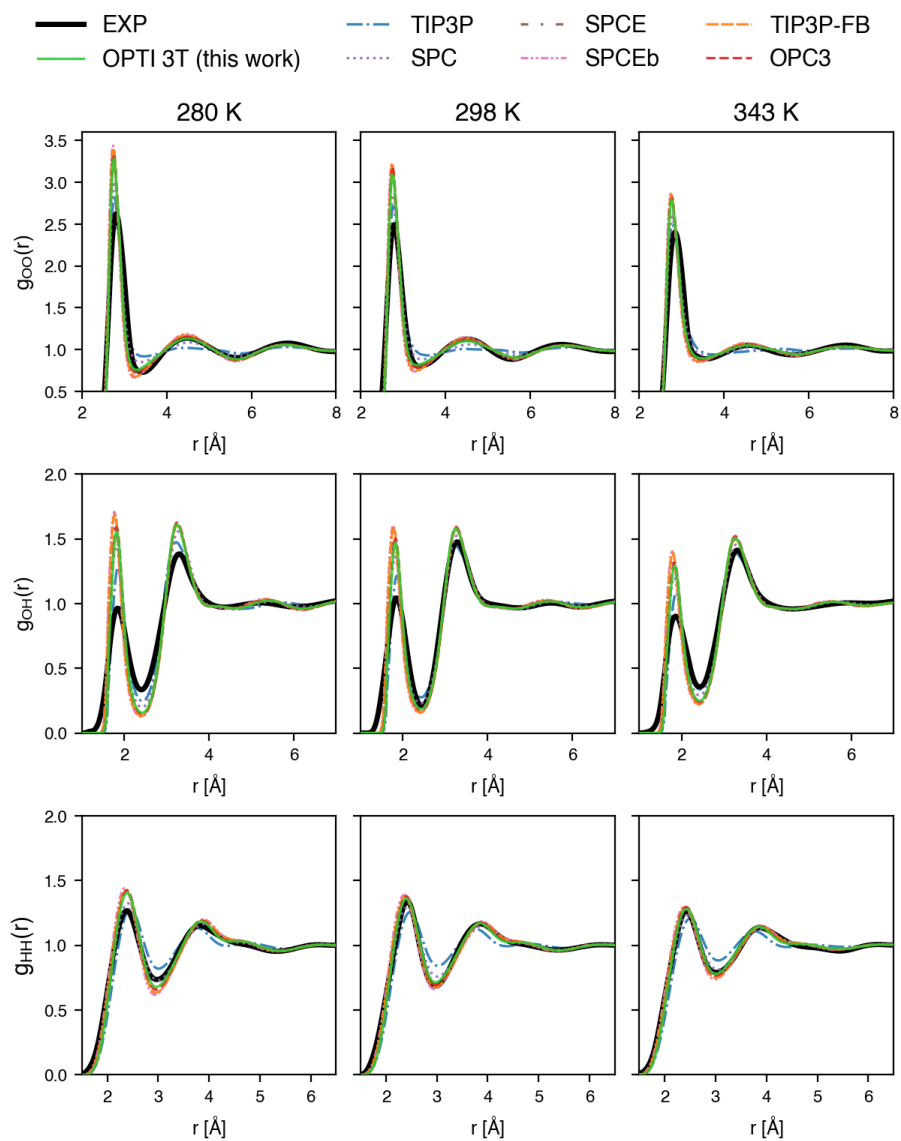


Figure S3: Radial distribution functions obtained of the model OPTI-3T along with a comparison with other models.

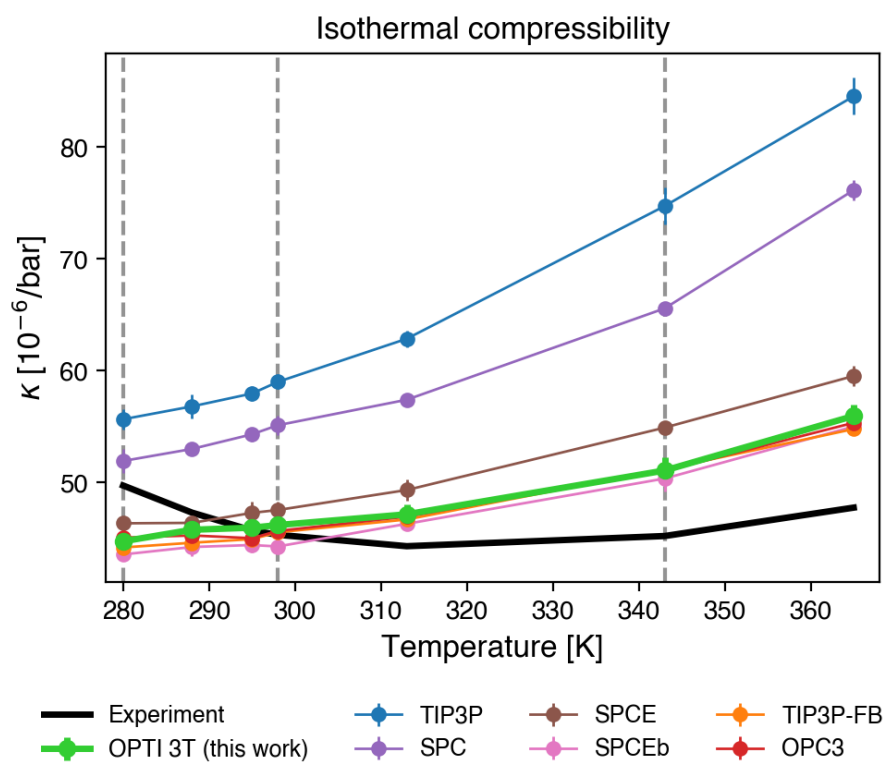


Figure S4: Isothermal compressibility as a function of temperature. Dashed vertical gray lines indicate the temperature at which the model was trained.

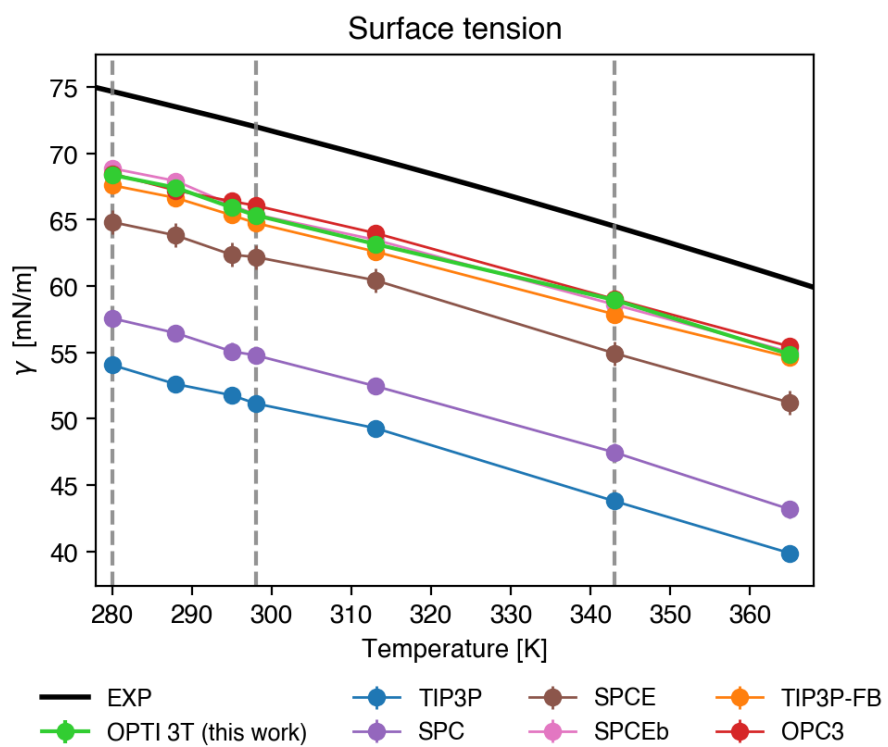


Figure S5: Surface tension as a function of temperature. Dashed vertical gray lines indicate the temperature at which the model was trained.

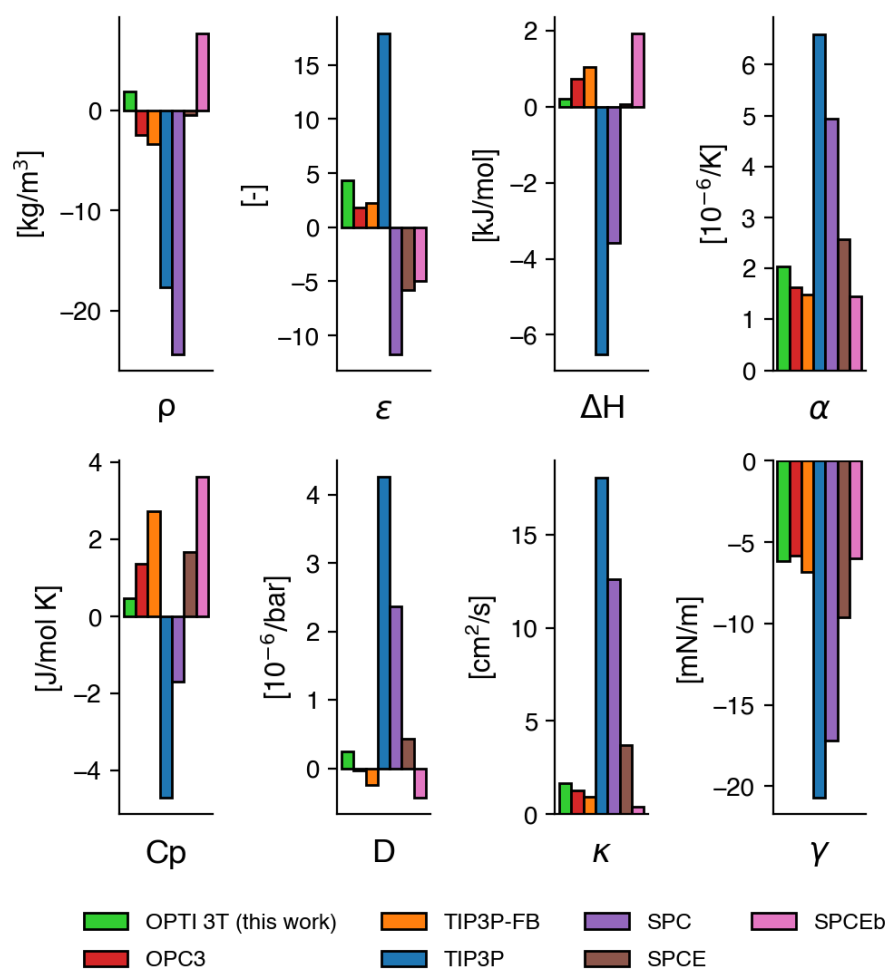


Figure S6: Average deviation of simulated observables with respect to experimental data in the liquid regime. The plots show water density, static dielectric constant, enthalpy of vaporization, thermal expansion coefficient, isobaric heat capacity, diffusion coefficient, adiabatic bulk modulus and surface tension.

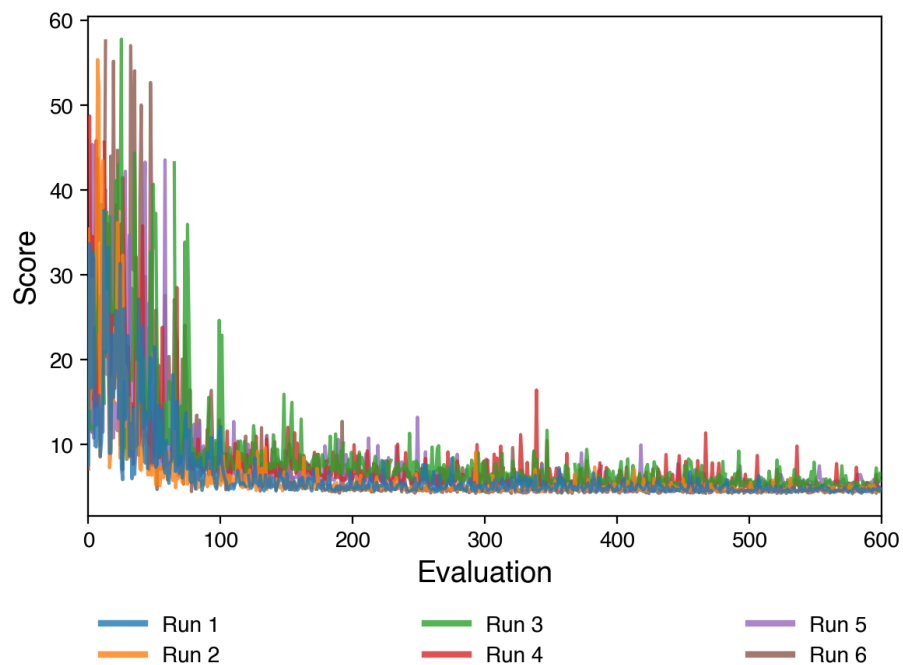


Figure S7: Score as a function of iteration number of the six optimizations. Each color represent an independent optimization run.

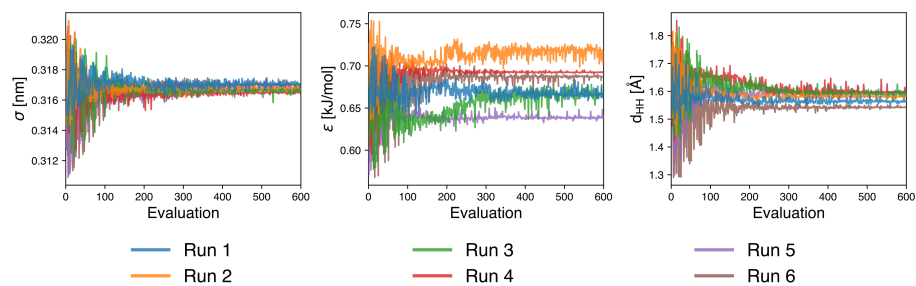


Figure S8: Values of sigma, epsilon and distance between the Hydrogens atoms as a function of iteration number. Each color represent an independent optimization run.



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