Molecular simulation of activity coefficients and the phase diagram of CH$_3$OH-CO$_2$ binary system

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Abstract

The interaction energies of five pairs of interactions: CH$_3$OH-CH$_3$OH, CH$_3$OH-CO$_2$, CO$_2$-CO$_2$, CH$_3$OH-CH$_3$CH$_2$OH, and CH$_3$CH$_2$OH-CH$_3$CH$_2$OH were obtained using MP2, B3LYP, and B3LYP-D3 methods. Activity coefficients of CH$_3$CH$_2$OH-CH$_3$OH and CH$_3$OH-CO$_2$ systems were calculated using the UNIQUAC equation and the phase diagrams of these binary systems were constructed based on the Raoult's law. The B3LYP-D3 methanol/ethanol isothermal phase diagram at 298K is in good agreement with the experimental data and the agreement is slightly better than the MP2 result and much better than the B3LYP result. Thus, the B3LYP-D3 method is promising to obtaining activity coefficients and illustrates the promise of molecular simulation of phase diagram of binary systems. A carbon dioxide/ethanol isothermal phase diagram at 230K was constructed. The current results showed that CO$_2$ and CH$_3$OH are hardly mixed under the simulation conditions.

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1. Introduction

For efficient and sustainable industrial processes, rational design of separation operations is essential. This requires knowledge of the activity coefficients of fluid mixtures and phase diagrams[1-4]. Therefore, although studies of catalysis[5-13], reactions[6, 14], various properties of molecules[15-20] and aggregates[21, 22] and design of new materials[23-28] for specific applications[29, 30] have garnered a lot of attention, considerable efforts have been devoted to the studies of binary systems[31, 32]. Investigations using various theories[33] and models[34] were carried out to obtain binary interaction parameter[35] and phase behaviors[36-38]. A particularly useful equation, UNIQUAC becomes a useful tool for process-design engineers to calculate fluid-phase equilibria as required in the design of distillation, extraction, and similar operations[39]. The UNIQUAC equation, using only two adjustable parameters per binary in addition to the pure-component parameters reflecting the sizes and outer surface areas of the molecules, is applied widely to calculate activity coefficients.

The UNIQUAC equations based on the representation of liquid-liquid interaction contain two parts: combinatorial and residual. The combinatorial part is a function of mole fraction, and molecular area and segment fractions. These structural parameters are based on the types and numbers of each component, and from the Van der Waals volume and surface areas of each component. The residual part of the activity coefficient is computed from mole fractions, molecular areas, and the energy of interactions between molecules. In this part, there are two parameters \( \tau_{ij} \) and \( \tau_{ji} \) representing the energy of interactions which are not able to be measured but must be evaluated empirically from vapor-liquid or liquid-liquid
equilibrium data[40]. Based on experimental data, Kato, et al[41] proposed that the values of the interaction energies can be correlated with a fourth-order function of \((q_j - q_i)/q_j\), where \(q_i\) and \(q_j\) denote the molecular surface area of the solute i and j, which represents binary alkane/alkane systems.

These interaction parameters could be obtained by fitting to experimental Vapor Liquid Equilibrium data, but it is not easy to obtain these parameters through experiments in the UNIQUAC equation. It is significant to find an alternative way to determine these parameters because considerable difficulty or even unfeasibility for some systems. Molecular simulation provides a promising approach to calculate the UNIQUAC parameters. Jonsdottir[42] and his coworkers found a new way to obtain these parameters, based on Molecular Mechanics (MM) with the consistent force field (CFF). In CFF model, two main types of interaction between atoms are considered: bonded interactions (chemical bonds) and non-bonded interactions (vander Waals and electrostatic), and parameters are optimized from experimental structural and spectroscopic data for pure components without any experimental data for mixtures. Interaction parameters for the UNIQUAC method have successfully been determined for \(n\)-butane/\(n\)-pentane, \(n\)-hexane/c-hexane, \(n\)-petane/\(n\)-hexane, \(n\)-pentane/acetone, acetone/cyclohexane, cyclohexane/cyclohexanone. Rasjuussen, et al[43] studied phase equilibria of carbohydrates in polar solvent with molecular mechanics method, the calculations applied in glucose/water and sucrose/water are in agreement with experiments.

Theoretical Interaction Energy Calculation (TIEC) was proposed by Cooke, et al[44] to model and measure solid-liquid and vapor-liquid equilibria of polyols and carbohydrates in aqueous solution. In this method, interaction parameters are determined between the solvent
and a model compound of similar size as the solvent molecule that contains one or two repeat units of the polymer molecule. The solubilities of monosaccharides xylose and galactose, and the disaccharides maltose monohydrate, cellobiose and trehalose dehydrate in water were measured at various temperatures in good agreement with experiment. Kjeld Rasmussen, et al[45] used this method to calculate solvent activities for the diethylketone/polypropylene system, giving results in good agreement with experimental values.

A search algorithm denoted as the Boltzmann Jump Procedure (BJP), which Klein, et al[46] and coworkers employed to determine UNIQUAC interaction parameters for alkane/amine system, was a more efficient method for sampling conformational space than the existing method before. The advantages of the BJP algorithm are that it uses a stochastic selection criterion, as in Monte Carlo simulations, in which up-hill jumps in energy are allowed, and it is thus possible to jump over energy barriers or saddle points. The temperature effect is included by calculating the Gibbs free energy at certain temperature for each pair of different locally stable conformers and using Boltzmann statistics to determine their relative probabilities. They also applied this method to carry out simulation for 1,2-ethanediol, 1,2-propanediol and glycerol, surrounded by water molecules[47]. This simulation was efficient and gave results in good agreement with the experimental data.

Due to the importance of CO₂ capture and utilization, many models for CO₂ adsorption[48-51] and capture[52-58] were proposed. Investigations were made on supercritical CO₂[59-62] and of CO₂ interacting with various organic molecules[63-86]. Studies were carried out for the CO₂-CH₃CH₂OH binary system using MD simulation[87, 88] or artificial neural network approach[89-92]. Studies of CO₂-CH₃OH binary system[93, 94]
were also reported, as CH₃OH plays an important role in the super critical CO₂ system. The role of CH₃OH has been to increase the polarity and solvent strength while retaining the sensitivity of solubility with respect to pressure and temperature. Moreover, CH₃OH can improve the selectivity of separation by preferentially interacting with one or more components and facilitating selective fractional separation[95]. Fluid phase equilibria of CO₂-CH₃OH[96] using Peng-Robinson and Soave-Redlich-Kwong mixing rules.

In this work, we utilize electronic structure theory calculations to study the interaction of two binary systems, CO₂-CH₃OH and CH₃OH-CH₃CH₂OH. The objectives of the work are to determine the accuracy of MP2, B3LYP, and B3LYP-D3 in calculating the interaction energy by comparing calculated phase diagrams of the CH₃OH-CH₃CH₂OH binary system and the experimental results, and to calculate the activity coefficients and phase diagram of CO₂-CH₃OH binary system to explore whether it is feasible to study the performance of co-solvent in supercritical system using molecular simulation.

2. Method

Activity coefficients and constructions of phase diagrams of binary systems rely heavily on the accuracy of the results of interaction energy, which is often the character of dispersion interaction. Dispersion interactions are generally defined as the interaction between instantaneous dipole moments within the electron distributions of two atoms or molecules[97, 98]. These interactions are ubiquitous, long-range attractive forces which act between separated molecules even in the absence of charges or permanent electric moments[99]. The dispersion interactions, so-called “weak interactions” are usually underestimated in most electronic structure based computations. However, their neglect may
sum up to significant relative contributions and then lead to qualitatively wrong conclusions. This will become more important with the size of the system, because dispersion effects have a much longer range than the opposite exchange repulsion.

Density functional theory (DFT) has gained attention as an ideally cost-effective method for calculations of many chemical systems. However, conventional DFT has problems in describing weak Van der Waals interactions. The general drawback of all common density functionals such as hydrids like the popular B3LYP, is that they cannot describe long-range electron correlations that are related for the dispersion forces[100]. The problem stems from the insufficiency of both exchange and correlation functional[101]. Lacks and Gordon[102] described the issue with the exchange functionals that the behavior of the exchange energy density at the low-density-high-gradient region had the crucial unignorable influence on the interaction energies.

Lot of effort has been exerted to develop a robust dispersion correction that can be applied to common standard density functionals without any specific fitting procedure. The approach should be as simple as possible, and in particular allow the easy calculation of energy gradients for efficient geometry optimization. DFT-D method (atom pairwise sum over $C_6R^6$ potentials[103, 104]) is this kind of dispersion correction, and has been improved by lots of researchers. The DFT-D3 (recent version of DFT-D) approaches have been quite successful in describing weak dispersion interactions among carbonic systems or organic materials[105-107]. The DFT-D3 method with B97-D or BLYP functionals yields interaction energies that are very close to the best CCSD(T) reference data. The approaches have been quite successful in describing weak dispersion interactions among carbonic systems or
organic materials. One of the improvements of DFT-D3 is that the complicated, non-additive and system-dependent exchange-repulsion, electrostatic, and induction effects are very accurately described by the DFT-D3 density functionals[108]. The development of DFT-D3 has provided chemists with a valuable tool for modeling a great number of systems.

In this work, MP2, B3LYP, and B3LYP-D3 calculations were performed with the 6-311+g(d,p) basis set using the Gaussian software to obtain the interaction energy of five pairs of interactions: CH$_3$OH-CH$_3$OH, CH$_3$OH-CO$_2$, CO$_2$-CO$_2$, CH$_3$OH-CH$_3$CH$_2$OH, and CH$_3$CH$_2$OH-CH$_3$CH$_2$OH. Convergence criteria are the same as these used in our previous work.[109-111] To obtain the most stable complex configuration, we used MD simulations to generate about 10 conformations for each pair and performed electronic structure calculations to determine the most stable ones.

The obtained interaction energies, $u_{ij}$, were used to calculate the activity coefficients from the UNIQUAC equations:

$$\ln \gamma_i = \ln \phi_i + \left(\frac{Z}{2}\right) q_i - \ln \frac{\phi_i}{x_i} + \frac{\phi_i}{x_i} \sum_j x_j l_j - q_i \ln \left(\sum_j \phi_j \tau_{ji}\right) + q_i - q_i \sum_k \theta_i \tau_{ijk}$$

$$l_i = \frac{Z}{2} (r_i - q_i) - (r_i - 1)$$

$$Z = 10$$

$$\phi_i = \frac{r_i x_i}{\sum_j r_j x_j}$$

$$\theta_i = \frac{q_i x_i}{\sum_j q_j x_j}$$

$$\tau_{ji} = \exp\left(-\frac{u_{ji} - u_{ij}}{RT}\right)$$

where R is the gas constant, T is temperature, $x_i$ is mole fraction of component i, $\gamma_i$ is activity coefficient of i, $r_i$ is a parameter related to the Van der Waals Volume, $q_i$ is a parameter related to the Van der Waals surface area, and $u_{ij}$ is the interaction energy between molecules i and j.
In order to obtain activity coefficients, in addition to the interaction energy mentioned above, two other fundamental structural parameters \( r_i \) and \( q_i \) are required. We used the method proposed by Bondi A, et al [112] to calculate them:

\[
\begin{align*}
    r_i &= V_i^*/(15.17 \text{cm}^3 \text{mol}^{-1}) \\
    q_i &\approx A_i^*/(2.5 \times 10^9 \text{m}^2 \text{mol}^{-1})
\end{align*}
\]

where \( V_i^* \), \( A_i^* \) are mole Van der Waal volume and surface area. These two parameters are obtained in Materials Studio. The three parameters, vdW Scale factor, Connolly radius, solvent radius, were needed for the calculations of these parameters. The vdW scale factor is a factor specified to uniformly modify all van der Waals radii, representing a hard shell into which a probe may not pass. The Connolly radius is the radius of the Connolly probe used in calculating the atom volume field. The solvent radius is the solvent probe radius used to display solvent and accessible solvent surfaces. The van der Waals volume and surface are the volume and surface that intersect with the vdw radii of the atoms in the structure. This is equivalent to a solvent surface with a solvent probe radius of zero, a Connolly surface with a Connolly probe radius of zero and vdW scale factor of 1. The phase diagrams were generated based on the Raoult’s law.

3. **Result and discussion**

3.1. The van der Waals parameters related to the volume \( (r_i) \) and area \( (q_i) \) of methanol, ethanol, and carbon dioxide

The van der Waals’ contours of three molecules, methanol, ethanol, and carbon dioxide were obtained from the Materials Studio and are depicted in Figure 1. The results of \( r_i, q_i \) are provided in Table 1. These parameters together with the interaction energies will be
used to determine the activity coefficients that are used to construct the phase diagram. As shown in Table 1, the values of both parameters decrease from ethanol to methanol then to carbon dioxide.

![Figure 1](image)

**Figure 1.** Description of ethanol (left), methanol (middle), and carbon dioxide (right).

<table>
<thead>
<tr>
<th>Molecule</th>
<th>r_i</th>
<th>q_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH$_3$CH$_2$OH</td>
<td>2.1281</td>
<td>1.9483</td>
</tr>
<tr>
<td>CH$_3$OH</td>
<td>1.4603</td>
<td>1.4279</td>
</tr>
<tr>
<td>CO$_2$</td>
<td>1.3213</td>
<td>1.2747</td>
</tr>
</tbody>
</table>

3.2. methanol-ethanol system

In this research, methanol-ethanol system was studied to examine the accuracy of interaction energy using three methods and the results are provided in Figures 2-5 and Tables 2-4. For the methanol-ethanol binary system, three $u_{ij}$ pairs are required: $u_{\text{ethanol-ethanol}}$, $u_{\text{ethanol-ethanol}}$, $u_{\text{ethanol-ethanol}}$. Take $u_{\text{ethanol-ethanol}}$ for an example, it represents the energy difference between the ethanol-ethanol stable dimer and the sum of two single ethanol molecules. Figures 2-4 show the stable structures of ethanol-ethanol, methanol-ethanol, and methanol-methanol pairs. For each pair, two different stable structures are shown after
optimizing plentiful different initial structures. It is revealed that the same character of the six stable structures is the H-bond formation. As such, the H-bond contributed dominantly to the interaction energy of the pairs. The difference between configuration a and b is on the spatial angles of alkyl groups. To illustrate each structure more clearly, we also show each structure at a different angle of view: in first one (for example a) the H-bond is parallel to the paper, while the second one (like a’) is perpendicular. The optimized energies from three methods are shown in Tables 2-4. For each pair, structure b is more stable than a with MP2 and B3LYP calculations; but when adding dispersion energy, structure a is more stable than b, and one exception is that MP2 and B3LYP results for methanol-methanol are almost the same under similar structures.

**Figure 2.** (a) the optimized structure of ethanol-ethanol interaction. (a’).the same structure to a, but from different angle of view. (b) the second optimized structure of ethanol-ethanol interaction. (b’).the same structure to b, but from different angle of view.
### Table 2. Interaction energy (kJ/mol) of ethanol-ethanol pair

<table>
<thead>
<tr>
<th></th>
<th>Structure a</th>
<th>Structure b</th>
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<tbody>
<tr>
<td>MP2</td>
<td>-22.09</td>
<td>-22.14</td>
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<tr>
<td>B3LYP</td>
<td>-12.69</td>
<td>-13.27</td>
</tr>
<tr>
<td>B3LYP-D3</td>
<td>-27.77</td>
<td>-21.87</td>
</tr>
</tbody>
</table>

![Image of ethanol-ethanol interaction structures](image)

**Figure 3.** (a) the optimized structure of methanol-methanol interaction. (a’) the same structure to a, but from a different angle of view. (b) the second optimized structure of methanol-methanol interaction. (b’) the same structure to b, but from a different angle of view.

### Table 3. Interaction energy (kJ/mol) of methanol-methanol pair

<table>
<thead>
<tr>
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<th>Structure b</th>
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</thead>
<tbody>
<tr>
<td>MP2</td>
<td>-18.91</td>
<td>-18.91</td>
</tr>
<tr>
<td>B3LYP</td>
<td>-14.97</td>
<td>-14.98</td>
</tr>
<tr>
<td>B3LYP-D3</td>
<td>-22.72</td>
<td>-20.36</td>
</tr>
</tbody>
</table>
**Figure 4.** (a) the optimized structure of methanol-ethanol interaction. (a’).the same structure to a, but from different angle of view. (b).the second optimized structure of methanol-ethanol interaction. (b’).the same structure to b, but from different angle of view.

<table>
<thead>
<tr>
<th></th>
<th>Structure a</th>
<th>Structure b</th>
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<tbody>
<tr>
<td>MP2</td>
<td>-19.99</td>
<td>-20.75</td>
</tr>
<tr>
<td>B3LYP</td>
<td>-13.27</td>
<td>-14.96</td>
</tr>
<tr>
<td>B3LYP-D3</td>
<td>-24.29</td>
<td>-21.87</td>
</tr>
</tbody>
</table>

**Table 4. Interaction energy (kJ/mol) of methanol-ethanol pair**

Based on the \(r_i\), \(q_i\), \(u_{ij}\) results and the Raoult's law, phase diagrams of methanol-ethanol system are generated. Figure 5 shows the isotherm phase diagrams under three calculation methods comparing to experimental data (for each \(u_{ij}\), the more optimized one is selected). From the diagrams, it is obvious that the MP2 and B3LYP-D3 calculation are in better agreement with the experimental data. The results from B3LYP-D3 is even better than that from MP2.
3.3 methanol-carbon dioxide system

The most stable structures for CO$_2$-CO$_2$ and CO$_2$-CH$_3$OH pairs are shown in Fig. 6. It is found that carbon atoms and oxygen atoms are on the same plane to form the most stable structures. Table 5 shows the interaction energy obtained by MP2, B3LYP, and B3LYP-D3, and MP2 results are more similar to that from B3LYP-D3 than that from B3LYP. We also show in Table 5 the MP2 results from different basis sets.
Figure 6. The optimized structure of (a) carbon dioxide-carbon dioxide interaction, (b) carbon dioxide-methanol interaction.

Table 5. Interaction energy (kJ/mol) of CO₂-CO₂ and CO₂-CH₃OH pair

<table>
<thead>
<tr>
<th>Interaction energy</th>
<th>CO₂-CO₂</th>
<th>CO₂-CH₃OH</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP2</td>
<td>-5.04</td>
<td>-13.8</td>
<td>This work</td>
</tr>
<tr>
<td>B3LYP</td>
<td>-1.42</td>
<td>-8.81</td>
<td>This work</td>
</tr>
<tr>
<td>B3LYP-D3</td>
<td>-4.95</td>
<td>-15.1</td>
<td>This work</td>
</tr>
<tr>
<td>MP2/CBS</td>
<td>-5.78</td>
<td></td>
<td>[113]</td>
</tr>
<tr>
<td>MP2/8s6p4d1f</td>
<td>-4.94</td>
<td></td>
<td>[114]</td>
</tr>
<tr>
<td>MP2/6-311+G(2df)</td>
<td>-5.40</td>
<td></td>
<td>[115]</td>
</tr>
</tbody>
</table>

Based on the parameters obtained from electronic structure calculations, we constructed the phase diagram for the methanol-carbon dioxide phase diagram. Figure 7 shows the Isothermal phase diagram of CO₂-CH₃OH system under 230K. Based on the results from methanol-ethanol binary system, the results from B3LYP-D3 prediction is the best. Indeed, the results in Fig. 7 shows that B3LYP-D3 predicted the low solubility, which is the case experimentally.[116]
Figure 7. Isothermal phase diagram of methanol-carbon dioxide system at 230K, where the red, green, and blue lines denotes the results based on the MP2, B3LYP, and B3LYP-D3 calculations, respectively. Lines with squares are for liquid phase and lines with circles are for gas phase.

Figure 8 summarizes all $u_{ij}$ calculation results from the five pairs obtained from the three methods. It is obvious that both the difference between B3LYP and MP2 and the difference between B3LYP-D3 and MP2 increase with the increment of interaction energy. It seems that B3LYP-D3 captures the H-bond better than MP2.
Figure 8. Comparison of the interaction energy among the MP2, B3LYP, and B3LYP-D3 calculations on five pairs of binary systems: 1:CO₂-CO₂, 2:CO₂-CH₃OH, 3:CH₃OH-CH₃OH, 4: CH₃OH-C₂H₅OH, and 5: C₂H₅OH- C₂H₅OH. Red, blue, and green lines denote the B3LYP, the MP2, and the B3LYP-D3 results, respectively.

4. Conclusions

Electronic structure calculations using MP2, B3LYP, and B3LYP-D3 methods were carried out to obtain interaction energies for five pairs of interactions: CH₃OH-CH₃OH, CH₃OH-CO₂, CO₂-CO₂, CH₃OH-CH₃CH₂OH, and CH₃CH₂OH-CH₃CH₂OH. Activity coefficients and phase diagrams were constructed for two binary systems: CH₃CH₂OH-CH₃OH and CH₃OH-CO₂. Comparison of CH₃CH₂OH-CH₃OH phase diagrams between the calculated and the experimental results indicate that B3LYP-D3 is the best to obtain the interaction energies. The general agreement between the B3LYP-D3 and the experimental phase diagram illustrates the promising feature of molecular simulation of
phase diagram of binary systems. Compared to the experimental data of CH$_3$CH$_2$OH-CH$_3$OH
system, we show that the molecular simulation approach is promising to obtaining activity
coefficients. The current results showed that CO$_2$ and CH$_3$OH are hardly mixed under the
current simulation conditions.

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