Cation disorder dominates the defect chemistry of high-voltage LiMn\textsubscript{1.5}Ni\textsubscript{0.5}O\textsubscript{4} (LMNO) spinel cathodes

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Abstract

High-voltage spinel LiMn\textsubscript{1.5}Ni\textsubscript{0.5}O\textsubscript{4} (LMNO) can exist in a Mn/Ni ordered \textit{P}4\textsubscript{3}32 or disordered \textit{Fd\textsubscript{3}m} arrangement with a majority of literature studies reporting improved electrochemical performance for the disordered phase. Through modifying synthesis conditions, the Mn/Ni ordering can be tuned, however oxygen and Mn\textsuperscript{3+} stoichiometries are also affected, making it difficult to decouple these responses and optimise performance. Here, we investigate all intrinsic defects in \textit{P}4\textsubscript{3}32 LMNO under various growth conditions, using density functional theory (DFT) calculations. We find that the majority of defects are deep and associated with small polarons (Mn\textsuperscript{3+}, Mn\textsuperscript{2+} and...
Ni$^{3+}$) formation. The tendency for cation disorder can be explained by the low formation energy of the antisite defects and their stoichiometric complexes. The intrinsic Fermi level of LMNO varies from moderately $n$-type under oxygen-poor conditions to weakly $p$-type under oxygen-rich conditions. Our work explains experimental observations (e.g. the Mn/Ni disorder) and provides guidelines for defect-controlled synthesis.

**Introduction**

High-voltage spinel LiMn$_{1.5}$Ni$_{0.5}$O$_4$ (LMNO) is a promising cobalt-free cathode material which can deliver high energy density ($650 \text{ W h kg}^{-1}$) due to its high operating voltage $\sim 4.7 \text{ V (vs Li}^+/\text{Li})$ arising from the Ni$^{2+}$/Ni$^{4+}$ couple. As an important derivative of the extensively studied LiMn$_2$O$_4$ mixed-valence spinel (LMO), LMNO has structural and chemical similarities with LMO. In both structures Li$^+$ occupy tetrahedral (Td) sites, and the transition metals (TM) occupy octahedral (Oh) sites of the cubic close-packed oxygen arrays. In addition, they have 3D diffusion channels where Li$^+$ migrate through “gate sites”: six-member cation rings perpendicular to the diffusion path. LMNO, however, has improved energy density and cycling stability compared to LMO partly due to a reduced prevalence of Mn$^{3+}$ in LMNO. Mn$^{3+}$ gives rise to pronounced Jahn-Teller (JT) distortion and Mn dissolution via disproportionation reactions ($2\text{Mn}^{3+} \rightarrow \text{Mn}^{2+} + \text{Mn}^{4+}$) leading to structural instability.

Depending on synthesis conditions, LMNO can show site disorder in the octahedral TM cation sites (see Figure 1). A lower synthesis/anneal temperature ($\sim 700 \text{ °C}$) leads to the formation of the ordered ($P4_332$) phase, where Ni and Mn are located at 4$b$ and 12$d$ Wyckoff sites. In the ideal $P4_332$ LMNO, all Mn species present as redox-inactive Mn$^{4+}$. The Ni$^{2+}$/Ni$^{4+}$ couple gives rise to a distinctive voltage at $\sim 4.7 \text{ V}$. Higher synthesis temperatures ($> 700 \text{ °C}$) promote the formation of the $Fd\bar{3}m$ LMNO with Mn and Ni disordered across the 16$d$ sites. High temperature synthesis may also drive oxygen vacancy formation, which is expected to be compensated by the reduction of Mn$^{4+}$ to Mn$^{3+}$, leading to an
off-stoichiometric composition \((LiMn_{1.5}Ni_{0.5}O_{4-\delta})\) with the formation of rock-salt phase (e.g. \(Li_xNi_{1-x}O\)) impurities.\(^{11,13}\) \(Mn^{3+}\) is redox active and the \(Mn^{3+}/Mn^{4+}\) redox reaction introduces an additional plateau at \(\sim 4\) V on the voltage profile.\(^4\) The disordered phase generally exhibits better cycling stability and rate capability than the ordered phase, however the connection between disorder and performance has remained unclear, with increasing attention placed on understanding the role of cation disorder.\(^{14-22}\)

\[\text{Figure 1: Representative structures (with Wyckoff positions) of the } P4_332 \text{ ordered and } Fd\overline{3}m \text{ disordered LMNO.}\]

While some studies found that Mn/Ni disorder increases with oxygen vacancy and Mn\(^{3+}\) formation, others have suggested that the “disordered” phase from different studies is likely to be samples with partial cation disordering, a varying degree of oxygen content and impurity phases.\(^{13,19,20,23-25}\) Regardless, the correlation between cation disorder and oxygen content has made it difficult to distinguish the key driving factor for improved performance, illustrating the need to decouple these effects and study their impacts independently.

Density functional theory (DFT) is a key tool for studying the defect chemistry of functional materials.\(^{26,27}\) Previous defect studies\(^{17,28,29}\) have primarily focused on understanding the effects of lithium and oxygen vacancies on the \(P4_332\) and \(Fd\overline{3}m\) LMNO with various Mn/Ni orderings using a 56-atom supercell. Although they have provided useful insights on defect formation and cation disorder, these studies have various deficiencies: (i) ignoring the effect of atomic chemical potentials on the defect formation energies; (ii) neglecting
to address the spurious long-range Coulomb interactions between charged defects inherent within the supercell approach; (iii) calculating defect properties within a small supercell corresponding to unrealistically large defect concentrations.\textsuperscript{30,31}

In this paper, we present a comprehensive study of the energetics and behaviour of intrinsic defects in $P4_332$ LMNO. We first systematically study the magnetic ordering in bulk LMNO, before computing bulk properties and all stable competing phases in the Li-Mn-Ni-O chemical space to determine the phase stability region. We then calculate all intrinsic defects in all possible charge states, using Bader charges, charge densities and Madelung site potential analysis to characterise charge compensation mechanisms and identify small polaron formation. We also investigate a range of likely defect complexes and the migration of lithium defects. Finally, we discuss the effect of synthesis conditions on defect chemistry and how this can explain experimental observations and guide defect-controlled synthesis.

**Methodology**

**Computational details**

Calculations were performed using spin-polarised plane-wave DFT as implemented in the Vienna \textit{ab initio} Simulation Package\textsuperscript{32–34} (VASP) code (version 5.4.4). PAW pseudopotentials\textsuperscript{35,36} (version PBE\_5.4) of Li\textsubscript{sv}, Mn\textsubscript{pv}, Ni\textsubscript{pv} and O were used to model the core electrons. Calculations were performed with the GGA(PBEsol) + $U$ method\textsuperscript{37,38} where $U$ values for $d$-orbitals of Mn and Ni were set to 3.9 and 6.0 eV, respectively, based on previous literature studies.\textsuperscript{39,40} The initial structure of $P4_332$ LMNO obtained from Materials Project\textsuperscript{41} contains 56 atoms in a primitive cell with 8 formula units of LiMn$_{1.5}$Ni$_{0.5}$O$_4$. The structure was optimised using a plane-wave cutoff of 550 eV and a $\Gamma$-centered Monkhorst-Pack\textsuperscript{42} $k$-point sampling of $3 \times 3 \times 3$ (kpoint-spacing of 0.2528 Å$^{-1}$) following convergence tests with respect to energy cutoff and $k$-points. Tolerances of $10^{-5}$ eV and $10^{-2}$ eV Å$^{-1}$ were applied to electronic and ionic convergence respectively for bulk and defect calculations.
Pymatgen (version 2020.12.31)\textsuperscript{43,44} was used to investigate the magnetic ordering of the bulk structure. Additional calculations were performed using the HSE06 hybrid functional\textsuperscript{45} on the ferrimagnetic ground state\textsuperscript{46} LMNO structure (Mn↑ Ni↓) to compare with the PBEsol + $U$ results. A denser $\Gamma$-centered $4 \times 4 \times 4$ k-point mesh was used to produce the electronic density of states (DOS) and band structure (path from Bradley and Cracknell\textsuperscript{47}).\textsuperscript{48}

Defect calculations were conducted using the PBEsol + $U$ functional on a cubic $2 \times 2 \times 2$ (448-atom) supercell with $\Gamma$-only k-point grid. Performing calculations using hybrid functional would be practically infeasible with our chosen supercell size due to the exorbitant computational cost. The ferrimagnetic spin configuration was initialised for all defects with fixed-volume relaxations.\textsuperscript{49} The force tolerance was raised to $2 \times 10^{-2}$ eV Å$^{-1}$ for interstitial defects. Defect calculations set up and analysis were facilitated by DOPED.\textsuperscript{50,51} A lean version of the ShakeNBreak\textsuperscript{52} approach was used to aid the location of the ground-state defect structures.\textsuperscript{53,54} Notably, standard defect relaxations without this symmetry breaking were found to give higher-energy metastable structures (often without correct polaronic localisation) for many defects. Displacement distributions with standard deviations of 0.02 Å and 0.05 Å were tested on individual defect structures and the lowest energy relaxed structures were taken for further analysis. Climbing-image nudged elastic band (cNEB) method was used to calculate point defect migration barriers.\textsuperscript{55,56} All crystal structure diagrams were prepared using VESTA.\textsuperscript{57}

**Defect Analysis**

The formation energy of a defect $X$ in a charge state $q$ with respect to the host lattice is calculated using\textsuperscript{26,49,58}

$$E_f(X^q) = E_{\text{tot}}(X^q) - E_{\text{tot}}(\text{host}) - \sum_i n_i \mu_i + q(E_{\text{vbm}} + \mu_e) + \Delta^q,$$  \hspace{1cm} (1)
where $E_{\text{tot}}(X^q)$ and $E_{\text{tot}}(\text{host})$ are the total energies of a defect supercell and the defect-free supercell respectively. $\mu_i$ is the atomic chemical potential of species $i$ and $n_i$ is the number of atoms of species $i$ which have been added ($n_i > 0$) or removed ($n_i < 0$) to form the defects. $\mu_i$ is subject to thermodynamic constraints and can be used to describe experimental conditions. $\mu_e$ is the electronic chemical potential (i.e. the Fermi level), referenced to the valence band maximum (VBM) of the host ($E_{\text{vbm}}$). $\Delta^q$ is a correction term to align the electrostatic potentials of the defect-free and defect supercells and to account for the finite-cell-size effect on the total energies of charged defects.\textsuperscript{30,58} The Freysoldt, Neugebauer and Van de Walle (FNV)\textsuperscript{31,59} charge correction scheme was used. This involved computing the static dielectric constant ($\epsilon_0$) with contributions from high-frequency electronic/optical ($\epsilon_{\text{optic}}$) and low-frequency lattice/ionic ($\epsilon_{\text{ionic}}$) contributions: $\epsilon_0 = \epsilon_{\text{optic}} + \epsilon_{\text{ionic}}$.\textsuperscript{60} (see Table S1-S3)

The chemical potential stability limits of LMNO are defined by the equality

$$\mu_{\text{Li}} + 1.5\mu_{\text{Mn}} + 0.5\mu_{\text{Ni}} + 4\mu_{\text{O}} = \Delta H^f(\text{LiMn}_{1.5}\text{Ni}_{0.5}\text{O}_4),$$  \hspace{1cm} (2)

where $\Delta H^f(\text{LiMn}_{1.5}\text{Ni}_{0.5}\text{O}_4)$ is the formation enthalpy of LMNO. The potential formation of Li-Mn-Ni-O competing phases imposes thermodynamic constraints on $\mu_i$, namely that the stoichiometric sum of $\mu_i$ of a competing phase must not be higher than its formation energy.\textsuperscript{30,49} For example, to avoid the formation of NiO would require

$$\mu_{\text{Ni}} + \mu_{\text{O}} \leq \Delta H^f(\text{NiO})$$  \hspace{1cm} (3)

to be satisfied.\textsuperscript{61,62} The range of Li, Mn, Ni and O chemical potential values where LMNO is thermodynamically stable can therefore be determined in this way using CPLAP.\textsuperscript{62,63} The atomic chemical potential diagrams\textsuperscript{64,65} for the host was constructed by exploring the stable competing Li-Mn-Ni-O phases in Materials Project (accessed date: 22 Nov 2020). Magnetic compounds were relaxed with the ground state magnetic ordering, according to previous
Elemental reference energies were obtained from the constituent elements in their standard states e.g. $O_2(g)$.

In a solid, defects typically occur in multiple charge states with positive/negative charge states involving removal/addition of electrons. The actual position of Fermi level ($\mu_e$) is determined by minimising the system energy under self-consistent charge neutrality condition using py-sc-fermi:$^{49,77,78}$

$$\sum_{X,q} qc(X^q) + n_h - n_e = 0,$$

where the net charge of a system takes into account all defect species ($X$) with charge $q$, free electrons ($n_e$) and free holes ($n_h$). The free carrier concentrations are determined according to the Fermi-Dirac distribution function.$^{79}$ The concentration $c$ of a defect (in thermodynamic equilibrium) at temperature $T$ is related to its formation energy ($E_f$):$^{49,58}$

$$c = N_{\text{states}} \exp \left( \frac{-E_f}{k_B T} \right),$$

where $N_{\text{states}}$ is the density of available microstates $N_{\text{states}} = N_{\text{sites}} N_{\text{config}}$. $N_{\text{sites}}$ is the number of symmetry-inequivalent sites in the lattice per unit volume where the defect can be incorporated, $N_{\text{config}}$ is the number of equivalent configurations (i.e. degeneracy) and $k_B$ is the Boltzmann’s constant.

Madelung potentials can suggest where ions in a structure are situated in the energy landscape and the likelihood of redox.$^{80}$ They are the electrostatic energies by approximating ions as point charges and can be expressed by a sum of pair-wise interactions: $E_M = \sum_{(i,j)} \frac{z_i z_j}{r_{ij}}$, where $z_i$ and $z_j$ are the formal charges on ion $i$ and $j$ respectively and $r_{ij}$ is the interionic distance. Cations have negative potentials and anions have positive potentials.$^{81}$ A higher potential corresponds to a greater attraction to negative charge (and thus electron polaron formation), and a lower potential to positive charge (thus hole polaron formation).

Defect complex calculations involved constructing supercells with two point defects at a range of pair distances (e.g nearest and farthest). We performed defect complex analysis.
on the lowest energy configurations, which are those with the smallest pair distance. A
generalised equation $R_1 + R_2 \iff P$ was used to represent their formation, where $R_1$ and
$R_2$ are reactants (isolated defects) and $P$ is the product (complex). The enthalpy change for
the forward reaction

$$\Delta H = E^f(P) - E^f(R_1) - E^f(R_2),$$

(6)

was used to approximate complex association energy, where $E^f(X)$ is the formation energy
of a corresponding defect (complex) species $X$.

The equilibrium concentration of complexes can be approximated according to the mass-
action law\textsuperscript{49,82} using

$$c_{AB} = \frac{c_A c_B}{N_{\text{states}}} \exp \left( -\frac{\Delta H}{k_B T} \right),$$

(7)

where $c_A$ and $c_B$ are concentrations of isolated defects $A$ and $B$, respectively.

The Gibbs free energy change $\Delta G$ for complex formation can be calculated following:

$$\Delta G = \Delta H - T\Delta S.$$

(8)

The dominant entropic contribution to the free energy of defects is the configurational en-
tropy, given by:

$$S = k_B \ln W = k_B \ln \left( N_{\text{states}} c_X \right),$$

(9)

where $W$ is the number of microstates and is related to the concentration of defect $X$
($c_X$) with potential microstates $N_{\text{states}}$. Accounting for the reduction in the configurational entropy upon complex association, and employing the assumptions of (i) equal $N_{\text{states}}$ for point and complex defects and (ii) majority and minority constituent point defects ($c_B \gg c_A$),
a high degree of complex association ($c_{AB} \geq c_A$) is expected when the magnitude of the association energy ($|\Delta H|$) is larger than the entropic cost ($-T\Delta S$), approximated by the
critical association energy $\Delta E_{\text{crit}}$. The relationship between critical association energy $\Delta E_{\text{crit}}$, temperature $T$, entropy $\Delta S$, and the Boltzmann constant $k_B$ is given by:

$$- T \Delta S \approx \Delta E_{\text{crit}} = k_B T \log \left( \frac{N_{\text{states}}}{c_B} \right).$$

(10)

Results and discussion

Bulk properties

Magnetic ordering

With the presence of open-shell TM cations in the structure, there are many ways to orient the unpaired electrons. Hence, the starting point towards obtaining the bulk properties of LMNO is to find the correct ground state structure. Erroneous magnetic ordering assignment can lead to an increased computational cost, incorrect ground state energy and electronic structure. We have considered all possible magnetic orderings in the 56 atom conventional $P4_332$ LMNO cell. The ground state magnetic ordering was calculated to be ferrimagnetic (FiM) with Mn↑ Ni↓, in line with experimental findings. More details can be found in Figure S1.

Crystal structure

The ground state FiM structure was optimised with PBEsol + $U$ and HSE06 methods separately and the calculated (cubic) lattice constants are in good agreement with experimental data (Table 1). The local coordination around Ni shows six oxygens at equal distances of 2.05 - 2.06 Å. The local Mn coordination is distorted with three inequivalent Mn-O distances and is a consequence of Mn coordinating to two symmetry-inequivalent oxygens in the $24e$ sites (surrounded by $2\text{Mn}^{4+}$, $\text{Ni}^{2+}$ and $\text{Li}^+$) and $8c$ sites (surrounded by $3\text{Mn}^{4+}$ and $\text{Li}^+$). This distortion ($\Delta d_{\text{Mn-O}} = 0.06$ Å) is minor compared to the pronounced JT distortion for Mn$^{3+}$ observed in LMO spinel, with $\Delta d_{\text{Mn-O}} = 0.28$ Å.85
Table 1: Structural parameters obtained from PBEsol + $U$, HSE06 calculations and from experimental data.$^{9,86}$

<table>
<thead>
<tr>
<th>Theory/Exp</th>
<th>PBEsol + $U$</th>
<th>HSE06</th>
<th>Exp</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lattice constant [Å]</td>
<td>8.17</td>
<td>8.15</td>
<td>8.166(3)</td>
</tr>
<tr>
<td>(Ni-O) × 6 [Å]</td>
<td>2.05</td>
<td>2.06</td>
<td>2.068(2)</td>
</tr>
<tr>
<td>(Mn-O) × 2 [Å]</td>
<td>1.88</td>
<td>1.87</td>
<td>1.870(4)</td>
</tr>
<tr>
<td>(Mn-O) × 2 [Å]</td>
<td>1.91</td>
<td>1.90</td>
<td>1.914(1)</td>
</tr>
<tr>
<td>(Mn-O) × 2 [Å]</td>
<td>1.94</td>
<td>1.93</td>
<td>1.932(4)</td>
</tr>
</tbody>
</table>

Electronic structure

Figure 2: The electronic density of states of $P4_332 \text{LiMn}_{1.5}\text{Ni}_{0.5}\text{O}_4$ obtained from (a) PBEsol + $U$ and (b) HSE06 calculations.

Figure 2 shows the electronic density of states (DOS) using PBEsol + $U$ and HSE06 functionals. Both functionals indicate that the VBM is dominated by Ni and O states in the spin-down channel and that the CBM is dominated by Mn and O states in the spin-up channel. The VBM has a larger contribution from O 2$p$ states than Ni 3$d$ states, even with HSE06 where the Ni contribution is increased (from 24% - 34%) (Table 2). This phenomenon is also observed in other compounds such as LiNiO$_2$ where oxygens are bonded to late-3$d$ metals with increased crystal-field splitting. This would decrease the tendency towards hole localisation and redox processes are likely to involve Ni-O hybridised bands strong in O character.$^{58,87}$ However, on a per-atom basis, the contribution from Ni states is at least two times
higher than that of O at VBM. This is consistent with experimental observations where Ni was shown to be redox-active.\textsuperscript{1}

Table 2: Percentage of transition metals (Mn and Ni) and oxygen contributions at the valence band maximum (VBM) and conduction band minimum (CBM) in $P4_332$ LiMn$_{1.5}$Ni$_{0.5}$O$_4$ from PBEsol + $U$ and HSE06 calculations.

<table>
<thead>
<tr>
<th>Spin channel</th>
<th>VBM spin ↓</th>
<th>CBM spin ↑</th>
<th>VBM spin ↓</th>
<th>CBM spin ↑</th>
</tr>
</thead>
<tbody>
<tr>
<td>O</td>
<td>71%</td>
<td>24%</td>
<td>61%</td>
<td>21%</td>
</tr>
<tr>
<td>Ni</td>
<td>24%</td>
<td>0%</td>
<td>34%</td>
<td>0%</td>
</tr>
<tr>
<td>Mn</td>
<td>4%</td>
<td>70%</td>
<td>4%</td>
<td>72%</td>
</tr>
</tbody>
</table>

The CBM has a greater proportion of Mn 3$d$ states than O 2$p$ states with both functionals. The presence of oxygen character at the CBM suggests polar covalent Mn-O bonding with hybridised states. Since Mn and Ni dominate the CBM and VBM respectively, charge localisation would occur on Mn (for electron polarons) and Ni (for hole polarons). The Mn 3$d$ states in the spin-up channel at the CBM also suggest that Mn would adopt a high-spin configuration when an additional electron is populated to form Mn$^{3+}$ ($d^4$). Liu et al.\textsuperscript{88} suggested that the 2$p$ unoccupied states from O is related to the O$_2$ release phenomena under thermal treatment, the formation of the rock-salt impurity phases Li$_x$Ni$_{1-x}$O and high-temperature phase $Fd\bar{3}m$ LiMn$_{1.5}$Ni$_{0.5}$O$_{4-\delta}$ where Mn$^{3+}$ is also redox-active. Ryoo et al.\textsuperscript{89} related the bonding characteristics of Mn-O and Ni-O, both hybridised with a similar degree of covalency, to the propensity for cation disordering in LMNO at elevated temperatures. In contrast, cation ordering is strongly preserved in $P4_332$ LiGe$_{1.5}$Ni$_{0.5}$O$_4$ despite Ge$^{4+}$ having the same ionic radius to Ni$^{4+}$, even at a very high temperature (e.g. 950 °C), due to the contrasting bonding motifs of Ge-O (ionic) and Ni-O (polar covalent).\textsuperscript{89}

Figure S2 shows the electronic band structure and both functionals show weak band dispersion, reflecting strong electron-electron correlations. LMNO is calculated to be a wide-gap semiconductor, with an indirect band gap of 1.76 and 3.17 eV determined using PBEsol + $U$ and HSE06 functionals, respectively. The hybrid approach, in theory, should give a more
reliable prediction of fundamental band gap compared to PBEsol+U, in which the magnitude of the band gap is influenced by the choice of $U$ values. There is limited experimental data on the fundamental band gap to compare with the current study, though an optical gap between 1.2-1.3 eV has been reported experimentally. The large difference between the fundamental and optical band gaps is likely due to the strong excitonic effects in flat $d$-band wide gap materials.

**Phase stability**

Thermodynamic stability of LMNO was determined by computing formation energies of all stable competing phases in the Li-Mn-Ni-O phase diagram, where $\text{Li}_2\text{Ni}_2\text{O}_3$ was obtained using structure prediction. Formation energies of all stable competing phases are summarised in Table S4-S5.

The host material is thermodynamically stable and there are 20 intersection points bounding the stability region in the chemical potential space. Each intersection point corresponds to a facet in the phase diagram where the corresponding phases are in equilibria with the host (see Table S6). O-rich ($\mu_O = 0$ eV) oxidizing conditions are usually Mn- and Ni-poor, and favour $p$-type (acceptor) defect formation. O-poor, Mn- and Ni-rich reducing conditions favour $n$-type behaviour, which can occur under higher temperature and/or reduced oxygen partial pressure and/or under the presence of oxygen-reducing agents (e.g. LiH).

The quaternary phase space of LMNO yields a 4D stability region, thus to allow visualisation on a 2D plot, we show in Figure 3 the stability region where $\mu_{\text{Li}}$ and $\mu_{\text{Mn}}$ were set to the average values over all atomic chemical potentials intersection points. LMNO is stable across a small $\mu_{\text{Ni}}$ range, a moderate $\mu_{\text{Li}}$ range and a larger $\mu_{\text{Mn}}$ range. The size of the stability field reflects the ease of synthesizing the target product experimentally and determines the range of variation in defect formation energy over all possible growth conditions. Thus, there is a greater sensitivity of defect concentrations to Mn-related than Ni-related growth environments. The ease of nickel oxide impurity phase formation upon synthesizing LMNO
Figure 3: The chemical potential space of $P4_{3}32 \text{LiMn}_{1.5}\text{Ni}_{0.5}\text{O}_{4}$, (a) where chemical potential of Li was set to an average value of \(-3.35\) eV; (b) where chemical potential of Mn was set to an average value of \(-6.42\) eV. Yellow/dark-blue colour shadings represent $p/n$-type conditions.

**Defect structure and energetics**

We investigated a range of intrinsic point defects in all possible charge states. This includes all vacancies ($V_{\text{Li}}$, $V_{\text{TM}}$ and $V_{\text{O}}$), antisites ($\text{Li}_{\text{TM}}$, $\text{TM}_{\text{Li}}$ and $\text{TM}_{\text{TM}}$) and interstitials ($\text{Li}_{\text{i}}$, $\text{TM}_{\text{i}}$ and $\text{O}_{\text{i}}$); ‘TM’ refers to a transition metal species (Mn or Ni) and vacancies on symmetry inequivalent oxygens in the 24e and 8c sites are denoted by $V_{\text{O},1}$ and $V_{\text{O},2}$ respectively. For interstitials, we tested six symmetry inequivalent interstitial positions and their corresponding Wyckoff positions are specified in Table S7. In addition, the potential formation of peroxide species from oxygen interstitials was tested by placing $\text{O}_{\text{i}}$ at distances $\sim 1.4$ Å from $\text{O}_{\text{host}}$ and relaxing the defect geometry from this structure. We also considered a range defect complexes including the antisite-pair defects ($\text{Li}_{\text{Mn}}-\text{Mn}_{\text{Li}}$, $\text{Li}_{\text{Ni}}-\text{Ni}_{\text{Li}}$ and $\text{Mn}_{\text{Ni}}-\text{Ni}_{\text{Mn}}$), vacancy-pair (Schottky) defects (e.g. $V_{\text{O}}-V_{\text{Li}}$, $\text{Mn}_{\text{Ni}}-V_{\text{Li}}$) and a lithium Frenkel-pair ($\text{Li}_{\text{i}}^{+}-V_{\text{Li}}^{-}$), due to the low formation energies (and thus high concentrations) of the constituent point defects.
Figure 4: Formation energies for each intrinsic point defect in the lowest energy charge state at the self-consistent Fermi levels over 20 chemical potential conditions for $P4_332$ LiMn$_{1.5}$Ni$_{0.5}$O$_4$. The chemical potential conditions ($A\to T$) are arranged in order of increasing self-consistent Fermi level determined from all intrinsic point defects. Interstitial defects are represented by dotted lines; antisite and vacancy defects are shown by solid lines.

To illustrate the complex nature of defect chemistry of LMNO, in Figure 4 we have plotted the formation energies of all intrinsic defects in their lowest energy charge states at the self-consistent Fermi levels over the 20 chemical potential conditions from O-rich to O-poor (increasing self-consistent Fermi levels). With the inverse-exponential relationship between defect concentration and formation energy (Equation 5), the large variation in formation energies across different chemical potentials indicates a strong sensitivity of defect concentrations to growth conditions in LMNO. Transitioning from O-rich to O-poor ($A\to T$), the formation energies of several $n$-type defects (e.g. oxygen vacancies) decrease sharply (by $\geq 0.5$ eV). Therefore, the formation of these defects are highly sensitive to the oxygen
and/or temperature environments. Formation energies of Mn$_{Ni}$ and Li$_{Ni}$ defects remain be the lowest in each chemical potential condition, thus the disordering in LMNO is greatly driven by those defects.

Figure 5: Formation energies as a function of Fermi level for intrinsic defects in $P4_332$ LiMn$_{1.5}$Ni$_{0.5}$O$_4$ under (a) O-rich (the most intrinsically $p$-type) condition A and (b) O-poor (the most intrinsically $n$-type) condition T. The self-consistent Fermi levels are denoted by the black dashed lines. Interstitial defects (dotted lines) and antisite and vacancy defects (solid lines) with the lowest energy charge states are shown, for clarity.

Figure 5 shows the defect transition levels under two contrasting conditions A (O-rich; $p$-type) and T (O-poor; $n$-type). The gradient of each line corresponds to defects’ charge states. The Fermi level where the formation energy of a defect $X$ in two charge states $q_1/q_2$ becomes equal is termed as the thermodynamic transition level $\epsilon(q_1/q_2)$ where $X^{q_1}$ and $X^{q_2}$ are in thermodynamic equilibrium, and is represented by a filled circle.

Below we discuss the lowest-formation-energy (highest concentration) defects at the self-consistent Fermi level, and their likely influences on material properties.
**Vacancies and antisites**

From Figure 5, it is clear that among defects with $E_f < 1$ eV, antisite defects dominate over other types of defects such as vacancies and interstitials. Under condition A, the formation energy of Ni\textsubscript{Mn}\textsuperscript{0} is the lowest, at 0.33 eV. Followed by that are Ni\textsubscript{Li}\textsuperscript{+} (0.64 eV) and Li\textsubscript{Ni}\textsuperscript{-} (0.75 eV). Under condition T, Mn\textsubscript{Li}\textsuperscript{+}, Mn\textsubscript{Ni}\textsuperscript{0}, Li\textsubscript{Ni}\textsuperscript{-} and Ni\textsubscript{Li}\textsuperscript{0} have similar formation energies (0.5-0.7 eV).

Antisite behaviour is found to depend on the change in local coordination environment for the TM involved (e.g. O\textsubscript{h} → O\textsubscript{h}, O\textsubscript{h} → T\textsubscript{d}). For Mn\textsubscript{Ni} and Ni\textsubscript{Mn} with no change in coordination (O\textsubscript{h} → O\textsubscript{h}) at the defect site, fully ionised charge states (+2 and -2 respectively) give Mn\textsuperscript{4+} and Ni\textsuperscript{2+} species (see Table 3), while other charge states correspond to the addition of localised electrons/holes to the $d$ orbitals of Mn/Ni, as confirmed by Bader charge density and the site-projected magnetisation analysis.\textsuperscript{98,99} For example, the magnetisation on the Ni\textsubscript{Mn}\textsuperscript{0} defect site is 0.1 $\mu_B$, suggesting no unpaired electrons and corresponds to Ni\textsuperscript{4+} ($d^6$) low-spin configuration. The local geometry near Ni\textsuperscript{4+} is slightly distorted with Ni\textsuperscript{4+}-O ranging between 1.85 - 1.88 Å which is shorter than that of Ni\textsuperscript{2+}-O = 2.06 Å (see Table 1). Similarly, the Mn\textsubscript{Ni}\textsuperscript{0} defect site corresponds to Mn\textsuperscript{2+} ($d^5$) high-spin of -4.5 $\mu_B$. The sign change in the magnetisation reflects a spin flip relative to Mn in the bulk and this is related to the complex orbital interactions exchanging the spin information.
Table 3: Antisite defects (Mn$_{Ni}$, Ni$_{Mn}$, Mn$_{Li}$ and Ni$_{Li}$) in different charge states, site projected magnetisation, Bader charge and the species identities at defect sites.

<table>
<thead>
<tr>
<th>Defect</th>
<th>Magnetisation [µB]</th>
<th>Bader charge$^a$</th>
<th>Species</th>
<th>Average TM-O bond lengths [Å]$^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mn$_{Ni}^{2+}$</td>
<td>3.1</td>
<td>1.84</td>
<td>Mn$^{4+}$ ($d^4$)</td>
<td>1.92</td>
</tr>
<tr>
<td>Mn$_{Ni}^+$</td>
<td>-3.7</td>
<td>1.70</td>
<td>Mn$^{3+}$ ($d^4$)</td>
<td>2.02</td>
</tr>
<tr>
<td>Mn$_{Ni}^{0}$</td>
<td>-4.5</td>
<td>1.46</td>
<td>Mn$^{2+}$ ($d^6$)</td>
<td>2.13</td>
</tr>
<tr>
<td>N$<em>i$Mn$</em>{2}^{-}$</td>
<td>1.7</td>
<td>1.22</td>
<td>Ni$^{2+}$ ($d^8$)</td>
<td>2.04</td>
</tr>
<tr>
<td>N$<em>i$Mn$</em>{-}$</td>
<td>-0.9</td>
<td>1.32</td>
<td>Ni$^{3+}$ ($d^7$)</td>
<td>1.95</td>
</tr>
<tr>
<td>N$<em>i$Mn$</em>{0}$</td>
<td>0.1</td>
<td>1.39</td>
<td>Ni$^{4+}$ ($d^6$)</td>
<td>1.87</td>
</tr>
<tr>
<td>Mn$_{Li}^{3+}$</td>
<td>4.4</td>
<td>1.50</td>
<td>Mn$^{2+}$ ($d^8$)</td>
<td>1.99</td>
</tr>
<tr>
<td>Mn$_{Li}^{2+}$</td>
<td>4.4</td>
<td>1.52</td>
<td>Mn$^{2+}$ ($d^8$)</td>
<td>1.98</td>
</tr>
<tr>
<td>Mn$_{Li}^+$</td>
<td>4.4</td>
<td>1.51</td>
<td>Mn$^{2+}$ ($d^8$)</td>
<td>1.98</td>
</tr>
<tr>
<td>Mn$_{Li}^{0}$</td>
<td>4.4</td>
<td>1.51</td>
<td>Mn$^{2+}$ ($d^8$)</td>
<td>1.98</td>
</tr>
<tr>
<td>N$<em>i$Li$</em>{3}^{-}$</td>
<td>-1.8</td>
<td>1.30</td>
<td>Ni$^{2+}$ ($d^8$)</td>
<td>1.94</td>
</tr>
<tr>
<td>N$<em>i$Li$</em>{2}^{-}$</td>
<td>-1.8</td>
<td>1.29</td>
<td>Ni$^{2+}$ ($d^8$)</td>
<td>1.94</td>
</tr>
<tr>
<td>N$<em>i$Li$</em>{+}$</td>
<td>-1.8</td>
<td>1.29</td>
<td>Ni$^{2+}$ ($d^8$)</td>
<td>1.94</td>
</tr>
<tr>
<td>N$<em>i$Li$</em>{0}$</td>
<td>-1.7</td>
<td>1.27</td>
<td>Ni$^{2+}$ ($d^8$)</td>
<td>1.93</td>
</tr>
</tbody>
</table>

$^a$ Bader charge in bulk LMNO (Wyckoff sites): Li$^+$ (8c): 0.89; Mn$^{4+}$ (12d): 1.81; Ni$^{2+}$ (4b): 1.21; O$^{2-}$ (24e): -1.06; O$^{2-}$ (8c): -1.05.

$^b$ Average TM-O bond length in bulk LMNO: Mn$^{4+}$-O: 1.91 Å; Ni$^{2+}$-O: 2.05 Å.

Moorhead-Rosenberg et al.$^{100}$ suggested the spin on Mn$^{3+}$ and Mn$^{2+}$ should be parallel to that of Ni$^{2+}$ due to the oxygen-mediated super-exchange interaction. This coincides with the spin-flip phenomena shown in Table 3. However, a spin flip is not observed on all defects and there is no clear trend whether a certain spin orientation is preferred. Magnetic interactions are influenced by the point defects and each defect has a complex potential energy surface, meaning that different defect structures and magnetic states may be found, depending on the initial structures, their magnetisation values and the choice of electronic minimisation algorithm. In each case, different degrees of structural perturbation (rattling) and magnetisation initialisations were tested, to aid the location of the true ground state defect.$^{52–54}$ One spin flip usually gives less than 0.1 eV difference in energy. However, multiple spin flips can lead to a large energy change (e.g. ~1 eV) and give erroneous results. All calculations have been checked and rerun to ensure there are no multiple spin flips on the TM cations.

For Mn$_{Li}$ and Ni$_{Li}$ defects with a change in coordination (O$_h$ → T$_d$) at the defect site,
we observe a strong preference of Mn and Ni species to adopt the lowest stable positive oxidation states (a closer charge to Li$^+$) for these element (i.e. Mn$^{2+}$ and Ni$^{2+}$) (see Table 3). Ni coordinating to O in a T$_d$ crystal field would favour a high-spin configuration as the size of ligand field splitting is reduced ($\Delta_{T_d} = \frac{4}{9} \Delta_{O_h}$), where $\Delta_{T_d}$ and $\Delta_{O_h}$ are the T$_d$ and O$_h$ crystal field splitting energy respectively.$^{101}$ The average bond lengths of T$_d$ Mn$^{2+}$-O is 1.98 Å, which is longer than the standard O$_h$ Mn$^{4+}$-O bond lengths between 1.87-1.94 Å (see Table 1). The energetic penalties for Ni to adopt Ni$^{4+}$ ($d^6$), Ni$^{3+}$ ($d^7$) and Ni$^{2+}$ ($d^8$) in a T$_d$ field are 2.13 $\Delta_{O_h}$, 1.26 $\Delta_{O_h}$ and 0.84 $\Delta_{O_h}$, respectively, while for Mn in Mn$^{4+}$ ($d^3$), Mn$^{3+}$ ($d^4$) and Mn$^{2+}$ ($d^5$), they are 0.84 $\Delta_{O_h}$, 0.42 $\Delta_{O_h}$ and 0 $\Delta_{O_h}$ respectively. Configurations with the lowest energetic penalties (i.e. Ni$^{2+}$ and Mn$^{2+}$) are favoured on the Li$^+$ site. In addition, the ionic radius of Mn$^{2+}$ in a four-coordination environment is 0.66 Å, which is closer to that of Li$^+$ (0.59 Å), compared to Mn$^{4+}$ (0.39 Å).$^{102}$ Literature studies$^{2,7,21,103}$ suggest the formation of Mn$^{2+}$ as a consequence of Mn$^{3+}$ disproportionation reactions. Mn$^{2+}$ are detrimental to performance as they easily dissolve in electrolytes leading to capacity fading.$^{23}$

Li$_{TM}$ defects involve changing the coordination of Li (T$_d$ $\rightarrow$ O$_h$). We found that fully ionised Li$_{Ni}^-$ is always the most stable charge state across all synthesis conditions at their self-consistent Fermi levels. In addition, the formation energies of Li$_{Ni}^-$ are lower than Li$_{Mn}$, indicating that Li$^+$ would preferentially form on the Ni sites over the Mn sites. Similar observations were found on Li-doped LMNO, where Li$^+$ sat in the $4b$ positions.$^{21}$ This inevitably replaces some Ni$^{2+}$, leading to reduced capacity.

Among vacancy defects, $V_O$ and $V_{Li}$ have much lower formation energies (thus exist in higher concentrations) than $V_{Mn}$ and $V_{Ni}$ under all conditions (see Figure 4). Therefore, oxygen and lithium vacancies are discussed here. For lithium vacancies, while $V_{Li}^-$ (removing a Li$^+$ ion) generates little disturbance to the system, $V_{Li}^0$ (removing a Li atom) reveals the delithiation mechanism.$^{16,85}$ A localised hole is observed on the Ni in proximity to the defect site, with a magnetisation of -0.9 $\mu_B$ which is nearly half of the original value (-1.7 $\mu_B$) for Ni$^{2+}$. This suggests that Ni$^{2+}$ ($d^8$) is oxidised to Ni$^{3+}$ ($d^7$) low-spin configuration, with the
Bader charge of 1.33 compared to 1.21 for bulk Ni\(^{2+}\) in agreement with the reduction in Ni-O bond lengths from 2.05 Å to 1.86-2.05 Å. The Ni-O bonds become more covalent and exhibit a JT distortion associated with \(d^7\) low-spin. Madelung potential analysis on the unrelaxed \(V_{Li}^0\) structure confirmed that the nearest Ni to the vacant lithium site is oxidised to Ni\(^{3+}\).\(^{104}\) The average Madelung site potential on Ni is -24.55 V, whereas the Ni\(^{3+}\) site has the most negative potential, with -27.38 V.

Oxygen vacancies are found to be non-negative \(U\) type (stable in the 0 and +1 charge states, likely aided by polaronic stabilisation of excess electrons), adopting the neutral state under most growth conditions.\(^{105}\) \(V_{O,1}\) are lower in energy than \(V_{O,2}\) at each charge state (Figure 5). This can be explained from their local coordination as mentioned previously, with \(V_{O,1}\) configuration having less unfavourable electrostatic repulsion between the cations than that of \(V_{O,2}\), reflected in the average Madelung potentials of 26.31 V (\(O_1\)) and 29.37 V (\(O_2\)). In the neutral state (\(V_{O,1}^0\) and \(V_{O,2}^0\)), there are two five-coordinated Mn\(^{3+}\) which acquire the electrons donated from \(V_O\), with magnetisation values between 3.6 - 3.7 \(\mu_B\) compared to \(\sim 3.0\) for the remaining Mn\(^{4+}\) species. Mn\(^{3+}\) have Bader charges \(\sim 1.6\) which are lower than that of Mn\(^{4+}\), with 1.83. In the +1 charge state, \(V_{O,1}^+\) has a localised electron on one of the five-coordinated Mn, whereas \(V_{O,2}^+\) has the additional electron delocalised over three five-coordinated Mn, confirmed by hybrid DFT calculations in a 56-atom supercell.

Overall, antisites and vacancies in LMNO are deep defects, generating localised states (that are distant from VBM and CBM) with small polarons on Mn sites for electrons and Ni for holes. The dopability of LMNO can be assessed by the size of the doping energy window, determined by the lowest energy compensating acceptor/donor defect at the CBM/VBM for \(n/p\)-type dopability.\(^{106}\) The lowest energy donor and acceptor is antisite Ni\(^{+}\)_\(Li\) and Li\(^{-}\)_\(Ni\), giving a dopability window of 0.05 eV (for \(p\)-type, O-rich) and 0.36 eV (for \(n\)-type, O-poor). Thus LMNO is potentially \(n\)-type dopable and not \(p\)-type dopable. This behaviour is driven by the antisite defects in LMNO rather than oxygen vacancies in other \(n\)-type oxides.\(^{107,108}\)
**Interstitials**

In $P4_332$ LMNO, there are two distinct types of vacant $O_h$ sites: $4a$ and $12d$ sites which are surrounded by $3\text{Mn}^{4+}$ and $3\text{Ni}^{2+}$ and $5\text{Mn}^{4+}$ and a $\text{Ni}^{2+}$, respectively.\(^{109}\) The lowest energy relaxed configurations of $\text{Li}_i$ and $\text{Ni}_i$ interstitials are in the $12d$ sites, whereas that of $\text{Mn}_i$ are in the $4a$ sites. While the electrostatic interactions form a key contributor to the observed interstitial site preference, we find charge localisation, coordination environment and (potential) dumbbell formation to also play a key role.

For $\text{Li}_i$, a configuration with a Li-Li dumbbell split between neighbouring $12d$ sites is around 0.07 eV lower than a lone $\text{Li}_i^+$ on the $12d$ site (see Figure S3). The preference for Li-Li dumbbell configuration was reported for the parent spinel compound LMO.\(^{85}\) The Madelung potentials for $\text{Li}^+$ in the $T_d$ 8c crystal site are around -9.54 V, whereas those in the interstitial $4a$ and $12d$ are -6.84 V and -6.40 V respectively. Neutral $\text{Li}_i^0$ gives an electron polaron on a neighbouring Mn (giving $\text{Mn}^{3+}$) beside the Li-Li dumbbell.

For $\text{TM}_i$, the charge compensation mechanism for non-fully-ionised states is the same as for other $n$-type defects where $\text{Mn}^{4+}$ in the proximity to the defect site get reduced. Having tested a range of charge states, we found $\text{Mn}^{3+}$ and $\text{Mn}^{2+}$ but no $\text{Mn}^{4+}$ species in $\text{Mn}_i$ defect supercells, whereas all Ni species always exist as $\text{Ni}^{2+}$ in $\text{Ni}_i$ defect supercells. When $q = 2$, interstitial Mn and Ni exist in $+$2 oxidation states. Adding more electrons in the systems ($q = 1$, $0$) led to the formation of more $\text{Mn}^{3+}$. Interstitial $\text{Ni}^{2+}$ prefers to be in the $12d$ sites, identical to the preference for $\text{Li}_i$. Interstitial $\text{Mn}^{2+}$, on the other hand, prefers to be in the $4a$ sites ($\sim$ 0.05 - 0.15 eV lower in energy). This could be partly due to a size effect. The $4a$ sites have larger octahedral volume than the $12d$ sites (Table S8).\(^{109,110}\) The ionic radii for $\text{Li}^+$, $\text{Ni}^{2+}$ and $\text{Mn}^{2+}$ in octahedral coordination are 0.76 Å, 0.69 Å and 0.83 Å, respectively.\(^{102,110}\) Ions with smaller ionic radii (i.e. $\text{Li}^+$ and $\text{Ni}^{2+}$) would have a smaller volume mismatch with the smaller $12d$ octahedral volume, while bigger $\text{Mn}^{2+}$ need sites with larger volume. Similar to the structural features observed in $\text{Li}_i$, Li-Ni and Mn-Li split-interstitial configurations are also seen for $\text{Ni}_i$ and $\text{Mn}_i$. This configuration reduces cation-cation repulsion and lattice
strain, facilitated by highly mobile Li\textsuperscript{+} which can easily migrate from T\textsubscript{d} to O\textsubscript{h} sites.\textsuperscript{111} Finally, the exchange type reaction Ni\textsubscript{i} $\rightarrow$ Li\textsubscript{i} + Ni\textsubscript{Li} is found to spontaneously occur, again reflecting the high mobility of Li\textsuperscript{+}.\textsuperscript{82}

All O\textsubscript{i} relaxed into dumbbell configurations with O-O distance within 1.4 Å.\textsuperscript{112} In addition, they are stable in the charge neutral state over all Fermi levels, with high-lying O-O anti-bonding states. They are also known as peroxide species (O\textsubscript{per}) and are seen in many more n-type oxides (e.g. SnO\textsubscript{2} and BaSnO\textsubscript{3}).\textsuperscript{53,107,113} We identified two distinct O\textsubscript{i} peroxide configurations with different TM environments: surrounded by 3Mn\textsuperscript{4+} and a Ni\textsuperscript{2+} (ground state) and surrounded by 2Mn\textsuperscript{4+} and 2Ni\textsuperscript{2+} (0.8 eV higher in energy).

Among all interstitial defects, Li\textsubscript{i} is always lower in energy than Mn\textsubscript{i} and Ni\textsubscript{i}, indicating that they are more likely to form in high concentration. The energetic ordering of Ni\textsubscript{i} and Mn\textsubscript{i} can vary depending on the chemical potential conditions. The formation energy of O\textsubscript{i} can become lower than that of Li\textsubscript{i} in O-rich synthesis conditions. Overall, Li\textsubscript{i}, Mn\textsubscript{i} and Ni\textsubscript{i} are deep donors and O\textsubscript{i} are ultra-deep acceptors with only neutral states stable across the band gap (see Figure 5).

**Defect complexes**

Given the low formation energies and thus high concentrations of cation antisites, lithium and oxygen vacancies, we further investigate the association of point defects, including three stoichiometric (antisite-pairs), and eight non-stoichiometric (vacancy-related) complexes. Since the formation energy on the oxygen vacancy in 24\textit{e} sites (O\textsubscript{1}) is always lower and O\textsubscript{1} has higher multiplicity than O\textsubscript{2} (in the 8\textit{c} sites), we only conduct detailed analysis on the association of V\textsubscript{Li} with V\textsubscript{O,1}.
Figure 6: Defect complex association energy $\Delta H$ for a selection of stoichiometric complexes (denoted by squares) and non-stoichiometric complexes (denoted by triangles) calculated at the self-consistent Fermi level over 20 different growth conditions from A (O-rich, $p$-type) to T (O-poor, $n$-type).

Figure 6 shows defect complex association energies $\Delta H$ calculated following Equation 6 over 20 chemical potential limits at their self-consistent Fermi levels. $\Delta H$ for a given complex can vary with chemical potentials, even for stoichiometric complexes, due to the indirect effect on the charge-balanced Fermi level. A negative $\Delta H$ indicates an exothermic enthalpy change upon defect association. The majority of complexes have negative $\Delta H$, and they feature point defects coulombically attracted to each other. The few complexes that feature defects of the same charge (i.e. both $p$-type defects) are coulombically opposed to each other, and hence have positive $\Delta H$ (e.g. $\text{Li}_{\text{Ni}} - V_{\text{Li}}$). Using $\Delta H$ alone to evaluate complex formation, however, neglects a key factor - the temperature dependent entropic cost of defect association. The actual formation of a defect complex and its concentration are influenced by a range of factors, including temperature, concentrations of the constituent defects and $\Delta H$.\cite{49,82} We considered the entropic effects in this study by approximating the entropic term
using Equation 10, where lower defect concentrations and higher temperatures correspond to larger entropic costs for association into defect complexes. Correspondingly, Figure 7 shows how $\Delta G$ increases with temperature, suggesting complex formation is sensitive to kinetic behaviour during synthesis and quenching. $\Delta G$ are generally more negative in the $n$-type condition than $p$-type, suggesting that there is a stronger driving force towards complex association even at high temperature in O-poor conditions.

Figure 7: Gibbs free energy change $\Delta G$ for complex formation as a function of temperature under (a) the most intrinsically $p$-type O-rich condition A and (b) $n$-type O-poor condition T, using Equation 10. Stoichiometric complexes are denoted by squares and non-stoichiometric complexes are denoted by triangles. A negative $\Delta G$ suggests complex association is likely, provided the constituent point defects are sufficiently mobile.

Figure 8 shows the approximated temperature-dependent entropic cost (calculated using Equation 10) for defect complexes with negative $\Delta G$. A high degree of association is achieved when the complex association energy magnitude $|\Delta H|$ is greater than $\Delta E_{\text{crit}}$, showing that one cannot use $\Delta H$ alone to evaluate complex formation. For example, under the intrinsically $p$-type environment (Figure 8a), the $\text{Mn}_{\text{Li}}-\text{Li}_{\text{Mn}}$ association energy $|\Delta H| = 0.4$ eV is only sufficient to achieve a high degree of association at 500 K but not at 900 K. Increased entropic cost of complex formation is observed for defects with lower concentration (see Figure 9).
The stoichiometric complexes have identical density of the available microstates \( N_{\text{states}} = 4.40 \times 10^{22} \text{ cm}^{-3} \), the differences in the calculated complex concentrations therefore arise from the concentrations of their constituent point defects. For instance, under condition T (Figure 9b), \([\text{Mn}_{\text{Li}}-\text{Li}_{\text{Mn}}]\) is lower than \([\text{Ni}_{\text{Li}}-\text{Li}_{\text{Ni}}]\) at the majority of temperature range despite that they have similar \( \Delta E_{\text{crit}} \) and that \( \Delta H \) for \( \text{Mn}_{\text{Li}}-\text{Li}_{\text{Mn}} \) is highly negative (see Figure 6). Although complex formation is more favourable at lower temperature, this requires isolated point defects to “find” each other. Point defects with high mobility – such as lithium vacancies and interstitials – and/or those that with high concentration are likely to achieve this criterion.

![Figure 8: Critical association energy \( \Delta E_{\text{crit}} \) (approximate entropic cost of complex formation; Equation 10) as a function of temperature under (a) the most intrinsically \( p \)-type O-rich condition A and (b) \( n \)-type O-poor condition T. Stoichiometric complexes are denoted by squares and non-stoichiometric complexes are denoted by triangles.](image)
Figure 9: Defect complex concentration as a function of temperature under (a) the most intrinsically $p$-type O-rich condition A and (b) $n$-type O-poor condition T, using Equation 7. Constituent point defect concentrations are fixed to the calculated concentrations at the anneal temperature ($T = 973$ K), following the typical frozen-defect approximation. Stoichiometric complexes are denoted by squares and non-stoichiometric complexes are denoted by triangles. Defect complex concentrations are capped at the concentration of the major constituent defect.

$\text{Mn}_{\text{Ni}} \text{Ni}_{\text{Mn}}$ has the highest concentration for maximum (>99%) complex formation (i.e. saturation concentration), reflecting the strong proclivity to $\text{Mn}_{\text{Ni}} \text{Ni}_{\text{Mn}}$ cation disorder in LMNO. Complexes (e.g. $\text{Mn}_{\text{Li}} \text{Li}_{\text{Ni}}$) with much lower predicted concentration contribute much less to the disorder. Since defect complex concentrations are higher in condition T than condition A, a greater range of disorder is induced by complex formation under O-poor conditions. In addition, there is a variable temperature-dependent entropic cost with complex formation, with higher defect concentrations corresponding to lower entropic costs of complex association. For example, $V_{\text{O,1}} - V_{\text{Li}}$ concentration is strongly dependent on temperature, whereas it is not the case for $\text{Ni}_{\text{Li}} - V_{\text{Li}}$. High synthesis/anneal temperatures will yield high point defect concentrations (Equation 7), which then increasingly favour complex formation as the material cools to room temperature, but with the ability to equilibrate by migrating and forming complexes also decreasing with temperature. Thus the final defect...
distribution in the material is predicted to be sensitively dependent on the thermal history (i.e. synthesis, annealing and cooling procedures), as witnessed experimentally.\textsuperscript{114}

The association of lithium vacancies with other defects can affect lithium diffusion. If $V_{Li}$ preferentially associates with a specific point defect, the additional species may function as a “trap” for diffusion lithium. Previous literature studies\textsuperscript{29,115} have specifically discussed the $V_{O,1} – V_{Li}$ complex, with Li$^+$ diffusion more difficult in disordered LMNO, as lithium vacancies preferentially form near oxygen vacancies – which have formation energies $\sim$1 eV lower in the disordered material. Our calculations indeed find strongly-favourable complex binding energies for $V_{O,1} – V_{Li}$, as well as the moderate binding for Mn$_{Li}$–$V_{Li}$ and Mn$_{Ni}$–$V_{Li}$ (Figure 7). Therefore, Li$^+$ mobility and its diffusion behaviour are affected by the competitive interactions between other point defects with strong association with lithium vacancies and are sensitive to the synthesis environment.

**Defect migration**

Diffusion of Li$^+$ is a fundamental process that occurs during charge/discharge of a cathode. In a spinel framework like LMNO, Li$^+$ migration occurs from a $T_d$ site to another $T_d$ site \textit{via} an empty $O_h$ (gate) site. There are two different gate sites ($4a$ and $12d$) in $P4_{3}32$ LMNO and thus two distinct migration paths for Li$^+$ (see Figure 10). Li diffusivity calculations have been used to explain the rate capabilities of LMNO. Most theoretical studies\textsuperscript{109,115,116} have considered the diffusion of Li$^+$ \textit{via} the vacancy mechanism, which involves hopping of a single Li$^+$ from one site to another ($V_{Li}$ hopping in the opposite direction). Our calculated Li$^+$ migration barriers \textit{via} the vacancy mechanism (Figure 11) are similar to previous studies.\textsuperscript{115,116} The activation energies for Li$^+$ to migrate through $4a$ (path 1) and $12d$ (path 2) gate sites are 0.34 eV and 0.30 eV, respectively. This is consistent with our findings that Li interstitials preferentially sit on the $12d$ sites.
Figure 10: A schematic showing 4$a$ and 12$d$ gate sites giving two distinct Li$^+$ migration paths in $P4_332$ LiMn$_{1.5}$Ni$_{0.5}$O$_4$. The 4$a$ site surrounded by 3Mn$^{4+}$ and 3Ni$^{2+}$; the 12$d$ sites surrounded by 5Mn$^{4+}$ and a Ni$^{2+}$. Path 1 and path 2 are denoted by yellow and green arrows, respectively. Oxygen atoms which define the polyhedral vertices are not shown for clarity.

We also considered Li$^+$ migration via an interstitialcy mechanism, which has not been previously studied in LMNO. This involves concerted migration of three Li$^+$ via the Li-Li dumbbell configuration. The path 1 end image is higher in energy, where one of the Li$^+$ in the Li-Li dumbbell sits on a 4$a$ site. The path 2 end image is an equivalent configuration to the initial image where both Li$^+$ in Li-Li dumbbell sit on 12$d$ sites. The activation energies to migrate through path 1 and path 2 are 0.14 eV and 0.12 eV, respectively. Again, migrating via 4$a$ sites is less favourable. Nevertheless, the activation energies for Li$^+$ to migrate via the interstitialcy mechanism are much lower than that via the vacancy mechanism. This suggests that the dominant diffusion mechanism may change during charge and discharge. Since the interstitialcy mechanism involves concerted migration of multiple Li$^+$ via the Li-Li dumbbell configuration, this migration is only possible when the concentration of Li$^+$ is sufficiently high and it has also been shown that migration barriers can vary with Li concentration due to effects on Ni valence and thus electrostatic interactions between Li and Ni.\textsuperscript{109,117}
Figure 11: (a) Migration barriers of lithium interstitials (Li$^+$) via interstitialcy mechanisms and lithium vacancies (V$_{Li}^-$) via vacancy mechanisms, calculated using the climbing-image nudged elastic band (cNEB) method. Path 1 and path 2 involve Li$^+$ migrating through the 4$a$ sites (surrounded by 3Mn$^{4+}$ and 3Ni$^{2+}$) and 12$d$ sites (surrounded by 5Mn$^{4+}$ and a Ni$^{2+}$), respectively. Schematic illustrations of the (single-ion) vacancy migration (b) and (concerted multi-ion) interstitialcy migration (c).

Finally, we show that the Li$^+$ interstitialcy migration makes the Li-Frenkel pair (Li$^+$ − V$_{Li}^-$) unstable to spontaneous recombination at short pair distances. To investigate this, a Li vacancy was placed with a Li interstitial over each of the six standard positions over a range of pair distances for each calculation. From the cNEB calculations, we determine a critical spontaneous recombination distance of $\sim 4.6$ Å, which is larger than the minimum separation (3.45 Å) between Li in their T$_d$ sites.

**Self-consistent Fermi levels**

While most discussions around defect chemistry are focused on two distinct O-rich($p$-type)/O-poor($n$-type) growth conditions, we would like to highlight that defect chemistry can be different in other intermediate growth conditions. Figure 12 shows the calculated self-consistent Fermi level in all growth conditions. Since the ideal synthesis/anneal temperature for the
ordered phase is at 700 °C, 973 K is used as the input temperature to determine the self-consistent Fermi levels.\textsuperscript{1,12,20,21} The chemical potential conditions A $\rightarrow$ T are arranged with increasing self-consistent Fermi level. To understand the defect behaviour at room/operating temperature, self-consistent Fermi levels were also calculated with constant non-lithium defect concentrations upon cooling (i.e. “frozen in”). While the kinetic barriers of non-lithium defects can be assumed to be large, the same assumption cannot be applied to lithium defects (Li\textsubscript{i} and V\textsubscript{Li}) as the facile movement of Li\textsuperscript{+} is one of the desirable features for a cathode.\textsuperscript{118} In addition, it has been suggested that some lithium vacancy-interstitial pairs would recombine (i.e. the reverse reaction of Frenkel pair formation) at lower temperatures.\textsuperscript{119,120} Two variations of the “frozen” approach were applied. The former approach only allowed V\textsubscript{Li} and Li\textsubscript{i} concentrations to vary, whereas the latter also allowed Li\textsubscript{Mn} and Li\textsubscript{Ni} concentrations to change.

The majority of conditions have Fermi levels $\pm$ 0.5 eV from the mid gap. Under the most O-poor conditions, the Fermi level is $\leq$ 0.24 eV away from the CBM. Therefore, the material is a weak semiconductor, and can be made more $n$-type than $p$-type. The intrinsic defects are not likely to generate significant quantities of free charge carriers that contribute to conductivity and electronic conduction would be mainly governed by the hopping of small polarons.\textsuperscript{85} Ni\textsuperscript{3+}/Mn\textsuperscript{3+} are the dominant charge compensating ions under all conditions. In addition, Mn\textsuperscript{2+} formation is frequently observed for $n$-type defects, suggesting a strong driving force for Mn\textsuperscript{3+} to undergo disproportionation reactions.
Figure 12: Self-consistent Fermi levels determined under all chemical potential conditions. The atomic chemical potential values can be found in Table S6. Two variant approaches assuming certain defects are “frozen in” during synthesis were adopted to simulate defect behaviours at room temperature (300 K).

The formation of oxygen vacancies and Mn$^{3+}$ is a dominant feature when LMNO is synthesized at a more O-poor (n-type) condition. The electrochemical performance of LMNO synthesized under those conditions can be improved due to the increase in cation disorder and the proportion of Mn$^{3+}$.\textsuperscript{12} While Mn$^{3+}$ species is redox active which improves the electronic conductivity, it is subject to JT distortion and promotes Mn dissolution. Therefore, [Mn$^{3+}$] should be controlled to achieve optimum performance. Ni-rich rock salt impurity phases are also likely to form under n-type conditions. This is not desirable as it leads to a decrease in capacity.

Since antisite defects are generally low energy over all chemical potential conditions, we propose that (local) cation ordering can be perturbed under many different synthesis conditions. This supports previous experimental studies\textsuperscript{13,20,21} where local Mn/Ni ordering
has been perturbed without significant concentrations of oxygen vacancies and/or Mn$^{3+}$. Theoretically, O-rich ($p$-type) conditions have a higher tendency towards Ni$^{3+}$ formation. For samples which undergo a series of heat treatment processes, a more $p$-type condition could oxidise the pre-formed Mn$^{3+}$ back to Mn$^{4+}$, reducing the total number of redox active centers and leading to reduced capacity. Synthesis techniques that can take the advantage of the inherent defect chemistry of the material are more likely to achieve the fine tuning of electrochemical performance with target features. For example, Lee et al.$^{21}$ utilised the preference for Li$_{Ni}$ antisite formation (on the 4$b$ sites) to block the Mn/Ni ordering transition pathways during the annealing process of the Li-doped LMNO samples. Their Li-doped LMNO “Mn$^{3+}$-free” samples annealed at 700 °C exhibited increased Mn/Ni disordering and showed enhanced cycling stability. In this case, the undesirable effects from Mn$^{3+}$ had been suppressed, as the Mn$^{3+}$ generated during the first (900 °C) treatment were re-oxidised to Mn$^{4+}$ when annealed at 700 °C. Unfortunately, the high concentration of Li$_{Ni}$ defects leads to a reduced capacity. The defect chemistry of LMNO therefore plays a crucial role in affecting the electrochemical performance. To design LMNO with optimum electrochemical performance, it is important to account for defect behaviour and effectively utilise it when developing synthesis methods.

**Conclusions**

The bulk properties of $P4_332$ LMNO and its intrinsic defect chemistry have been investigated using DFT. To correctly capture the magnetic properties and ground state electronic structure, we demonstrated the importance of a systematic evaluation of the magnetic ordering in the open-shell TM system. Analysis of the electronic structures indicated a high degree of hybridisation of Ni-O and Mn-O states in LMNO. The thermodynamic stability of LMNO is more sensitive to Ni chemical potentials, while there is a greater flexibility to vary the Li and Mn chemical potentials, leading to different defect properties.

The intrinsic defect chemistry suggests LMNO a weak semiconductor and can be made
more $n$-type. The formation of deep defects and small polarons indicated electronic conduction proceeds via the polaron hopping mechanism. The low formation energies for antisite defects explain the tendency towards cation disordering. There exist several defect pair complexes with strong association energies and they are sensitive to the experimental conditions and thermal history of the material. The investigation of Li defects migration revealed that Li$^+$ can migrate via a much lower energy concerted multi-ion interstitialcy mechanism than the previously proposed vacancy mechanism. However, this is achieved via low energy Li-Li dumbbell configuration and requires sufficiently mobile Li$^+$. The Li$^+$ mobility is influenced by the interactions with point defects.

A comprehensive analysis of the defect chemistry over all growth conditions shows a tendency toward O-poor ($n$-type) conditions where Mn$^{3+}$ formation is encouraged. This may explain why many experimental studies find it challenging to study solely the effect of Mn/Ni ordering on the electrochemical performance. Our work provides insights into the complex defect chemistry of LMNO which can be utilised when developing synthesis methods to fine-tune electrochemical performance. This knowledge is also crucial for understanding the charge compensation mechanisms with the presence of extrinsic defects.

Conflicts of Interest

There are no conflicts to declare.

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