Our theoretical treatment of electronic structure in coordination complexes often rests on assumptions of symmetry. Experiments rarely provide fully symmetric systems to study. In solution, fluctuation in solvation, variations in conformation, and even changes in constitution occur and complicate the picture. In crystals, lattice distortion, energy transfer, and phonon quenching is in play, but we are able to identify distinct symmetries. Yet the question remains: How is the real symmetry in a crystal compared to ideal symmetries? Moreover, at what level of detail do we need to study a system to determine, if the electronic structure behaves as if it has ideal symmetry? Here, we have revisited the Continues Shape Measurement (CShM) approach developed by Ruiz-Martínez and Alvarez to evaluate the structure of ten-coordinated europium(III) ions in a K₅Na[Eu₂(SO₄)₆] structure. By comparing the result of the symmetry deviation analysis to luminescence data, we are able to show the effect of small deviations from ideal symmetry. We suggest using a symmetry deviation value, σ_{ideal}, determined by using our updated approach to Continues Shape Measurements, via our AlignIt code. AlignIt includes normalization and relative orientation in the symmetry comparison, and by combining the calculated values with the experimentally determined energy level splitting, we were able place the first point on a scale that can show how close to ideal an experimental structure actually is.
Europium(III) has 3003 distinct electronic energy levels. Electron-electron repulsion and spin-orbit coupling separates these into 327 groups of states, each described by Russell-Saunders spectroscopic term symbol such as $^5D_0$. Each term (group of states) is split by the ligand field into a distinct subset of the 3,003 electronic energy levels. Among the trivalent lanthanides, Eu(III) is unique as the ground state ($^7F_0$) and the main emitting state ($^5D_0$) both are in a group of states with only a single electronic energy level. This makes the information from optical spectroscopy easier to interpret. Further, the number of the electronic energy levels represented by a spectroscopic term can be directly observed as lines in a high-resolution optical spectrum, see Table 1.

<table>
<thead>
<tr>
<th>Symmetry classes</th>
<th>$^5D_0 \rightarrow ^7F_J$</th>
<th>Octagonal</th>
<th>Tetragonal</th>
<th>Orthorhombic</th>
</tr>
</thead>
<tbody>
<tr>
<td>Point groups</td>
<td></td>
<td>$D_{5h}, C_{5v}$</td>
<td>$D_{4h}, D_{4v}$</td>
<td>$D_{3h}, D_{2h}$</td>
</tr>
<tr>
<td>$J = 0$</td>
<td></td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$J = 1$</td>
<td></td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$J = 2$</td>
<td></td>
<td>3</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>$J = 3$</td>
<td></td>
<td>4</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>$J = 4$</td>
<td></td>
<td>6</td>
<td>7</td>
<td>9</td>
</tr>
</tbody>
</table>

It is well established from Laporte’s parity selection rules that 4f-4f transitions are forbidden, and a decrease in symmetry around the central ion increases orbital mixing and, consequently, increases the probability of transition. That is, high symmetry around the lanthanide ion is expected to increase the luminescence lifetime, and – in the absence of quenching – luminescence intensity. In the crystal structure of $K_5Na[Eu_2(SO_4)_6]$, the Eu(III) ion is ten-coordinated by all-oxygen donors. A crystal structure of $K_5Na[Ce_2(SO_4)_6]$ has been reported by Eriksson et al., in which this conformation is also mentioned as unique for cerium(III) sulfates. Here we present the synthesis, structure, and luminescence properties of $K_5Na[Eu_2(SO_4)_6]$. In this compound, the Eu(III) coordination geometry can be viewed as a slightly distorted bicapped square antiprism (bcSAP), but this description is inadequate for investigating the effect of symmetry on the optical properties. Thus, we calculated deviations of the Eu(O)$_{10}$ moiety from idealized geometries and other highly
symmetric f-element structures in order to evaluate the extent of the distortion from the ideal bicapped square antiprismatic structure. K₅Na[Eu₂(SO₄)₆] crystallizes in the C2/m space group as rhombohedral blocks. Ten oxygen atoms (see Figure 1) coordinate to the Eu(III) ion. The unit cell contains four Eu(III) ions in dimers of two edge-sharing polyhedra bridged by two sulfate ions (see Figure 1). Crystallographic information is available as supporting information.

In order to estimate how close the Eu(O)₁₀ polyhedron is to an ideal bcSAP shape, we applied the Continuous Shape Measures (CShM) approach for ten-coordinated compounds as described by Ruiz-Martínez and Alvarez. The approach is incorporated it into the program SHAPE, which is readily available for public use. The program calculates a CShM value that describes the degree of shape deviation of a given structure from an ideal structure.

Building on this, we have used a normalization term (1/N) to normalize with the number of atoms in the structure, and a manual pre-alignment of the structures, to verify the optimization before we use equation 1 to calculate the symmetry deviation value, $\sigma_{\text{ideal}}$.

$$\sigma_{\text{ideal}}(Q) = \frac{\sum_{k=1}^{N}|Q_k - P_k|^2}{\sum_{k=1}^{N}|Q_k - Q_0|^2} \times \frac{100}{N} \quad \text{eq. (1)}$$

Here $Q$ is the position vector of a ligating atom in the structure, $P$ is the position vector of the corresponding ligating atom in an ideal polyhedron, and $Q_0$ is the zero point placed in origo.

However, to achieve transparent optimization of the intra-atomic distances and rotation in both real and ideal polyhedral, we incorporated equation 1 along with relative orientation and bond length scaling into a program we call AlignIt. Rotational optimization is done through Mercury before using AlignIt. A full description of AlignIt is available in Supporting Information.

Al-Karaghoulı and Wood have previously discussed the relative stability of common idealized ten-vertex structures based on calculations of ligand-ligand repulsion energies. Their work revealed that the bicapped square anti-prism (bcSAP) is the most stable ligand arrangement for ten-coordination, closely followed by the bicapped dodecahedron (bcDod) in stability of complexes with coordination number of ten.

To evaluate deviation from ideal symmetry, we first need to define a scale for our symmetry deviation value, $\sigma_{\text{ideal}}$, in order to relate the determined value for K₅Na[Eu₂(SO₄)₆] to ideal symmetries. To generate the scale, we used six ten-vertex ideal polyhedral. The bicapped dodecahedron (bcDod) and the staggered dodecahedron (SDod) are similar in shape to bcSAP, while the pentagonal prism (PP), pentagonal antiprism (PAP) and octagonal bipyramid (OBPy) were selected as shapes that are very different from bcSAP, bcDod and SDod. The
ideal models are spherical idealized, meaning all M-O bond lengths in the structure are equal. The scale only contains ten-vertex polyhedra that to some extent considers ligand-ligand repulsion. Ideal polyhedra that deviates more from bcSAP than PP, PAP and OBPy exist, but are not included here. Comparative symmetry deviation values, $\sigma_{\text{ideal}}$, between the ten-vertex polyhedra are shown in Table 2, along with the corresponding CShM values.$^{25}$ Note that $\sigma_{\text{ideal}}$ values below the diagonal are left out because they are consistent with the inverse to a second decimal. The full table can be found in Supporting Information. We leave it to the reader to compare the two scales, and only conclude that $\sigma_{\text{ideal}}$ takes values from 0 (identical) to 24 (very different). To test how a non-zero $\sigma_{\text{ideal}}$ value is expressed in terms of physical properties experimental evidence is needed for which we turn to K$_5$Na[Eu$_2$(SO$_4$)$_6$].

Using AlignIt to calculate the symmetry deviations for the Eu(O)$_{10}$ polyhedron in K$_5$Na[Eu$_2$(SO$_4$)$_6$] we show that it closely resembles the bcSAP geometry. The calculated $\sigma_{\text{ideal}}$ value is shown in Table 3 along two examples that previously have been described as nearly perfect real bcSAP coordination polyhedra, namely thorium(IV) oxalate and uranium(IV) oxalate.$^{28,29}$ The symmetry deviation values of Eu(O)$_{10}$ in relation to thorium(IV) oxalate (Th(O)$_{10}$) and uranium(IV) oxalate (U(O)$_{10}$) were calculated assuming that the most optimal coordination symmetries for ten-coordination polyhedra are bcSAP, bcDod and SDod.

<table>
<thead>
<tr>
<th></th>
<th>bcSAP</th>
<th>bcDod</th>
<th>SDod</th>
<th>PP</th>
<th>PAP</th>
<th>OBPy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$D_{4d}$</td>
<td>$D_2$</td>
<td>$D_2$</td>
<td>$D_{5h}$</td>
<td>$D_{5d}$</td>
<td>$D_{8h}$</td>
</tr>
<tr>
<td>bcSAP</td>
<td>0</td>
<td>2.31</td>
<td>7.93</td>
<td>14.46</td>
<td>17.71</td>
<td>16.14</td>
</tr>
<tr>
<td></td>
<td>(2.30)</td>
<td>(6.52)</td>
<td>(13.94)</td>
<td>(15.52)</td>
<td>(14.65)</td>
<td></td>
</tr>
<tr>
<td>bcDod</td>
<td>0</td>
<td>10.48</td>
<td>16.02</td>
<td>18.76</td>
<td>14.26</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(2.13)</td>
<td>(9.15)</td>
<td>(8.95)</td>
<td>(11.75)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SDod</td>
<td>0</td>
<td>17.74</td>
<td>15.93</td>
<td>24.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(10.38)</td>
<td>(6.82)</td>
<td>(16.83)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PP</td>
<td>0</td>
<td>8.00</td>
<td>17.27</td>
<td>22.06</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(7.85)</td>
<td>(20.84)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PAP</td>
<td>0</td>
<td>16.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OBPy</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$a$ Coordinates for the SDod, PP, PAP, OBPy models are reported by Ruiz-Martínez et al.$^{11}$ Coordinates for bcSAP and bcDod were created in Mercury from description by Al-Karaghouli et al.$^{26,27}$ Values in bold are calculated with AlignIt, and values in parenthesis are calculated using SHAPE by Lluenell et al.$^{25}$
Table 3. Symmetry deviation values, σ_{ideal} calculated with AlignIt for ten-vertex polyhedra^{a,b}

<table>
<thead>
<tr>
<th></th>
<th>bcSAP</th>
<th>bcDod</th>
<th>SDod</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eu(O)_{10}</td>
<td>1.15</td>
<td>5.29</td>
<td>7.58</td>
</tr>
<tr>
<td></td>
<td>(1.14)</td>
<td>(1.47)</td>
<td>(5.81)</td>
</tr>
<tr>
<td>Th(O)_{10}</td>
<td>0.58</td>
<td>2.63</td>
<td>9.36</td>
</tr>
<tr>
<td></td>
<td>(0.58)</td>
<td>(2.61)</td>
<td>(5.44)</td>
</tr>
<tr>
<td>U(O)_{10}</td>
<td>0.53</td>
<td>2.53</td>
<td>9.88</td>
</tr>
<tr>
<td></td>
<td>(0.52)</td>
<td>(2.53)</td>
<td>(6.20)</td>
</tr>
</tbody>
</table>

^{a} Pink polyhedron = Eu(O)_{10} in K_{5}Na[Eu_{2}(SO_{4})_{6}]. Orange polyhedra = Ideal structure models (same as Table 1). Blue polyhedra = Th(O)_{10}, U(O)_{10}. Coordinates for the polyhedra were isolated from the reported crystal structures. The values in parenthesis are CShM values calculated with SHAPE.

Th(O)_{10} and U(O)_{10} have high symmetry with a very low σ_{ideal} < 0.6 (and CShM) and are thus well represented as bcSAPs. Eu(O)_{10} has σ_{ideal}(bcSAP) = 1.15, which is slightly higher. Considering the symmetry deviation value of σ_{ideal}(bcDod) = 5.29 and σ_{ideal}(SDod) = 7.58, the structure is clearly best represented by bcSAP. Note that Th(O)_{10} and U(O)_{10} also have a low symmetry deviation value from the bcDod structure. Thus, the Th(O)_{10} and U(O)_{10} is closely related to bcSAP, with distortions maintaining the similarity to bcDod yet differentiating more towards SDod as a high symmetry deviation value is determined here (see Figure 2). For the Eu(O)_{10} polyhedron, the case is reversed.

The low symmetry deviation value of Eu(O)_{10} indicates the symmetry around the Eu(III) ion is close to $D_{4d}$ (bcSAP), with a reduction in symmetry towards $D_{2}$ (SDod). We investigated the loss of symmetry via the optical properties of the system at room temperature and 77 K. To evaluate the transition probabilities, the solid state quantum yield of luminescence from K_{5}Na[Eu_{2}(SO_{4})_{6}] was determined to be 38.5 %, with a luminescence lifetime of 2399 µs. Due to lack of an adequate quantum yield reference for solid state measurement, the dye ATTO 390 dissolved in glycerol was used as a reference to mimic the higher refractive index of the crystal compared to aqueous solution (see Supporting Information for details). We note that the procedure is not optimal; however, we considered it the best of the ones available.
The radiative lifetime can be found from eq. 2,

\[ \tau_0 = \frac{\tau}{\phi_{lm}} \quad \text{eq. (2)} \]

Where \( \tau_0 \) is the radiative lifetime, \( \tau \) is the observed lifetime of the luminescence and \( \phi_{lm} \) is the observed luminescence quantum yield. The radiative lifetime for \( \text{Eu}(\text{O})_{10} \) in the crystal is determined to \( \tau_0 = 6200 \mu s \) and the rate constant for non-radiative deactivation of the luminescent state is \( k_{nr} = 260 \text{s}^{-1} \) (see supporting information for details). For this low-phonon matrix, the luminescence quantum yield is high and the non-radiative rate constant is low as expected. The highly symmetric nine-coordinated \( [\text{Eu}(\text{D}_2\text{O})_9]^3^+ \) complex has a quantum yield of 44 ± 10 %, a \( \tau_0 = 3900 \mu s \) and a \( k_{nr} = 140 \text{s}^{-1} \) in solution.\(^{30} \) The quantum yields and non-radiative rate constants are comparable, however, the radiative lifetime of \( \text{Eu}(\text{O})_{10} \) in the crystal is significantly longer than for \( [\text{Eu}(\text{D}_2\text{O})_9]^3^+ \), which even considering the higher refractive index of the crystal, indicates an overall higher symmetry environment for \( \text{Eu}(\text{III}) \) in the crystal than nonaqua coordinated \( \text{Eu}(\text{III}) \) in solution.

The ligand field perturbation gives rise to the sublevel splitting shown in Table 1. In an ideal bcSAP polyhedron, the central atom will have \( D_{4d} \) point group symmetry.\(^{26} \) However, when the emission slit size is decreased it becomes evident that the \( ^5\text{D}_0 \rightarrow ^7\text{F}_1 \) band split into at least three sublevels, as opposed to the two sublevels expected from \( D_{4d} \) symmetry (see Figure 4).

Further resolving the spectra reveals the splitting of the supposedly degenerate \( m_J \pm 1 \) states.\(^{3,19,34} \) In \( D_{4d} \) symmetry, the three \( ^5\text{D}_0 \rightarrow ^7\text{F}_1 \) transitions would be assumed to reduce to two, one from the single state of \( ^5\text{D}_0 \) to the \( ^7\text{F}_1 \) \( m_J = 0 \) state, and one from the single state of \( ^5\text{D}_0 \) to the two degenerate \( ^7\text{F}_1 \) \( m_J = \pm 1 \) states.\(^{3,19,34} \) As three lines clearly are observed, none of the three states described in the \( ^7\text{F}_1 \) term are degenerate.

The local symmetry around \( \text{Eu}(\text{III}) \) is closely related to the sign of the ligand-field parameters, \( B_0^2 \).\(^{32} \) The ligand-field parameters, \( B_0^2 \) and \( B_2^2 \), can proposedly be determined directly from the \( ^5\text{D}_0 \rightarrow ^7\text{F}_1 \) transition band in the emission spectrum by use of eq. 3 and 4.\(^{32,35,36} \)

\[
\Delta = -\frac{3}{10} B_0^2; \quad \delta = -\frac{\sqrt{6}}{5} B_2^2 \quad (3,4) \]

In axial (e.g. octagonal, tetragonal) symmetry, the \( B_2^2 \) parameter is not present as \( m_J = \pm 1 \) are degenerate. In low symmetry systems, the \( B_2^2 \) can easily exceed 150-200 cm\(^{-1}\).\(^{32} \)
The splitting between the $m_J = \pm 1$, is $\delta = -28$ cm$^{-1}$ (see Figure 4, inset) and between the barycenter of the $m_J = \pm 1$ states and $m_J = 0$ states, is $\Delta = -106$ cm$^{-1}$. The sign of $\Delta$ and $\delta$ is negative if the $m_J = \pm 1$ states are lower in energy than the $m_J = 0$, corresponding to the $m_J = \pm 1$ states being at higher energies than the $m_J = 0$ in the emission spectrum. Therefore, the ligand field parameters in this formalism are $B_0^D = 353$ cm$^{-1}$ and $B_2^Z = 57$ cm$^{-1}$. The positive sign of the $B_2^Z$ parameter is a result of an axial contribution to the ligand field potential. The magnitude of the parameters is a direct measure of deviation from symmetry in Eu(O)$_{10}$, which we compare to $\sigma_{\text{ideal}}$(bcSAP) = 1.15.

CONCLUSION

The coordination environment around europium(III) in K$_5$Na[Eu$_2$(SO$_4$)$_6$] is highly symmetric and closely resembles a bicapped square antiprism (bcSAP) in geometric shape. To evaluate the deviation from ideality, we present the symmetry deviation value, $\sigma_{\text{ideal}}$, which builds on the Continuous Shape Measure approach. The calculated symmetry deviation value for K$_5$Na[Eu$_2$(SO$_4$)$_6$] of $\sigma_{\text{ideal}}$(bcSAP) = 1.15, which we consider to be low. The symmetry deviation value further allows for assigning the direction of distortion, which in this system is towards a staggered dodecahedron (SDod). The photophysics of the europium(III) centers in the crystal allowed an experimental correlation to the quantitative distortion reflected in the shape deviation. Ligand-field splitting of states in $D_{4d}$ must be symmetric, however, the splitting in the $^7F_1$ band ($\Delta m_J(\pm 1) = 28$ cm$^{-1}$, $\Delta m_J(\pm 1, 0) = 106$ cm$^{-1}$), and a relative low radiative lifetime ($\tau_0 = 6200$ µs) show that the low distortion reported by the symmetry deviation analysis has a significant impact on the physicochemistry of the lanthanide(III) center.

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Author Contributions

The manuscript was written through contributions of all authors. All authors have given approval to the final version of the manuscript.

Supporting Information

AlignIt program files, AlignIt manual, CIF and CheckCIF files, Example and Input files, Experimental and Crystallographic details, Optical spectra and Diffractoframs are available as supporting information.

The Supporting Information is available free of charge on the website.

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SYNOPSIS

Symmetry deviation analysis is revisited to evaluate the coordination environment in a ten-vertex europium(III) polyhedron in a K$_5$Na[Eu$_5$(SO$_4$)$_6$] crystal. After determining the symmetry deviation from ideal polyhedra, the effect of even minor distortions from ideal symmetry is investigated from the splitting of the solid-state luminescence emission of the crystal. The ligand-field perturbation of the energy levels in the lanthanide(III) ion as a results of asymmetry is estimated from the ligand-field parameters.