Visualizing the Avogadro Number

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Abstract

Background

On the occasion of the redefinition of the Avogadro constant in May 2019, a brief history and some didactic reflections on its magnitude are presented.

Purpose

Some analogies are reviewed and others are suggested to help visualize the extent of its magnitude, and their usefulness is assessed.

Design/Method

These analogies are set in the teaching context of the first and second courses of the degrees in several scientific and technic disciplines. Their effectiveness is discussed for the first time on the basis of a questionnaire filled by the corresponding students.

Results

The suggestions for educating and learning are that the most helpful models, following the opinion of the students, are those related to more substantial items, for example, neurons, individuals, planets, above analogies on geometric constructions.

Conclusions

Challenging current thought, pictorial descriptions are not all the times so advantageous.

1 INTRODUCTION

1.1 On analogies in science

Analog learning involves transferring familiar ideas on one topic to another related topic when both share the same underlying conceptual structure (Sidney & Thompson 2019). Analogies are key tools in science understanding (Shapiro 1985). Thus, the importance of knowing how to build successful analogies to illustrate elusive concepts has been highlighted in various publications from many different points of view. However, the potential of analogies as a teaching tool has not always been fully exploited (Treagust et al. 1989, Treagust 1993, Harrison 2001).

A non-exhaustive literature review of research on analogies is presented here. Examples of classic studies could be that of Levinson and Carpenter (1974), who investigated analogical reasoning in individuals aged 9 to 15 years and proposed a strategy to develop this ability, and Gentner (1983) who has presented the theoretical analysis of analogy that underlies most research on analogical learning: the structure-mapping theory. This framework describes how the meaning of the analogy is derived from the meanings of its parts, and the rules followed by mapping knowledge from a base domain to a target domain. More recently, Thiele & Treagust (1991) have explored the potential of analogies in teaching chemistry, their definition, types and uses. Glynn (1991) has exemplified the use of analogies by the scientists and has given guidelines for teaching with analogies. Sutton (1993) has placed analogy and metaphor within the broader context of figurative language, studied its evolution in the development of new scientific ideas, and traced the relationship between this type of expression and the direct description of phenomena. Treagust (1993) has proposed the model FAR (Focus, Action, Reflection) to guide the analogy teaching approach. Ruef (1998) has presented a program to develop critical thinking skills with the invention of analogies by the students. Markman & Moreau (2001) have discussed the role of the analogies in decision making. Coll et al. (2005) have reviewed the previous literature on analogies and underlined their relevance and motivational potential. They have delved into aspects such as criticism of the scope and limitations of the student's own analogies and those of others, and explored the social interaction through the group's work. Aubusson et al. (2006) have reviewed the principles and use of analogies and metaphors in educational research. Goswami (2007) has extensively reviewed the studies on the development of

analogical reasoning from early childhood to learn about the world. She reveals its pervasiveness in all creative activity, including artistic expression, problem solving, and analogies in science. Glynn (2008) has deeply studied the analogy in teaching strategies and its various steps, noting that it basically consists of comparing a topic with which the students are already familiar, with the new topic to be introduced. Raviolo & Garritz (2009) have reviewed and analyzed the use of analogies in the teaching of chemical equilibrium. Mair et al. (2009) have systematically reviewed the literature on non-trivial problem solving by experts based on analogy and have highlighted that analogical reasoning plays an important role in problem solving and that this skill is what characterizes an expert. Bellocchi (2009) has reviewed and discussed the usefulness and limitations of analogies in science. Gentner (2010) has analyzed the development of cognition in children on the basis of the analogical ability and the possession of a symbol system, that are mutually supportive. Etzion & Ferraro (2010) have studied the role of analogies in argumentative reasoning, applied to institutional change in the framework of public organizations. Petrucci (2011) has proposed building bridges between science and humanities through scientific visual analogies for cross-disciplinary teaching and research. Klahr & Chen (2011) have given a discussion of analogical transfer in general, as appropriate to school contexts. Day & Goldstone (2012) have reviewed and articulated different visions of knowlegde transference that motivates our conceptualization of the implicit analogy. Mozzer & Justi (2013) have studied and analyzed how chemistry teachers apply their analogical reasonings and ellaborate analogies. Vendetti et al. (2015) have provided an overview of analog reasoning studies that could be applied to classroom learning and they also present an explicit analogical support case. English (2013), and Sidney & Thompson (2019) have reviewed the research on analogies in the field of mathematics.

The documents analyzed implicitly affirm that the knowledge of nature is different from the analogies used to transmit it. But perhaps this knowledge is just another deeper analogy since it is based on models, and these can evolve as scientific knowledge expands. In the authors' opinion, analogies are not only the way knowledge is transmitted, but even the way the brain processes knowledge. This raises the philosophical question of what we can know about nature. It seems clear that things are learned through our perception. So can we understand the reality of what things are without representing them internally by analogies?

1.2 About Avogadro's number

Attempts to visualize the magnitude of the Avogadro constant has been a direct subject of didactic journals (Cervellati et al. 1982, Goh et al. 1994, Pekdağ and Azizoğlu 2013). The concept of mole, as well as the amount of substance are central for the definition of the Avogadro constant, even if they are not always clearly understood and need further clarification (Schmidt 1997, Giunta 2015, Davis 2015, Rees et al. 2018, Mweshi et al. 2019)

During 2017, the measurements made to calculate a more precise value for the Avogadro constant set its value to $6.022140758(62) \cdot 10^{23}$ mol^{-1}. On May, 20 2019 took effect the redefinition of SI units for this constant, designated with the symbol N_A, and its value is fixed to exactly $6.02214076 \cdot 10^{23}$ mol^{-1}. The Avogadro number, instead, is a dimensionless quantity, and has the same numerical value of the Avogadro constant (Wikipedia contributors 2020a). Actually, the revised 2019 definition of mole breaks the link to the kilogram by making a mole a specific number of entities of the substance in question: the mole, symbol mol, is the SI unit of amount of substance. One mole contains exactly $6.02214076 \cdot 10^{23}$ entities.

Moreover, Avogadro's law, sometimes called hypothesis (Nernst 1904, Chang 1994), states that at constant pressure and temperature the volume of an ideal gas is directly proportional to the number of atoms or molecules regardless of the nature of the gas, i.e., $V = kn$ (k = proportionality constant). Thus, in a reaction between two gases we have: $V_1 : V_2 = n_1 : n_2$.

This study presents a brief story of the Avogadro number, then reviews some analogies and their types to reach understanding of its magnitude, propose new ones, and performs a survey among university students to gain insight into their degree of perception of the different types of analogies, for the first time.

1.3 A little bit of History

Lorenzo Romano Amedeo Avogadro di Quarenga e di Cerreto (1776-1856), born in Turin, became Doctor of Law in 1796 and, in 1806 he taught physics in a college in his home-town, where he afterwards became professor of mathematical physics. He was removed from his chair in 1822 soon after the European turmoil against absolutist regimes that reached Turin in 1821. He was restored to it in 1835, but he continued to

be little known in Italy and still less abroad. His important study appeared in French in 1811 and in 1814, and passed completely unnoticed. Stanislao Cannizzaro (1826-1910) rediscovered it (and used it to state that the hydrogen molecule was composed of two atoms) and made it known at the Karlsruhe meeting in December, 1860. Cannizzaro had already published his 46 page letter *Sunto di un corso di filosofia chimica* (Summary of a course in chemical philosophy) in 1858 in *Nuovo Cimento* (Cannizzaro 1858). This rediscovery allowed Cannizzaro to derive a clear definition for the concept of molecule and the calculation of more precise atomic and molecular weights. Lothar Meyer (1830-1895), chemistry professor at the University of Tübingen (1876–1895), had attended the Karlsruhe Congress and made the Avogadro's law one of the basis of his *Modernen Theorien der Chemie*, published in 1864 (Meyer 1864, Partington 1989). Thus, finally, in 1864, Avogadro's studies reached the international scientific community.

The French physicist Jean Baptiste Perrin made the first determination of the number of molecules contained in one mole of gas under normal conditions by several methods and, in 1909, proposed naming this constant after Avogadro (Jensen 2007). A detailed description of all this advent can be found in Morselli 1984.

2 BACKGROUNG

2.1 Literature review

Thomson and Opfer (2010) have studied the numerical cognition in children and have concluded that it is more difficult as the order of magnitude increases. After our experience in teaching, we have arrived to the conclusion that the students are not aware of the magnitude of a unit followed by 23 zeroes. With this purpose in mind, we devised a set of concrete examples to visualize the concept of mole. These activities were presented to students of the first and second courses in chemistry, Environmental Sciences, Biotechnology, and Biology of the Universities of Girona and Valencia, and the results are presented at the end of this paper.

We should point out that 'mole' pertains to the same category of nouns as 'pair', 'trio' and 'dozen', that is, collective numerals, that specify a concrete grouping of entities. In our case, due to language abuse and for short, we say 'one mole of water', where we should say 'one mole of water molecules'. In fact, we use it correctly when say 'one mole of photons' or 'of electrons'.

A brief revision of the analogies found in previous literature is here presented:

Poskozim et al. (1986) reviewed the former literature and collected a series of analogies, classified in six main groups, with the goal to give an idea of the utter immensity of the Avogadro's number. These groups are based, respectively, on objects, counting, people, water, money, and computing. The analogies do not equal the ones presented in the present paper, even if they have a similar structure and intent. Notice that Poskozim et al. do not give any introductory historical information about the Avogadro number. The authors argue that when these analogies use numbers of lesser magnitude, they are more meaningful. The classification of analogies presented in that paper can be very useful and are adopted in the present paper, and probably they are not only based on different topics, but also on different degrees of abstraction, as we shall see later on.

After this review, other works of the same type have appeared, and they are here glossed. The work of van Lubeck (1989) lays out three analogies of 'objects' type: one related to ants (one mole would occupy 1000 Earths), other related to 0.3 mm (0.012 in) sand grains (one mole would fill a cube of 30 km side with $3 \cdot 10^{19}$ kg weight, or 2 m deep in the Sahara Desert), and one about finance (in one second, the interest of $ 1 mole, would render $200,000 for each one of 5 billion people).

Diemente (1998) has also explored the topic of the sand grains of the Sahara Desert, where the grains are 1/100 of an inch on edge. He has also proposed the one-mole times volume increase of a 6-inch ball, that would equal the Earth size, and has also studied the arraying of atoms in one, two and three dimensions. This subject that will be covered in this article as well.

Uthe (2002) has presented an analogy of 'counting' type. According to this, one mole of seconds is over four million times the age of the earth and over a million times the age of the universe.

For additional information, the reader can refer to Furió et al. (2002). This work reviews the difficulties in learning the concepts of amount of substance and mole, as well as the didactic strategies to overcome these, including published examples of analogies for the Avogadro number.

We can begin our exploration by considering some other huge numbers to gain insight in these magnitudes, such as the grains of sand in our planet, or the number of stars in the observable universe. Planet Earth contains around $7.5 \cdot 10^{18}$ grains of sand and even if it is said (Krulwich 2012) that this is a big figure, it is less than the

Avogadro number. Following the European Space Agency (2019), there are something like 10^{11} to 10^{12} stars in our Galaxy, and there are something like 10^{11} or 10^{12} galaxies, and that means something like 10^{22} to 10^{24} stars in the Universe. This is only a rough number, as not all galaxies are identical, just like on a beach the depth of sand at different sites is not the same. It is remarkable that Archimedes (288-212 BCE) in his work entitled *The Sand-Reckoner* calculated the total number of grains of sand the universe could contain (Harrison 2000). He first estimated the volume of his known universe, using a grain of sand as his unit of volume, and he found that the universe had a volume of 10^{63} grains of sand. Remarkably, this volume of sand had a mass equal to our present estimates of the mass of the observable universe (National Solar Observatory 2001), which is about $6 \cdot 10^{51}$ kg. Let us assume that we are dealing with quartz grains, then each weighs $\sim 1.1 \cdot 10^{-2}$ g (Sepp 2011). Dividing this result by the mass of a nucleon ($1.6726 \cdot 10^{-24}$ g) we have that each grain of sand contains $7 \cdot 10^{21}$ nucleons, and that means $\approx 10^{63+22} = 10^{85}$ nucleons in the observable Archimedean universe. The actual number of atoms in our observable universe is estimated to be in between 10^{78} and 10^{82} atoms (Villanueva 2018).

2.2 Imagining the magnitude of one mole

Going back to the concept of mole, let us visualize the dimensions of a thread, of a surface, and of a cube that are made of N_A consecutive atoms each with diameter 10^{-8} cm. Actually, dimensionality plays a key role in this example. In passing to two, and then to three dimensions we notice a drastic reduction of the "size", that seem to be rather counterintuitive. A simple calculation for a unidimensional thread gives a length of $6.022 \cdot 10^{10}$ km (1 km = 0.6214 miles). As a first comparison, the furthest distance Earth-Pluto is about $7.5 \cdot 10^9$ km (when the two bodies are on the opposite sides of the Sun). Note that the distance Earth-Sun is *only* about $1.5 \cdot 10^8$ km and the next star near to Earth is Proxima Centauri, at $4 \cdot 10^{13}$ km (or 4.24 light-years). If we build a square with N_A consecutive atoms with diameter 10^{-8} cm placed in a rectangular fashion (i.e., not dense packing) one gets a square with a 77.6-meter side (6023 m^2 equivalent to 1.2 soccer fields of 100×50 m^2). If we build a cube with the previously described N_A atoms, placing them in a cubic arrangement (again a non-compact disposition), the cube will be of 0.84 cm side! This dimension gives a human-scale entity that all of us could manipulate.

Another image to visualize the concept of mole can be constituted by a prism of square base of 100 km (62.14 miles) of side and height of 60 km (37.28 miles). That structure would contain $6 \cdot 10^{23}$ cubic millimeters. Think that 100 km is the average width of Jutland Peninsula in Europe, and nearly the distance from Washington, DC to Hagerstown, Maryland. Sixty km is instead, the height of the mesosphere, where the main part of the meteorites that enter the atmosphere are burned, a height that only rockets can reach. In any case, the fact that the entire global structure and their elements are impossible to visualize simultaneously is worth thinking about. Now, let us suppose a building similar to the Atomium in Brussels (the monument representing a unit cell of iron but magnified about $1.65 \cdot 10^{11}$ times), but containing one mole of spheres instead of just nine. The diameter of each sphere is 18.0 m and the length of the cube edge tubes is 29.0 m (Pinto 2012). Thus, in an edge you find the center of one sphere with a radius of 47 m. The edges will contain as many spheres as the cubic root of the Avogadro number, so the length of the edge should be $8.45 \cdot 10^7 \cdot 47$ m $\cong 4 \cdot 10^6$ km $\cong 2.5 \cdot 10^6$ miles. Keep in mind that the diameter of the Sun is $1.4 \cdot 10^6$ km ($8.7 \cdot 10^5$ miles).

The following example concerns the ancient Indian legend about the discovery of the game of chess and the request of its inventor that is related to a geometric progression of rice or wheat grains (Wikipedia contributors 2020b): 1, 2, 4, 8, ... The number of grains in the last square is 2^{63} = 9,223,372,036,854,775,808 and the needed ones to fill the entire chessboard is 2^{64}-1 = 18,446,744,073,709,551,615, i.e., a 32646-th of the number of Avogadro. If the process to fill one single chessboard needs an impressive number of grains, in order to reach the N_A quantity one needs to fill 32646 boards! This tells us that we would need a bigger chessboard (i.e., with more squares) in order to apply the constructive process for a single chessboard and reach N_A. The number of chess table squares to advance in order to reach the N_A is $1+\log_2 N_A$= 1+ 78.99 $\approx$ 80, that is, we need a 9×9 board, or a Sudoku grid, and one single square will remain empty. The sum of the grains collected in the first n successive squares is 2^n-1, so the number of Avogadro grains fill, according to the series, the first 79 squares. As single chess table is not enough, we can try to change the legend a little and consider a new constructive progression having a ratio of 3. In this case the series of grains put in each square is 1, 3, 9, 27, ... So, Avogadro's number is reached in box number $1+\log_3 N_A$ = 50.84 $\approx$ 51. As the sum of the first n terms of this progression is given by $(3^n$-1)/2, and the N_A grains are used when filling also the 51-th square.

The last image turns around people on Earth. The Earth surface is approximately (r(Earth) = 6370 km = 3960 miles) 510,100,000 km^2 = 5.101·10^8 km^2 = 5.101·10^{14} m^2. If a person occupies $^1/_4$ of m^2, we need "several" layers of people in order to fit N_A humans. We do not take now into account that increasing the height of this human mountain will, at least theoretically, lead for more room for people at each layer (we are filling a spherical surface, not a flat one). Within a plane model, that is, a planar surface with the same area than our planet, the number of needed layers is impressive: 295,138,208. Taking as mean height of a person 1.70 m, this gives a block of people of 501,735 km (311,764 miles) high, i.e., humans would have by far surpassed the Moon [d(Earth-Moon) = 384,400 km = 238,900 miles]. A more accurate estimation could be obtained considering that each layer of humans provides a new floor (above their heads) with increased spherical area. In this case, 19,449,493 layers are needed, resulting in an Earth with a spherical shell with a height of 33,064 km (20,545 miles) above the Earth floor, roughly a tenth of the Earth-Moon distance.

Finally, as a curiosity, we can estimate how many years would be needed to reach a population of N_A persons with the present growth rate (Wikipedia contributors 2020c). The following equation can be deduced:

$$7.7 \cdot 10^9 \, (1 + 0.0114)^t = 6.022 \cdot 10^{23}$$

The result is rather amazing, $t = 2822$ years, not a lot of time. Now, a similar calculation could be considered in relation to the US national debt. Current debt is about 20 trillion dollars. This value is uncertain and very variable depending on the source of information, but this is not so relevant related to our goal to obtain an approximation of the year the nation will reach the N_A debt in dollars. Estimating a constant growth rate of 1% (2020), the equation is now

$$20 \cdot 10^{12} \, (1 + 0.01)^t = 6.022 \cdot 10^{23}$$

The result is even more astonishing: $t = 2425$ years.

3 METHODS

3.1 Participants

The study was conducted at the Universities of Girona and Valencia, Spain. Ethical authorization, when needed, was received from the management of the two universities to carry out the study. Students were fully informed about the study and that the anonymous interview data was to be recorded and analyzed.

At the University of Girona, the questionnaire was presented to 92 students of the first course of the Degrees in Chemistry, Environmental Sciences, Biotechnology, and Biology, and students of the second course of the Degree in Chemistry. At the University of Valencia, it was released to 41 students of Degree in Chemistry, in the context of the first course curriculum 'Informatics for Chemistry', and the second course matter 'Physical Chemistry Laboratory I". Male/female ratio was 42/58 for Girona and 37/63 for Valencia. The two universities are public and the rest of demographic data for their respective students are very close. Only 5 % of students had completed pre-university studies abroad, and the same percent of them worked sporadically for three months or less. Regarding the parents of the students, 40% had higher education, 57% had studied primary education only and 2% had no education. The difference between fathers and mothers was not great: the relation fathers/mothers for the three previous levels was, respectively, 39/41, 56/56, 5/0. The students were 18-20 years old and with adequate scientific background. Their participation in the study was optional and anonymous.

3.2 Questionnaire

One analogy was chosen among the ones presented in this study and four from the bibliography. The criterion followed for their selection was the maximum diversity according to the groups proposed by Poskozim et al. (1986), excepting the computer-related topic. This was set apart due to the fact that the technological evolution tends to make these analogies readily obsolete.

The question was formulated as:
Rate the following analogies from 5 to 1 according to their usefulness to make you understand the magnitude of Avogadro's number (5, very useful; 1, not useful):

1- A square base prism of 100 km side and 60 km high would approximately contain an Avogadro number of cubic millimeters.
2- For $1.9 \cdot 10^{16}$ years a human would have to count a grain of sand per second to match Avogadro's number. This number is more than four million times the age of the earth (4.6 billion years) and more than one million times the age of the universe (about 14 billion years).

3- Assuming that the Earth has 7 billion people and that each person has 10 billion neurons, on 8,600 planets with the population of the Earth there would be an Avogadro number of neurons.

4- There are approximately two Avogadro numbers of milliliters of water on Earth.

5- Suppose we have a Avogadro number of euros. The bank offers an annual interest of 5%. If we divide the interest received in one second among the entire population on earth (7 billion), each individual will get about 140,000 euros.

The five analogies can be sketched as 1: Objects, 2: Counting, 3: People, 4: Water, 5: Money. Table 1 shows them with their respective references.

Table 1 Analogies included in the questionnaire

Analogy #	Type	Phrasing	Reference
1	Objects	A square base prism of 100 km side and 60 km high would approximately contain an Avogadro number of cubic millimeters.	This paper
2	Counting	For $1.9 \cdot 10^{16}$ years a human would have to count a grain of sand per second to match Avogadro's number. This number is more than four million times the age of the earth (4.6 billion years) and more than one million times the age of the universe (about 14 billion years).	Uthe 2002
3	People	Assuming that the Earth has 7 billion people and that each person has 10 billion neurons, on 8,600 planets with the population of the Earth there would be an Avogadro number of neurons.	Poskozim et al. 1986
4	Water	There are approximately two Avogadro numbers of milliliters of water on Earth.	Poskozim et al. 1986
5	Money	Suppose we have an Avogadro number of euros. The bank offers an annual interest of 5%. If we divide the interest received in one second among the entire population on earth (7 billion), each individual will get about 140,000 euros.	van Lubeck 1989

4 RESULT AND DISCUSSION

Figure 1 shows the results independently obtained by the students of the two Universities. The high similarity between the two graphs reveals the significance of the

outcome. Despite the fact that the analogies' scores are numerical, they can be also understood as being levels. In this case all the Chi-squared significance tests performed among the tables of results gave very low p-values. This means that there are significant differences between the analogies and their levels. The complete data and their statistics are included in the supplementary material.

Curiously, it seems that the most rated analogies correspond to the most familiar concepts, and the less rated with the most abstract ones. Thus, it seems advisable when teaching N_A to use analogies related to people or countable entities, rather than those related to sizes of geometrical figures or high finance.

With respect to the previous literature, Skorstad et al. (1987), Faries & Reiser (1988) and Sidney & Thompson (2019) have related the usefulness of analogies on the close connection, between analogy and target. In our case, this principle does not apply since all the analogies studied related to N_A in the same way, that is, through the representation of quantity.

Duit (1991) has classified analogies into different groups including verbal, pictorial, personal, bridging and multiple. Treagust (1991) has concluded that the first three types are the most useful for educational purposes. Personal analogies approximate abstract concepts to student's real-world circumstances, and this type seems to best define the highest-scoring analogy in this study.

Shapiro (1985) has emphasized the importance of visualizations in scientific analogies, and Thiele & Treagust (1991) have studied the usefulness of analogies in secondary chemistry teaching. They concluded that, since one of the main emphases of using analogies in chemistry education is to make abstract concepts more easily grasped by the underperforming student, the use of a diagram or image to present the analog is considered more advantageous. In pictorial analogies, an image of a familiar real-life situation is central to the analogy. A pictorial analogy should prevent the student from mentally creating attributes not present in the explained concept and also avoid the need for long prose to describe the analog. However, in our study, the most pictorial representation, the giant prism of analogy #1 was the less rated. A pictorial representation should increase the likelihood that the analogue is familiar to the learner (Duit 1990), and this is not the case here. This could be due to the fact that this visual representation, in our case, is so huge that students cannot assimilate it.

Tobin (2006) has concluded that metaphors are useful if they are consistent with socially shared views in one field. Previously, Tenney & Gentner (1985) had emphasized that greater familiarity with the base domain of an analogy improved the usefulness of the analogy. In line with this, Dagher (1995) has reviewed studies on the effectiveness of analogies and synthesized their findings. She has also proposed teaching strategies in the application and evaluation of analogies. Her conclusion is that the familiarity of the source domain and its accessibility or lower complexity than the target one, determines the usefulness of the analogy. This could be in accordance with the results obtained in this study regarding the familiarity of the concepts involved. People are more familiar than numbers or water, and these, more familiar than geometry or finance.
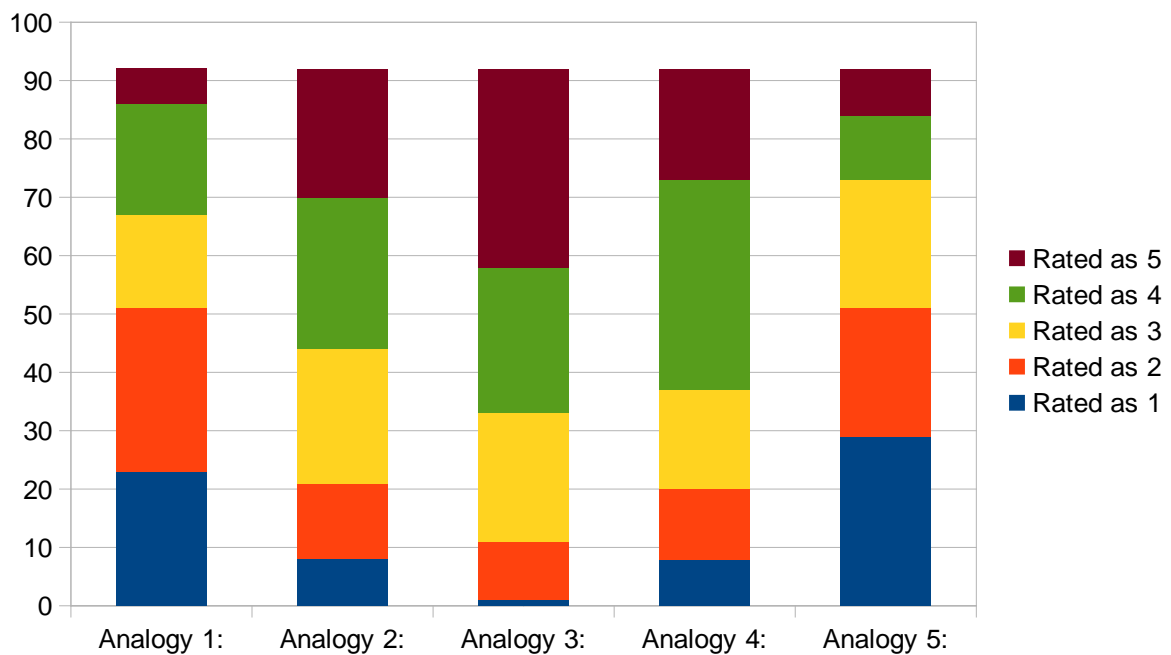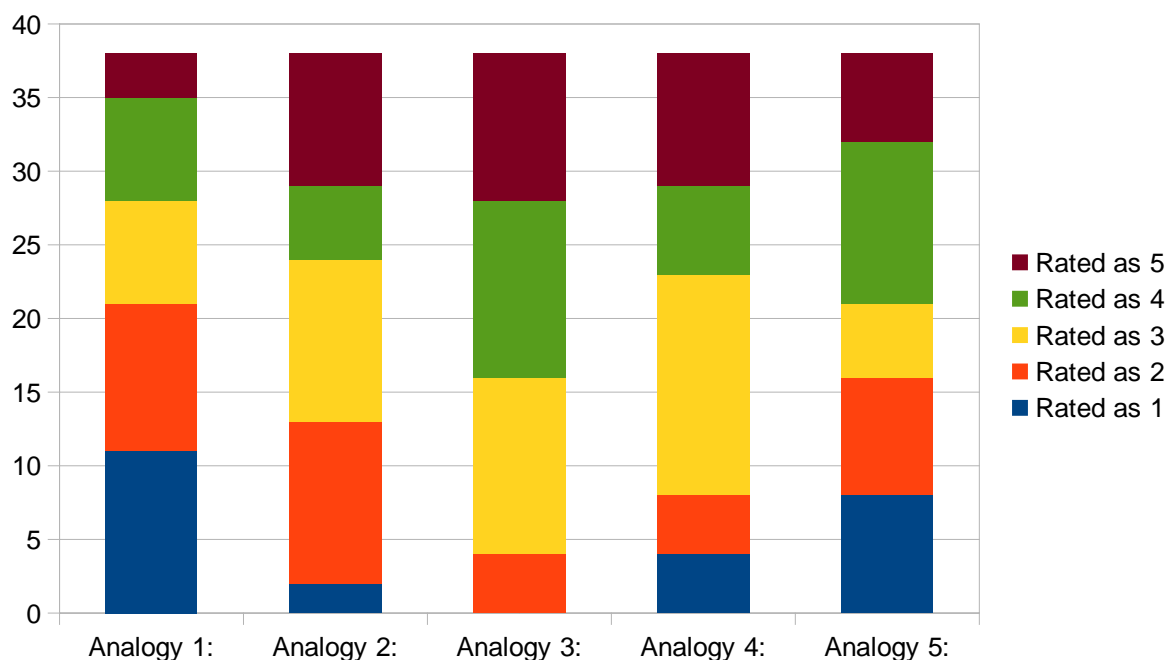
Figure 1. Distribution of the answers to the questionnaire. Upper: University of Girona. Lower: University of Valencia.

4 CONCLUSIONS

When handling large numbers, it is always worth trying to visualize them as analogies, as it helps to get a more accurate idea of their meaning. Throughout the present paper, we have reviewed the topic and have tried to give an idea of how great N_A is with a series of concrete examples. Imagining these magnitudes gives us a more accurate idea of what huge numbers of the Avogadro's order mean. Our paper evaluates students' perceptions of analogies for Avogadro number. The implications for teaching and learning are that the most useful examples, following the opinion of the students, are those related to more tangible objects such as neurons, people, planets, above the analogies about geometric constructions. Thus, in contrast current thinking, pictorial representations are not always so convenient. They are useful whether increase the familiarity of the target.

To sum up, regarding the analogies for Avogadro number:

a) The best analogies are those that imply familiar concepts. Thus, personal analogies approximate abstract concepts to student's real-world circumstances, in line with the highest-scoring analogy in this study.

b) The principle of connection analogy-target does not apply since all the analogies studied illustrate the representation of a quantity.

c) Pictorial representations based on geometric objects are not good.

SUPPLEMENTARY MATERIAL

The original questionnaire and spreadsheets with their complete data and results.

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References

Aubusson, P. J., Harrison, A. G., & Ritchie, S. M. (2006). Metaphor and analogy. In *Metaphor and analogy in science education* (pp. 1-9). Dordrecht, Netherlands: Springer.

Avogadro, A. (1811). Essai d'une manière de déterminer les masses relatives des molécules élémentaires des corps, et les proportions selon lesquelles elles entrent dans ces combinaisons. *Journal de Physique - Paris*, *73*, 58-76. doi:10.1007/bf02835690

Bellocchi, A. (2009). *Learning in the third space: a sociocultural perspective on learning with analogies* (pp. 19-45). Doctoral dissertation, Queensland University of Technology, Brisbane, Queensland, Australia.

Cannizzaro, S. (1858). Lettera del prof. Stanislao Cannizzaro al prof.S. De Luca; sunto di un corso di filosofia chimica, fatto nella R. Universita di Genova. *Il Nuovo Cimento*, *7*(1), 321–368. doi:10.1007/bf02827711

Cervellati, R., Montuschi, A., Perugini, D., Grimellini-Tomasini, N., & Balandi, B. P. (1982). Investigation of secondary school students understanding of the mole concept in Italy. *Journal of Chemical Education*, *59*(10), 852. doi:10.1021/ed059p852

Chang, R., & Goldsby, K. A. (2016). *Chemistry* (12th ed., pp. 183-184). New York, NY: McGraw Hill Education.

Coll, R. K., France, B., & Taylor, I. (2005). The role of models and analogies in science education: implications from research. *International Journal of Science Education*, *27*(2), 183-198.

Dagher, Z. (1995). Review of studies on the effectiveness of instructional analogies in science education. *Science Education*, *79*(3), 295-312.

Davis, R. S. (2015). What is a kilogram in the revised international system of units (SI)? *Journal of Chemical Education*, *92*(10), 1604–1609. doi:10.1021/acs.jchemed.5b00285

Day, S. B., Goldstone, R. L. (2012). The import of knowledge export: connecting findings and theories of transfer of learning. *Educational Psychologist*, *47*, 153–176.

Diemente, D. (1998). Demonstrations of the enormity of Avogadro's number. *Journal of Chemical Education*, *75*(12), 1565. doi:10.1021/ed075p1565

Duit, R. (1990). On the role of analogies, similes, and metaphors in learning science. In *Paper presented at the Annual Meeting of the American Educational Research Association, Atlanta.*

Duit, R. (1991). On the role of analogies and metaphors in learning science. *Science education*, *75*(6), 649-672.

English, L. D. (2013). Analogies, metaphors, and images: vehicles for mathematical reasoning. In *Mathematical Reasoning* (pp. 11-26). New York, NY: Routledge.

Etzion, D., & Ferraro, F. (2010). The role of analogy in the institutionalization of sustainability reporting. *Organization Science*, *21*(5), 1092-1107.

European Space Agency (2019). How many stars are there in the Universe? Retrieved from http://www.esa.int/Science_Exploration/Space_Science/Herschel/How_many_stars_are_there_in_the_Universe

Faries, J. M., & Reiser, B. J. (1988). *Access and use of previous solutions in a problem solving situation* (No. CSL-29). PRINCETON UNIV NJ COGNITIVE SCIENCE LAB.

Furió, C., Azcona, R., & Guisasola, J. (2002). The learning and teaching of the concepts 'amount of substance' and 'mole': a review of the literature. *Chemistry Education Research and Practice*, *3*(3), 277–292. doi:10.1039/b2rp90023h

Gentner, D. (1983). Structure-mapping: a theoretical framework for analogy. *Cognitive Science*, *7*, 155–170.

Gentner, D. (2010). Bootstrapping the mind: analogical processes and symbol systems. *Cognitive Science*, *34*(5), 752-775.

Giunta, C. J. (2015). The mole and amount of substance in chemistry and education: beyond official definitions. *Journal of Chemical Education*, *92*(10), 1593–1597. doi:10.1021/ed5007376

Glynn, S. M. (1991). Explaining science concepts: a teaching-with-analogies model. *The psychology of learning science*, 219-240.

Glynn, S. M. (2008). Making science concepts meaningful to students: teaching with analogies. Four Decades of Research in Science Education-from Curriculum Development to Quality Improvement: From Curriculum Development to Quality Improvement, 113. Münster, Germany: Waxmann.

Goh, N. K., Subramanian, R., & Chia, L. S. (1994). A more direct feeling for Avogadro's number. *Journal of Chemical Education*, *71*(8), 656–657. doi:10.1021/ed071p656

Goswami, U. (2007). Analogical reasoning in children. In *Children's Learning in Laboratory and Classroom Contexts* (pp. 73-88). New York, NY: Routledge.

Harrison, E. R. (2000). *Cosmology: the science of the universe* (2nd ed.). Cambridge, UK: Cambridge University Press.

Harrison, A.G. (2001). How do teachers and textbook writers model scientific ideas for students? *Research in Science Education*, *31*, 401–435.

Jensen, W. B. (2007). How and when did Avogadro's name become associated with Avogadro's number? *Journal of Chemical Education*, *84*(2), 223. doi:10.1021/ed084p223

Klahr, D., Chen, Z. (2011). Finding one's place in transfer space. *Child Development Perspectives*, *5*, 196–204.

Krulwich, R. (2012). *Which is greater, the number of sand grains on earth or stars in the sky?* Retrieved from https://www.npr.org/sections/krulwich/2012/09/17/161096233/which-is-greater-the-number-of-sand-grains-on-earth-or-stars-in-the-sky

Levinson, P. J., & Carpenter, R. L. (1974). An analysis of analogical reasoning in children. Child development, 857-861.

Lubeck, H. V. (1989). How to visualize Avogadro's number. *Journal of Chemical Education*, *66*(9), 762. doi:10.1021/ed066p762

Mair, C., Martincova, M., & Shepperd, M. (2009). A literature review of expert problem solving using analogy. In: 13th International Conference on Evaluation & Assessment in Software Engineering, 20-21 April 2009, Durham, UK.

Markman, A. B., & Moreau, C. P. (2001). Analogy and analogical comparison in choice. *The analogical mind: Perspectives from cognitive science*, 363-399.

Meyer, L. (1864). *Die modernen Theorien der Chemie und ihre Bedeutung für die chemische Statik* (1st ed.). Breslau, Prussia: Maruschke & Berendt.

Morselli, M. (1984). *Amedeo Avogadro, a scientific biography* (pp. 87-271). Dordrecht, Netherlands: D. Reidel.

Mozzer, N. B., & Justi, R. (2013). Science teachers' analogical reasoning. *Research in science education*, *43*(4), 1689-1713.

Mweshi, E., Munyati, O., Nachiyunde, K. (2019). Teacher's mole concept pedagogical content knowledge: developing the model for the mole concept content representations framework. *Journal of Education and Practice.* *10*(8), 51–65. doi:10.7176/jep/10-8-08

National Solar Observatory (2001). Mass, size, and density of the universe. Retrieved from https://people.cs.umass.edu/~immerman/stanford/universe.html

Nernst, W. (1904). *Theoretical Chemistry from the Standpoint of Avogadro's Rule & Thermodynamics.* (2nd ed., pp. 39-42). London: Macmillan & Co. Ltd.

Partington, J. R. (1989). *A Short History of Chemistry* (3rd ed.). New York, NY: Dover Publications.

Pekdağ, B., & Azizoğlu, N. (2013). Semantic mistakes and didactic difficulties in teaching the "amount of substance" concept: a useful model. *Chemistry Education Research and Practice*, *14*(1), 117–129. doi:10.1039/c2rp20132a

Petrucci, M. (2011). Scientific visualizations: bridge-building between the sciences and the humanities via visual analogy. *Interdisciplinary Science Reviews*, *36*(4), 276-300.

Pinto, G. (2012). An example of body-centered cubic crystal structure: the Atomium in Brussels as an educative tool for introductory materials chemistry. *Journal of Chemical Education*, *89*(7), 921–924. doi:10.1021/ed200841y

Poskozim, P. S., Wazorick, J. W., Tiempetpaisal, P., & Poskozim, J. A. (1986). Analogies for Avogadro's number. *Journal of Chemical Education*, *63*(2), 125.

Raviolo, A., & Garritz, A. (2009). Analogies in the teaching of chemical equilibrium: a synthesis/analysis of the literature. *Chemistry Education Research and Practice*, *10*(1), 5-13.

Rees, S., Kind, V., & Newton, D. (2018). The development of chemical language usage by "non-traditional" students: the interlanguage analogy. *Research in Science Education*, 1-20. doi:10.1007/s11165-018-9801-0

Ruef, K. (1998). The private eye: (5X) looking/thinking by analogy. Seattle, WA: The Private Eye Project.

Schmidt, H. (1997). An alternate path to stoichiometric problem solving. *Research in Science Education* **27,** 237. doi:10.1007/BF02461319

Sepp, S. (2011). Brain games with sand grains. Retrieved from https://www.sandatlas.org/brain-games-with-sand-grains

Shapiro, M. A. (1986). Analogies, visualization, and mental processing of science stories. *Annals of the International Communication Association*, *9*(1), 339-355.

Sidney, P. G., & Thompson, C. A. (2019). Implicit analogies in learning: supporting transfer by warming up. *Current Directions in Psychological Science*, *28*(6), 619-625.

Skorstad, J., Falkenhainer, B., & Gentner, D. (1987). Analogical processing: a simulation and empirical corroboration. *Proceedings AAAI*, **6**, 322-326.

Sutton, C. (1993). Figuring out a scientific understanding. *Journal of Research in Science Teaching*, *31*(10), 1215-1227.

Tenney, Y., & Gentner, D. (1985). What makes analogies accessible: experiments on the water-flow analogy for electricity. In R. Duit, W. Jung, & C. von Rhoneck (Eds.), *Aspects of understanding electricity*. Kiel: IPNiSchmidt & Klaunig, 31 1-318.

Thiele, R. B., & Treagust, D. F. (1991). Using analogies in secondary chemistry teaching. Retrieved from https://eric.ed.gov/?id=ED356137

Thompson, C. A., Opfer, J. E. (2010). How 15 hundred is like 15 cherries: effect of progressive alignment on representational changes in numerical cognition. *Child Development*, *81*, 1768–1786.

Treagust, D. F., Duit, R., Lindauer, I., & Joslin, P. (1989). Teachers' use of analogies in their regular teaching routines. *Research in Science Education*, *19*(1), 291-299.

Treagust, D. F. (1993). The evolution of an approach for using analogies in teaching and learning science. *Research in Science Education*, *23*(1), 293-301.

Uthe, R. E. (2002). For mole problems, call Avogadro: 602-1023. *Journal of Chemical Education*, *79*(10), 1213. doi:10.1021/ed079p1213

Vendetti, M. S., Matlen, B. J., Richland, L. E., Bunge, S. A. (2015). Analogical reasoning in the classroom: insights from cognitive science. *Mind, Brain, and Education*, *9*, 100–106.

Villanueva, J. C. (2018). How many atoms are there in the universe? Retrieved from https://www.universetoday.com/36302/atoms-in-the-universe

Wikipedia contributors. (2020a). Avogadro constant. Retrieved from https://en.wikipedia.org/w/index.php?title=Avogadro_constant

Wikipedia contributors. (2020b). Wheat and chessboard problem. Retrieved from https://en.wikipedia.org/wiki/Wheat_and_chessboard_problem

Wikipedia contributors. (2020c). World population. Retrieved from

https://en.wikipedia.org/w/index.php?title=World_population