PoreMatMod.jl: Julia package for in silico post-synthetic modification of crystal structure models

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Abstract

PoreMatMod.jl is a free, open-source, user-friendly, and documented Julia package for modifying crystal structure models of porous materials such as metal-organic frameworks (MOFs). PoreMatMod.jl functions as a find-and-replace algorithm on crystal structures by leveraging (i) Ullmann’s algorithm to search for subgraphs of the crystal structure graph that are isomorphic to the graph of a query fragment and (ii) the orthogonal Procrustes algorithm to align a replacement fragment with a targeted substructure of the crystal structure for installation. The prominent application of PoreMatMod.jl is to generate libraries of hypothetical structures for virtual screenings via molecular simulations. For example, one can install functional groups on the linkers of a parent MOF, mimicking post-synthetic modification. Other applications of PoreMatMod.jl to modify crystal structure models include introducing defects and correcting artifacts of X-ray structure determination (adding missing hydrogen atoms, resolving disorder, and removing guest molecules).
Keywords: porous materials, crystal structure models, high-throughput computational screening, substructure search

1 Introduction

Nanoporous materials have adsorption-based applications for the separation, storage, detection, and production of chemicals. Advanced classes of nanoporous materials, such as metal-organic frameworks (MOFs), offer highly tunable pore environments for guest adsorbates. A critical task is to determine the chemical structure that endows the material with an optimal property.

The adjustability of MOFs largely stems from their modular synthesis: metal atoms or metal-oxo clusters ["secondary building units" (SBUs)] coordinate to organic linker molecules to form an extended network. The abundance of compatible metals/SBUs and linkers, assembling within different topologies, gives many possible structures. Postsynthetic modification (PSM), building block replacement (BBR), and mixing linkers and/or metals grant further structural and chemical tunability. E.g., in covalent PSM, a reaction on the MOF can install a new chemical functionality on its linkers.

Among the practically unbounded number of possible structures, high-throughput computational screenings can accelerate the discovery of novel porous materials with optimal properties for a given application. By (i) constructing a library of atomistic models of material structures, then (ii) using molecular models and simulations to predict the property of each structure in the library, we can guide experimental campaigns by (a) providing a ranked list of materials and (b) elucidating structure-property relationships.

The computation-aided discovery of novel materials demands methods to construct predicted crystal structure models of hypothetical porous materials. Several algorithms to generate structure models of porous materials mimic de novo synthesis by computationally stitching together/ as- sembling molecular building blocks into a topological network/template. Molassembler generates molecules from graph representations and allows modification of them, but does not pertain to crystalline systems.

In this work, we present PoreMatMod.jl, a new, free, open-source, tested, and documented software package, written in the Julia programming language, to modify crystal structure models of porous materials. In contrast to prior structure generation algorithms that mimic de novo synthesis, PoreMatMod.jl produces hypothetical structures by modifying existing crystal structure models, mimicking PSM or BBR. PoreMatMod.jl functions, effectively, as a find-and-replace algorithm on labeled (by the chemical species) graph and point cloud representations of porous crystals. The user specifies a parent crystal structure, a query molecular fragment/moiety, and a replacement molecular fragment/moiety. PoreMatMod.jl then (1) applies Ullman's algorithm to search for all subgraphs of the parent graph that are isomorphic to the query graph, (2) uses orthogonal Procrustes to align the replacement fragment on the matching substructure of the parent structure, then (3) removes the query fragment and installs the replacement fragment in an optimal geometry, giving the child structure. When multiple subgraphs of the parent
are isomorphic to the query, PoreMatMod.jl grants precise control over the distribution of the replacements. We demonstrate that PoreMatMod.jl is useful for:

- tuning the chemistry of existing crystal structure models of porous materials (e.g., curated databases of experimentally-reported structures in Refs. [46][46]) to generate a library of hypothetical materials for computational screening (e.g., [47][50])
- generating heterogeneous, multi-linker MOFs with precise control of functional group placement (e.g., [51])
- repairing artifacts in crystal structures determined from X-ray diffraction (XRD) patterns, such as missing hydrogen atoms, disorder, and the presence of solvents (e.g., [41])
- introducing missing-linker and missing-SBU defects into MOFs to enable computational studies on the influence of such defects on properties (e.g., [52][54])
- searching for subgraphs in libraries of crystal structure models to e.g., filter structures or characterize chemical diversity (e.g., [55]).

See [github.com/SimonEnsemble/PoreMatMod.jl](https://github.com/SimonEnsemble/PoreMatMod.jl) for the source code and documentation for PoreMatMod.jl. Registered as an official Julia package under an MIT license, PoreMatMod.jl is easy to install and runs on Windows, Mac, and Linux operating systems.

### 2 Overview of the find-and-replace task in PoreMatMod.jl

Given a parent crystal structure, we wish to (1, find) search for all query fragments in the parent structure, then (2, replace) replace specified instances of the query fragments with the replacement fragments to produce a child crystal structure. For example, suppose we wish to replace all 1,4-benzedicarboxylate (BDC) linkers in IRMOF-1 with trifluoromethyl-BDC, mimicking BBR. The PoreMatMod.jl code:

```julia
# read crystal structure of the parent MOF
parent_xtal = Crystal("IRMOF-1.cif")

# read query fragment. masked atoms, to be replaced, marked with !
query_fragment = moiety("p-phenylene.xyz")
# read replacement fragment
replacement_fragment = moiety("tfm-p-phenylene.xyz")

# (1) search parent structure for query fragment
# (2) replace occurrences of query fragment with replacement fragments
#   (with randomly chosen orientations)
child_xtal = replace(parent_xtal, query_fragment => replacement_fragment)
```

produces a crystal structure model of a hypothetical, functionalized IRMOF-1 child shown in Fig. 1

Below, we explain the implementation of PoreMatMod.jl and illustrate more use cases. The source code, links to detailed documentation, and code for all use cases illustrated below are hosted at [github.com/SimonEnsemble/PoreMatMod.jl](https://github.com/SimonEnsemble/PoreMatMod.jl)
Figure 1: Overview of find-and-replace operations in PoreMatMod.jl to, for example, construct a hypothetical MOF. Here, we replace \( p \)-phenylene query fragments in the IRMOF-1 parent with trifluoromethyl-\( p \)-phenylene replacement fragments (in random orientations) to give a function-alized child structure. The masked atom of the query fragment to be replaced is annotated with \(!\).

3 The inner-workings of PoreMatMod.jl

PoreMatMod.jl functions as a find-and-replace tool for crystal structures.

3.1 Representing crystal structures and chemical fragments

During the “find” stage, PoreMatMod.jl works with node-labeled, simple graph representations (nodes: atoms, edges: bonds, labels: chemical species) of crystal structures and chemical fragments. Edges for (periodic) crystal structures also join atoms bonded across the unit cell boundary. During the “replace” stage, PoreMatMod.jl works with labeled point cloud representations, where each atom is represented by a point in 3D space, labeled by the chemical species. In the case of crystal structures, periodic boundary conditions are imposed over the 3D space composed of the unit cell. The 3D coordinates are needed to align and install the replacement fragment onto the parent structure in a reasonable geometry.

By default, we infer the graph representation of crystal structures and chemical fragments from their labeled point cloud representations provided in a crystallographic information file (.cif) and XYZ file (.xyz), respectively. We assign a bond between a pair of atoms if the (periodic, in the case of crystal structures) distance between them is less than the sum of their covalent radii taken from Refs. [56][57]. PoreMatMod.jl allows (i) alteration of the bonding rules and/or (ii) manual bond assignment.

N.b., one must construct the query and replacement chemical fragments for the find-and-replace task at hand by (i) cutting them from the parent structure and/or (ii) building them in an atom editor (e.g., Avogadro [58]).
3.2 “Find”: search for subgraphs of the parent graph that match the query graph

The goal of the “find” stage is to search for all subgraphs of the labeled graph representing the parent crystal structure that are isomorphic to the labeled graph representing the query fragment. Colloquially, two graphs are [exactly] isomorphic if they have the same number of nodes (atoms) and can be overlaid with each other to “match” in terms of both the connectivity of the nodes (bonding pattern) and the node labels (atomic species).

To precisely describe the [exact] subgraph isomorphism problem [38], let \( G_p = (V_p, E_p) \) and \( G_q = (V_q, E_q) \) be the graph representations of the parent crystal structure and query fragment, respectively, with \( V \) and \( E \) the set of nodes and edges, respectively. Let \( s : V_p \cup V_q \rightarrow \{ \text{H, He, Li, Be, \ldots, No, Lr} \} \) be the node-labeling function that maps a node to the chemical species it represents. A subgraph \( G_p' = (V_p', E_p') \) of the parent graph \( G_p \) (subgraph \( \implies \) \( V_p' \subseteq V_p \) and \( E_p' \subseteq E_p \)) is isomorphic to the query graph \( G_q \) if there exists a bijection \( \theta : V_p' \rightarrow V_q \) such that \( \{ v_{p,i}', v_{p,j}' \} \in E_p' \iff \{ \theta(v_{p,i}'), \theta(v_{p,j}') \} \in E_q \) and \( s(v_{p}') = s(\theta(v_{p}')) \) for all \( v_{p}' \in V_p' \). The subgraph isomorphism problem is to find all subgraphs \( G_p' \) of \( G_p \) isomorphic to \( G_q \).

PoreMatMod.jl searches for all subgraphs \( G_p' \) of the parent crystal structure graph \( G_p \) that are isomorphic to the query graph \( G_q \) using Ullmann’s subgraph isomorphism algorithm [37]. The set of subgraph isomorphisms often includes symmetry-equivalent subsets of bijections with the same domain \( G_p' \). Each bijection in a symmetry-equivalent set maps the nodes in \( V_p' \) to the nodes in \( V_q \) via a different permutation. E.g., there are four symmetry-equivalent isomorphisms between a subgraph of the BDC linker and the \( p \)-phenylene fragment (see Fig. 2a). PoreMatMod.jl groups the search results—the set of isomorphisms between subgraphs \( \{ G_p', i \} \) and \( G_q \)—by symmetry-equivalence.

3.3 “Replace”: align and install a replacement fragment on a substructure of the parent crystal structure

The goal in the “replace” stage is to align a replacement fragment with a targeted substructure of the parent crystal structure matching the query fragment, remove this substructure, then install the replacement fragment on the parent in its place to give the child crystal structure (e.g., in Fig. 1).

At this juncture, we have identified a subgraph \( G_p' \) of the graph of the parent crystal structure that is isomorphic to the graph of the query \( G_q \) via the bijection \( \theta : V_p' \rightarrow V_q \). Now, to translate and align the replacement fragment onto this substructure of the parent crystal, we must find an injective mapping from a subset of the nodes on the graph of the replacement fragment \( G_r \) to corresponding nodes of the parent subgraph \( G_p' \).

First, a subset of the atoms of the query fragment must be (manually) flagged as masked atoms in the XYZ input file by appending an exclamation mark ! to their atomic species labels (e.g., C → C! for a carbon atom). A masked atom in the query fragment implies that the corresponding atom of the parent crystal structure (i) must be removed [e.g., to make room for replacement with a different functionality] but (ii) does not correspond with an atom on the replacement fragment and thus
that is isomorphic to the induced subgraph of the query \(I\) where \(V\) is the
problem. (i) replacement fragment, and (ii) the corresponding substructure of the
parent graph—this directly informs the alignment of the replacement fragment onto the
parent structure.

Using the mapping \(\theta^{-1}(\phi(V'))\), we now aim to determine the optimal alignment (rotation and translation) of the replacement fragment onto the substructure of the parent crystal that matched the query fragment. For this task, we rely on labeled point cloud representations to specify the geometry in which the replacement fragment is installed. First, we center the 3D coordinates of the (i) replacement fragment and (ii) the corresponding substructure of the parent structure at the origin of a Cartesian coordinate system. If the substructure of the parent is split across a periodic boundary, we reconstruct it using the nearest images of atoms so that Euclidean distance is equal to periodic distance. Let \(x_v \in \mathbb{R}^3\) be the centered 3D coordinates of the atom represented by node \(v \in V_p' \cup V_r\) of the graph of the parent substructure or replacement fragment. Second, we find the \(3 \times 3\) orthogonal (rotation) matrix \(Q_{opt}\) that optimally rotates the replacement fragment about the origin to align it with the matching parent substructure by solving the orthogonal Procrustes problem \(^{[39]}\):

\[
Q_{opt} := \arg \min_{Q: Q^\top Q = I} \sum_{v' \in V_r} \|Qx_{v'} - x_{\theta^{-1}(\phi(v'))}\|^2, \tag{1}
\]

where \(I\) is the identity matrix. Here, \(Qx_{v'}\) is the position of atom \(v'\) of the rotated (by \(Q\)) replacement fragment, and \(x_{\theta^{-1}(\phi(v'))}\) is the position of the corresponding atom of the parent crystal structure. The objective in eqn. \(^1\) expresses the closeness of the atoms of the rotated replacement fragment to their corresponding atoms of the parent structure. The coordinates of each atom \(v_r \in V_r\) of the aligned replacement fragment are then:

\[
x_{v_r, \text{aligned}} := Q_{opt} x_{v_r} + x_0 \tag{2}
\]

where \(x_0\) is the center (accounting for periodic boundary conditions) of the coordinates of the substructure of the parent having correspondence with atoms in the replacement fragment (specifically, atoms \(\theta^{-1}(\phi(V'))\)).

Finally, we install the replacement fragment on the parent crystal structure by (i) removing the substructure of the parent that matched the query fragment, then (ii) augmenting the parent with the replacement fragment at its aligned coordinates in eqn. \(^2\). We also introduce bonds between the installed replacement fragment and the parent structure: if we removed a bond \(\{v_p, v'_p\} \in E_p\) such that \(v_p \in V_p' \setminus V_p'\) and \(v_p' \in V_p'\), we may introduce a new edge \(\{v_p, v'_p = \phi^{-1}(\theta(v'_p))\}\) with the installed replacement fragment.
There may exist multiple, symmetry-equivalent (from the graph perspective) bijections \( \theta^{-1} \circ \phi \) from the subset of atoms of the replacement fragment to a subset of the atoms on the substructure of the parent matching the query fragment. In this case, we solve the orthogonal Procrustes problem for all of them and select the bijection for the installation procedure that gives the lowest alignment error in eqn. [1] As opposed to choosing one of the graph-symmetry-equivalent bijections at random, this procedure to select the bijection with the optimal alignment can be important. E.g., when replacing a two-fold disordered pyridine rotamer with a corrected pyridine ligand, there are a total of twelve bijections between the pyridine replacement and the two-fold disordered pyridine query, four of which describe spatially correct [flat] rings and eight of which describe spatially incorrect [taco-shaped] rings.

The unit cell of the parent crystal is preserved after replacement. It is currently not possible to replace a query fragment in the parent with a replacement fragment that requires, to accommodate it, (i) expansion/contraction of the unit cell and/or (ii) parent atoms not belonging to but surrounding the query fragment to move.

4 Use cases of PoreMatMod.jl

We provide illustrative examples of several practical uses of PoreMatMod.jl.

4.1 Subgraph matching

Suppose we wish to search for subgraphs of the IRMOF-1 parent crystal structure that are isomorphic to a p-phenylene query fragment. The PoreMatMod.jl code:

```julia
# read in crystal structure of parent MOF
parent_xtal = Crystal("IRMOF-1.cif")
# read in query fragment
query_fragment = moiety("p-phenylene.xyz")
# search for isomorphic subgraphs
search = query_fragment ∈ parent_xtal
```

finds a set of 96 isomorphisms between subgraphs of the parent IRMOF-1 graph and the p-phenylene query graph, composed of 24 groups of four symmetry-equivalent isomorphisms with 24 distinct subgraphs in the IRMOF-1 unit cell (one on each BDC linker). See Fig. 2a Notably, PoreMatMod.jl finds the query fragments that are split across the periodic boundaries. search.isomorphisms is a nested array, where the sets of symmetry-equivalent bijections between a particular \( \mathcal{V}_p \) and \( \mathcal{V}_q \) are grouped together. We provide helper functions to count the total number of isomorphisms found, the number of unique subgraphs \( \mathcal{V}_p \) involved in the isomorphisms, and the number of bijections for each distinct \( \mathcal{V}_p \).
Figure 2: Use cases of PoreMatMod.jl. (a) Searching for subgraphs of IRMOF-1 that match $p$-phenylene, including those crossing the periodic boundary. (b) Creating a hypothetical, multi-variate MOF, by finding $p$-phenylene fragments in IRMOF-1 and partially replacing them at randomly selected positions with fluoro-$p$-phenylene or acetylamido-$p$-phenylene fragments. (c) Resolving disordered ligands and removing acetylene adsorbates in an XRD-determined SIFSIX-2-Cu-i structure [60] to afford a simulation-ready structure. (d) Introducing missing-BDC-linker defects in UiO-66, replacing them with two capping groups.
4.2 Appending functional groups to a MOF

Suppose we wish to construct a hypothetical MOF derived from a parent MOF by appending functional groups to its linkers. To accomplish this, we construct a query fragment matching a fragment of the incumbent linker in the parent structure and a replacement fragment as the functionalized version of it. Via a find-and-replace operation, PoreMatMod.jl produces a child structure with a new chemical functionality installed on the linker of the parent, in a reasonable geometry for (i) a warm-start for a geometry optimization and/or (ii) molecular simulations to predict its properties.

Illustrated in the overview in Sec. 2 Fig. 1 shows the child IRMOF-1 structure produced by effectively appending one trifluoromethyl functionality on each BDC linker of the parent IRMOF-1 structure. With four possible substitution sites for the trifluoromethyl group on each BDC linker—reflected by the four symmetry-equivalent ismorphisms with the p-phenylene query fragment for each BDC linker of the parent structure—PoreMatMod.jl by default chooses the substitution site on each linker which minimizes the root-mean-square deviation of coordinates for nodes $v'_r = \phi^{-1}(\theta(v'_p))$. However, PoreMatMod.jl grants us precise control over (i) which linkers are functionalized and (ii) which substitution site on each linker is functionalized. Specifically, the replace function accepts keyword arguments to control the distribution of replacement fragment installations: the functional group can be installed: (i) at random substitution sites on (a) all linkers, (b) a specified number of randomly-chosen linkers, or (c) a specified subset of the linkers or (ii) at specified substitution sites on specified linkers.

The controls over the distribution of the replacement fragment installations enable PoreMatMod.jl to generate multivariate (multi-linker) MOFs with precise distributions of functional groups on the linkers. E.g., suppose we wish for partial functionalization of the BDC linkers of IRMOF-1 with acetamido and fluoro groups. We accomplish this by defining a p-phenylene query fragment, tagged with ! at one of its hydrogen atoms, and two replacement fragments, acetylamido-p-phenylene and fluoro-p-phenylene. Executing two sequential replacement operations with incomplete replacement gives the multi-variate MOF in Fig. 2b.

4.3 Repairing artifacts of experimentally-resolved MOF crystal structures

Crystal structures of MOFs experimentally determined from XRD studies often exhibit artifacts such as missing hydrogen atoms, crystallographic disorder, and unwanted guest molecules in the pores [21,42]. These artifacts are obstacles to conducting molecular simulations of gas adsorption, which require a chemically valid structure resembling the activated MOF. PoreMatMod.jl can repair these artifacts of XRD to give a simulation-ready crystal structure. Fig. 2c illustrates.

Unwanted solvent molecules or adsorbates in the pores of a MOF structure can be removed by searching for the graph of the solvent/adsorbate and replacing it with nothing (i.e., deleting it). To prevent necessary structural components from also being removed, the user may specify an option in the substructure search to return subgraphs of the parent graph that are isomorphic to the query fragment and are disconnected from the remainder of the nodes in the parent graph.

Disordered moieties, such as components of organic linkers with rotational freedom, manifest in crystal structures by presenting a multiplicity of conformations. To convert such a multiplicity of
conformers into a single conformer to give a simulation-ready structure, we can (i) extract the disordered substructure from the MOF and define it to be the query fragment then (ii) execute a find-and-replace with a replacement fragment constructed as a single conformation of the disordered moiety.

We can append missing hydrogen atoms to a parent structure via a find-and-replace operation with (i) the query fragment as the fragment of the parent with the missing hydrogen atom(s) and (ii) the replacement fragment as the query fragment with hydrogen atoms appropriately appended to it.

4.4 Introducing defects into MOFs

Missing-SBU/metal and missing-linker defects can be engineered into MOFs, which affect their properties. To use molecular models and simulations to study how defects influence the properties of MOFs, we wish to construct structure models of MOFs featuring defects. PoreMatMod.jl can introduce both missing-linker/SBU defects into MOFs with precision by searching for and deleting the linker/SBU, then replacing it with nothing or a capping group. Fig. 2d illustrates the introduction of a missing-linker defect in UIO-66. The precise controls over replacement operations in PoreMatMod.jl enables computational studies on how the number, location, and distribution of defects in the MOF structure influence its properties.

5 Discussion

We presented PoreMatMod.jl as a free, open source, user-friendly, tested, and documented Julia package for modifying crystal structures. PoreMatMod.jl functions as a find-and-replace algorithm on crystal structures: given a parent crystal structure, PoreMatMod.jl (i, via Ulmann’s subgraph isomorphism algorithm) searches for subgraphs of the parent that match the graph of a query fragment then (2, via orthogonal Procrustes) aligns and installs replacement fragments in place of the substructures of the parent matching the query fragment, giving a child crystal structure. PoreMatMod.jl grants precise control over the distribution and orientation of the replacement fragments on the parent. We demonstrated PoreMatMod.jl as a useful tool for: (1) installing functional groups on the ligands of MOF structures to generate a library of hypothetical MOFs for virtual screening (Figs. 1 and 2b); (2) repairing artifacts of XRD structure determination, such as missing hydrogen atoms, disorder, and solvent in the pores (Fig. 2c); (3) introducing missing-node/linker defects in MOFs to facilitate studies on how defects influence their properties (Fig. 2d); and (4) filtering databases of crystal structures by searching the structures for subgraphs (Fig. 2a).

In addition to MOFs, PoreMatMod.jl can modify other atomistic systems, whether periodic, such as semi-conductors, or non-periodic, such as drug molecules. The latter is achieved by assigning an arbitrary unit cell box. Depending on the parent and query structures, run times for subgraph matching range from milliseconds to tens of seconds. E.g., the subgraph search for BDC in the replicated UIO-66 unit cell shown in Fig. 2d runs in ca. 20 s (subsequent replacement operations, however, require only ca. 5 ms). In Fig. S1 we analyze the run times for an example subgraph matching task over a large database...
of MOFs. To reduce computational expense in the subgraph matching routine, (1) the parent crystal structure, where possible, should be provided as the minimal representation of the unit cell and (2) the query fragments should be provided as the minimal needed to effect the find-and-replace operation.

We now discuss limitations of the (1) subgraph matching algorithm and (2) replacement routine as implemented in PoreMatMod.jl. First, the subgraph matching algorithm is unable to distinguish among stereoisomers or conformations of flexible moieties, as the graph representation of a fragment is invariant to stereoisomerism and conformation changes. By imposing other geometry-based constraints on matches, such as a budget on the sum of pairwise distances between the corresponding atoms after aligning the query with its matching substructure of the parent—thereby quantifying the quality of the spatial overlay—it is possible to distinguish among stereoisomers and conformations. Second, replacement of a query fragment with a replacement fragment bearing no structural commonalities (“overlap”) with it is presently not possible because there is no basis for automatically defining the optimality of the alignment of the replacement fragment with the parent crystal structure. Third, as the child structure inherits the unit cell of the parent structure, PoreMatMod.jl cannot currently handle replacements that require expansion/contraction of the unit cell. E.g., it cannot replace a linker of a MOF with a shorter or longer linker, as then the unit cell would need to contract/expand to accommodate it.

Future work to improve PoreMatMod.jl includes: improve the efficiency of the subgraph matching algorithm by parallelization and/or implementation of faster algorithms [66]; implement inexact subgraph matching (subgraph similarity searching) [67]; handle replacement fragments which (a) require unit cell expansion/contraction and/or (b) have no overlap with the query fragment; consider edge metadata (e.g., bond order) in the definition of a subgraph match; include spatial overlay criteria in the matching algorithm to discern among spatial isomers and conformations; and leverage crystallographic point group symmetries to, when enumerating e.g. functionalized MOF analogues, prevent generation of symmetry-degenerate structures.

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References


[42] Yongchul G. Chung, Emmanuel Haldoupis, Benjamin J. Bucior, Maciej Haranczyk, Seulchan Lee, Konstantinos D. Vogiatzis, Sanliang Ling, Marija Milisavljevic, Hongda Zhang, Jeff S. Camp, Ben


