

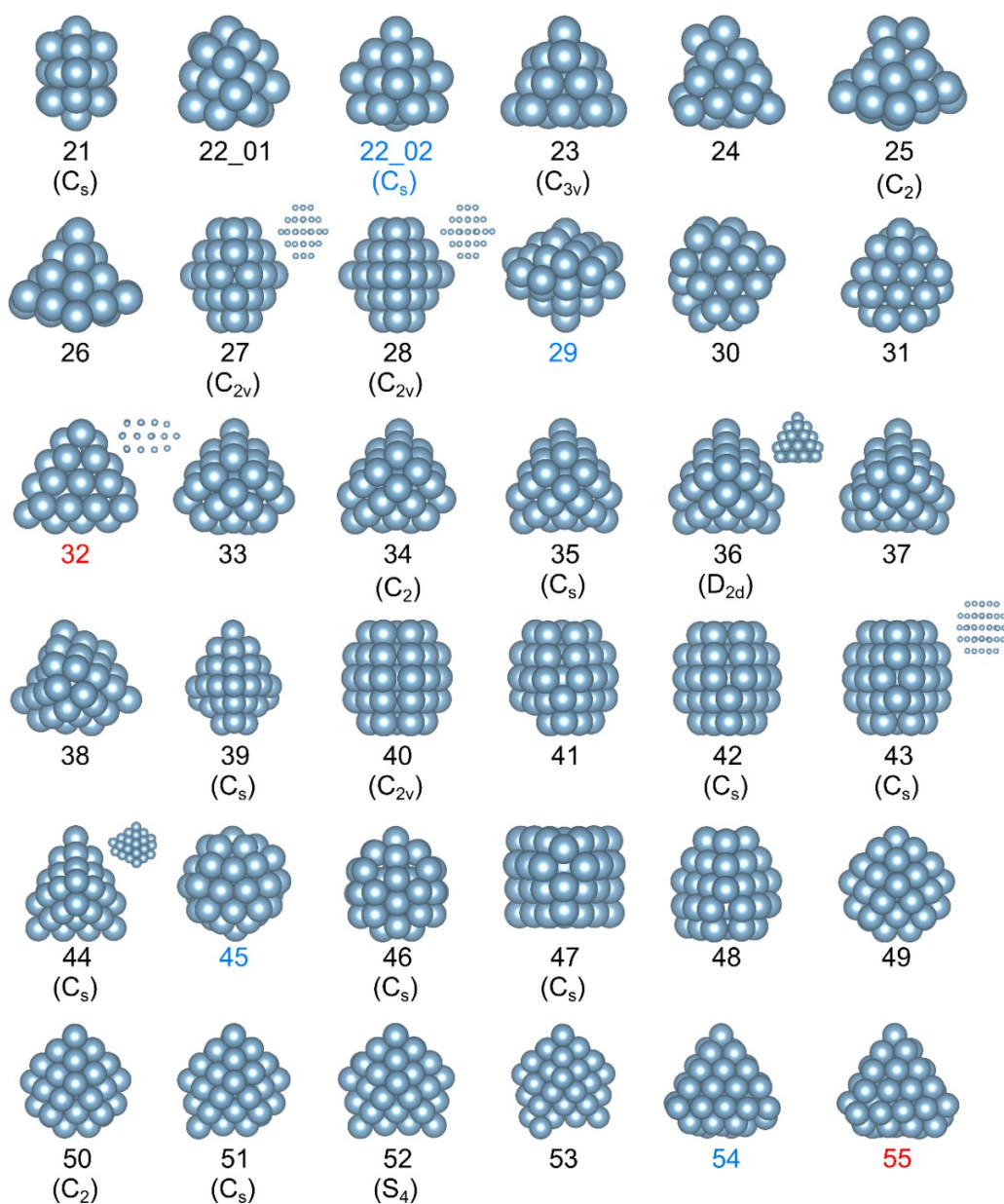
# Accelerated Prediction of Atomically Precise Cluster Structures Using On-the-fly Machine Learning

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**KEYWORDS:** *nanocluster, genetic algorithm, machine learning interatomic potential, active learning, transferrable potential, structure search*



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**ABSTRACT:** The chemical and structural properties of atomically precise nanoclusters are of great interest in numerous applications, but predicting the stable structures of clusters can be computationally expensive. In this work, we present a procedure for rapidly predicting low-energy structures of nanoclusters by combining a genetic algorithm with interatomic potentials actively learned on-the-fly. Applying this approach to aluminum clusters with 21 to 55 atoms, we have identified structures with lower energy than any reported in the literature for 25 out of the 35 sizes. Our benchmarks indicate that the active learning procedure accelerated the average search speed by more than an order of magnitude relative to genetic algorithm searches using only density functional calculations. This work demonstrates a feasible way to systematically discover stable structures for large nanoclusters and provides insights into the transferability of machine-learned interatomic potentials for nanoclusters.

## INTRODUCTION

Nanoclusters have drawn much attention due to their special physical and chemical properties<sup>1,2</sup> which are distinct from molecules or bulk crystal materials. These properties make them useful in diverse research fields including catalysis,<sup>3-6</sup> chemical sensing,<sup>7</sup> fluorescence,<sup>8,9</sup> and medicine.<sup>10</sup> The unique properties of nanoclusters are largely the consequence of distinct size-dependent atomic structures, quantum finite size effects, and very large surface-to-volume ratios.<sup>11,12</sup> These properties are generally not smooth functions of cluster sizes and can fluctuate with the addition or removal of a single atom.

Computational screening is a promising way to identify nanoclusters with desirable properties, but to predict the properties of a nanocluster from first principles it is necessary to first identify the low-energy atomic structures of the cluster. Many optimization methods have been proposed to perform global structure searches for nanoclusters, including the basin hopping method,<sup>13,14</sup> unbiased random sampling<sup>15</sup>, particle swarm optimization,<sup>16,17</sup> simulated annealing<sup>18,19</sup> and genetic algorithms (GA).<sup>20-24</sup> Each of these methods involves the evaluation of the energies of a large number of candidate structures, which makes it critically important to evaluate structure energies with a method that is both fast and sufficiently accurate to distinguish between competing

structures. Density functional theory (DFT) provides a high level of accuracy, but its speed and scalability typically limit the search to nanoclusters of sizes up to only several dozen atoms.<sup>25-28</sup> Classical interatomic potentials, which typically have simple functional forms derived from fundamental physics, are several orders of magnitude faster than DFT and have been used to search for ground state structures with up to a few hundred atoms.<sup>13,29,30</sup> However, classical interatomic potentials often lack the accuracy required to resolve the energy differences between competing candidate structures, especially for the low-lying local minima on the potential energy surface (PES) that are often only tens of meV apart.<sup>20,31</sup>

In recent years, an alternative type of interatomic potential has emerged in the form of machine-learned interatomic potentials (MLIPs),<sup>32-36</sup> which are parameterized by fitting to a set of training data. Examples of MLIPs are neural network potentials,<sup>37-39</sup> Gaussian approximation potentials (GAP),<sup>40-42</sup> spectral neighbor analysis potentials (SNAP),<sup>43,44</sup> moment tensor potentials (MTP),<sup>45-47</sup> the Atomic Cluster Expansion (ACE),<sup>48</sup> and potentials found through symbolic regression.<sup>49</sup> Although they may be slower than traditional interatomic potentials by an order of magnitude or more, MLIPs are generally more accurate and are still orders of magnitude faster than *ab initio* calculations.<sup>50</sup>

Here we demonstrate how machine-learned interatomic potentials, in the form of moment tensor potentials, can be used to significantly accelerate a genetic algorithm search for low-energy cluster structures. Moment tensor potentials have been shown to have a good balance between accuracy and speed for bulk materials,<sup>50</sup> and we demonstrate that they also work well for small atomic clusters. One of the challenges in using machine-learned interatomic potentials to search for ground state structures is that because the ground state is unknown, it is difficult to ensure that the potential is constructed in a way that will yield accurate ground state energies. To address this challenge we use active learning, in which the potential is trained adaptively with new data generated during the search. The idea of refining an interatomic potential model during the structural search was introduced by Hartke using classical interatomic potentials and proved to be capable of discovering new low-energy silicon clusters.<sup>51</sup> Recently, similar strategies have also been successfully applied using MLIPs for global optimization of bulk crystalline materials<sup>52-55</sup> and nanoclusters.<sup>56,57</sup>

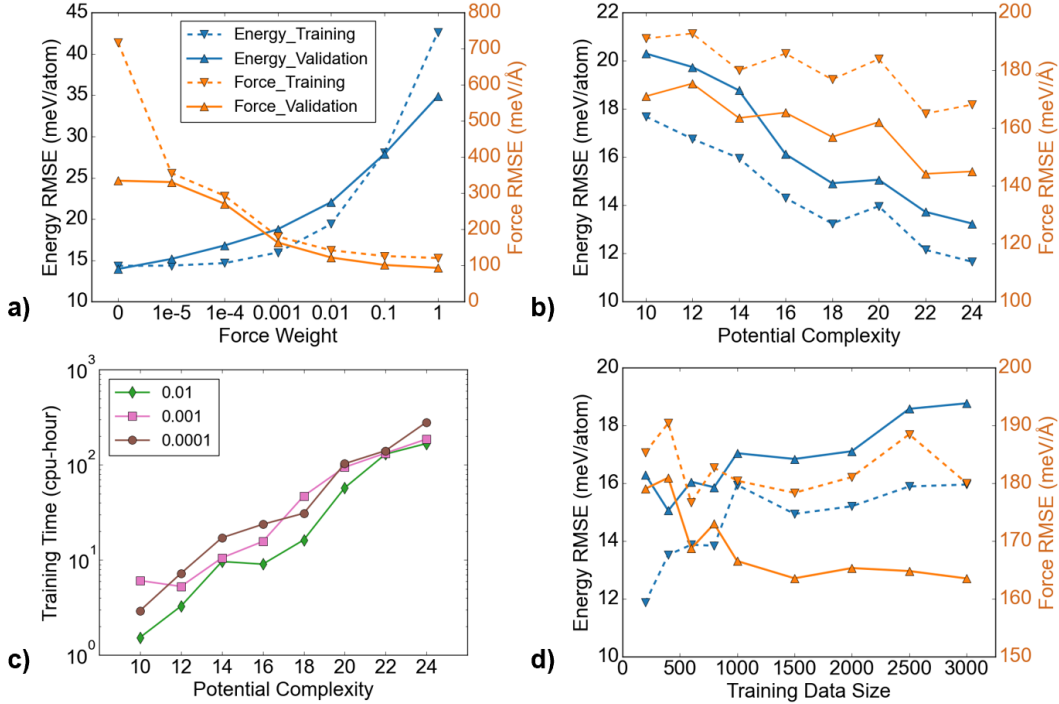
We demonstrate our approach by searching for low-energy structures for aluminum clusters of between 21 and 55 atoms. Using this approach, we have discovered new cluster structures for 25 out of the 35 sizes that are at least 1 meV/atom lower in DFT-calculated energy than the lowest-energy structures we have found in the literature.<sup>28,29,58-60</sup> New low-energy structures for an additional two sizes were discovered by DFT-only genetic algorithms used for benchmarking. Our approach, described in detail below, provides a template that can be used to significantly accelerate the computational design of atomic clusters, and paves the way for determining atomic structures of large nanoclusters.

## RESULTS

### Hyperparameter selection for Moment Tensor Potentials

Existing benchmarks of MTP on bulk crystalline structures<sup>50,61</sup> give generally good sets of parameters for training reliable MTP, but little information is available on good parameters for training clusters. To identify a good set of parameters for our calculations we used Al clusters with 24 atoms as a model system and tested various combinations of hyperparameters, including potential complexity (defined by the parameter  $lev_{max}$ ),<sup>47</sup> the amount of training data, and the weight for force components (the “force weight”) relative to the weight for energies. One quarter of the structures were randomly selected for validation, with the rest used to train the potential. Additional details about the construction of this dataset are provide in the Methods section and Section 2.1 of the Supplementary Information.

For a fixed training set, both energy and force errors decrease steadily as increasingly complex potentials are used (Figure 1b). However, such a gain is at the expense of an exponential growth in training costs (Figure 1c). Additional analysis of other combinations of hyperparameters (Figure S4 to Figure S7 of Supplementary Information) shows similar trends as in Figure 1. To balance accuracy and training costs we used and  $lev_{max} = 14$  and a force weight that was 1/1000 that of the energy weight for all subsequent active learning genetic algorithm (GA\_AL) runs. Using these parameters, we found that force and energy errors plateau after the training set exceeded about 1000 structures (Figure 1d).



**Figure 1. Benchmarks of hyperparameters for training MTP potentials.** Training (dashed lines) and validation (solid lines) root-mean-squared errors for both energies and force components are plotted against **a)** force weights relative to the energy weight, **b)** potential complexity  $lev_{max}$ , and **d)** the number of structures in the training set. **a), b)** and **d)** share the same legend as shown in **a)**. Training costs of potentials with different relative force weights and complexities are shown in **c)**. Where not specified, the training data contained 3000 clusters,  $lev_{max} = 14$ , and the force weight was  $1/1000^{\text{th}}$  that of the energy weight.

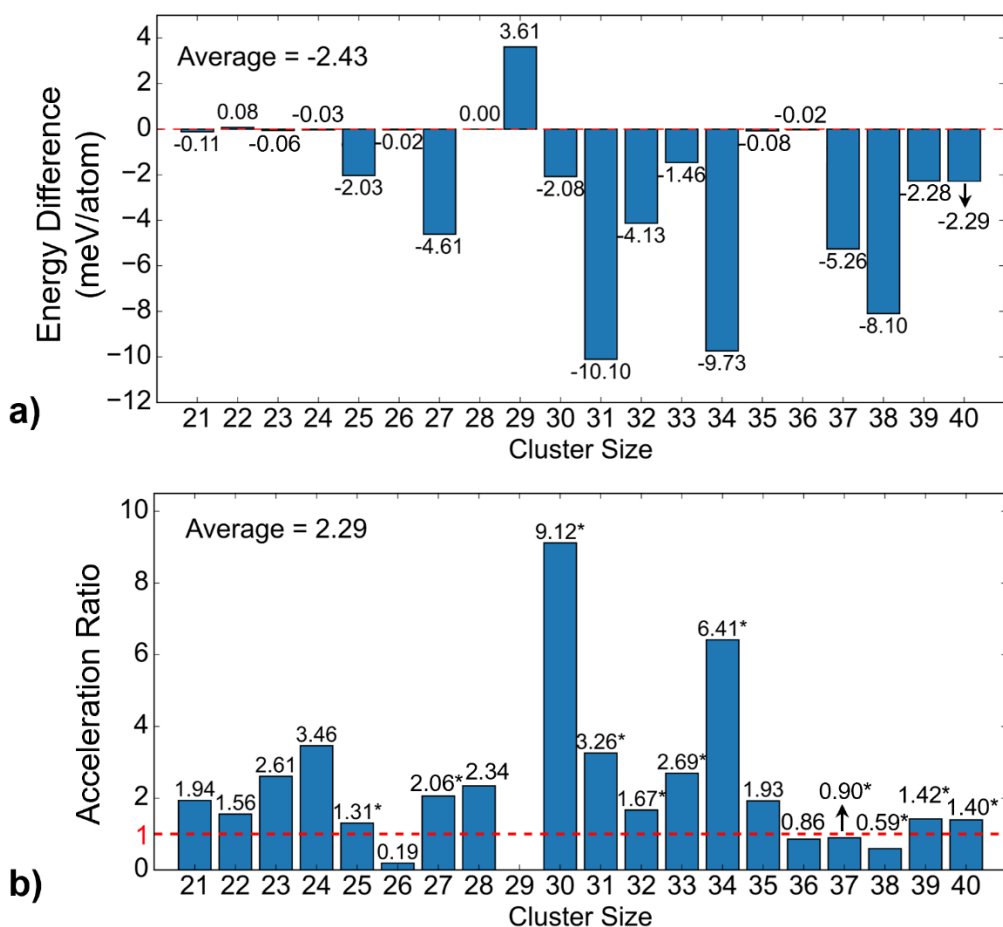
### Prediction of cluster structures for Al clusters with 21–40 atoms

We evaluated our approach by predicting structures of aluminum nanoclusters with 21 to 40 atoms (Figure 2). The performance of the GA\_AL algorithm was evaluated by comparing it with a genetic algorithm that used only DFT to calculate energies (GA\_DFT), where both algorithms were run for the same amount of computing time. For our initial evaluation, GA\_AL was initialized with untrained potentials and a new potential was trained at each cluster size. Details of how we performed the comparison are provided in the Methods section.

For 8 cluster sizes (21, 23, 24, 26, 28, 33, 35 and 36), mostly among the smaller clusters, GA\_AL and GA\_DFT found essentially the same lowest-energy clusters, with similarity scores, a measure of geometrical differences, below 0.3 (see the Methods section). For 10 out of the 20 sizes (25, 27, 30 – 32, 34, 37 – 40), GA\_AL found clusters that were lower in energy than those found by GA\_DFT by at least 1 meV / atom, with an average energy difference of -5.06 meV/atom (or -169.64 meV/cluster). For clusters of 22 atoms, GA\_AL identified a distinct cluster with a calculated energy within 0.1 meV / atom of the lowest-energy cluster identified by GA\_DFT. The

lowest-energy 33-atom cluster found by GA\_AL is 1.47 meV/atom lower in energy than the one found by GA\_DFT, but it is structurally similar based on both the similarity score and visual inspection (see Figure S14 of Supplementary Information). Therefore it is not counted as a new lowest-energy cluster. There was only one cluster size (29 atoms), for which GA\_DFT found a distinct cluster with lower energy than that found by GA\_AL. For this size the cluster found by GA\_DFT was lower in energy by 3.61 meV/atom (104.69 meV/cluster). On average, the energies of structures found by GA\_AL are lower by 2.43 meV/atom (82.27 meV/cluster).

To quantify how much more quickly the GA\_AL approach finds low-energy structures, we define the “acceleration ratio” as the ratio of time it took GA\_DFT to find its lowest-energy structure divided by the time it took GA\_AL to find a structure with at least as low of the energy. In the case of size 29, GA\_AL failed to discover better or equivalent configurations, so the ratio is set to 0. The average acceleration ratio across all 20 sizes is 2.29 (Figure 2b). GA\_DFT often did not find a cluster with energy as low as that found by GA\_AL (Figure 2b), suggesting that if the acceleration ratio were based on the time required to find the lowest-energy structure it would be larger. Additional data illustrating the acceleration of GA\_AL relative to GA\_DFT are provided in Figure S11 and Figure S12 of the Supplementary Information.



**Figure 2. Performance benchmarks between GA\_AL initialized with an untrained potential and GA\_DFT. a)** DFT-calculated energy differences between the lowest-energy structures found in GA\_AL and GA\_DFT. Negative values indicate GA\_AL discovered structures with lower energies. **b)** Acceleration ratios of GA\_AL relative to GA\_DFT. The sizes for which GA\_AL discovered better structures are marked with \*, suggesting that they could be larger if GA\_DFT were allowed to run for longer time.

### Size-transferable interatomic potentials for nanoclusters

The results presented in the previous section were obtained by training a new potential at every cluster size, as there is a risk that a potential trained on one cluster size might not work well for clusters of another size due to the fact that the properties of atomic clusters can change discontinuously with the number of atoms in the cluster. However using a potential trained at one size to find structures of a different size could significantly speed up the structure search by reducing the total amount of training data that must be generated. In particular, using potentials trained with smaller clusters to predict the structures of larger clusters can have significant performance advantages, as the cost of generating training data using DFT typically scales as approximately the cube of the number of valence electrons in the cluster.<sup>62</sup>

We examined how accurately potentials trained on clusters of a range of small sizes are able to predict the energies of clusters with larger sizes. Training data was separated into a group of 3000 clusters with an even number of atoms (22, 26, 30, 34, and 38) and another group of 3000 clusters with an odd number of atoms (21, 25, 29, 33, and 37), as DFT calculations indicate that even-sized clusters and odd-sized clusters have distinct ground state magnetic moments (see Section 4 of the Supplementary Information). A third training set was created by combining the even and odd sets. The validation sets were composed of about 3000 clusters for each cluster size between 50–55 atoms. Details of the construction of the training and validation sets can be found in the Methods section.

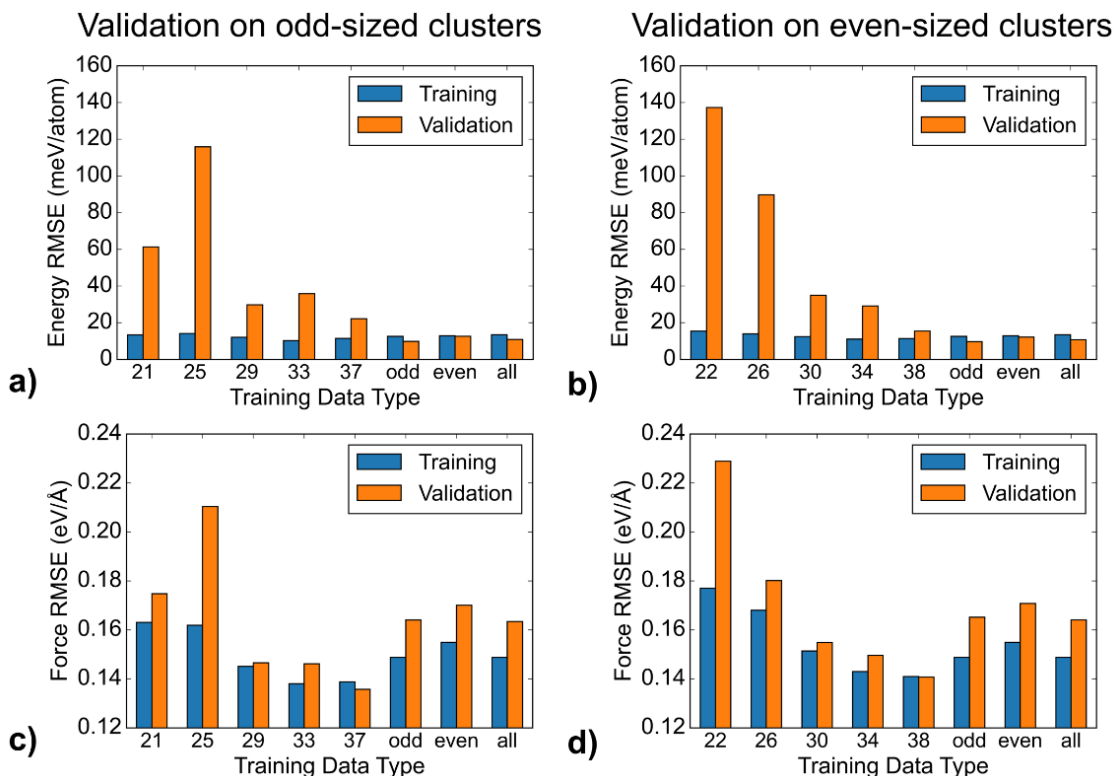
All three mixed-size potentials predicted energies of the large clusters with validation errors ( $\sim 10$  meV/atom) comparable to training errors (Figure 3a and Figure 3b). The errors in the predicted forces ( $\sim 165$  meV/Å) were slightly worse than the fitting errors. These errors are similar to the training and validation errors achieved when all of the training and validation data consisted of clusters of 24 atoms (Figure 1). Mixing training data with different magnetic moments did not have a significant adverse effect on model predictions (Figure 3). The potential trained on clusters with an odd number of atoms has slightly smaller prediction errors for both force and energy than potential trained on clusters with an even number of atoms regardless of whether the validation set contains even-sized or odd-sized clusters. The potential trained with both even and odd clusters has larger energy training errors than both of the even and odd potentials, but energy validation errors between the validation errors for even and odd potentials. For forces, the potential trained with both even and odd clusters has the lowest training and validation errors in all cases.

For comparison, we evaluated the ability of potentials trained on clusters of a single size. For each size, the training data consisted of 3000 dissimilar clusters, with exception of clusters with 21 atoms (the smallest size) for which our training set only had 2136 clusters after removing structurally similar clusters (see Methods section). For potentials trained on clusters of a single size, training and validation errors were similar for forces, and potentials trained on a single size may predict forces with significantly lower errors than the potentials trained on a mixed set of sizes. However validation errors for energies are notably worse than the training errors, especially for potentials trained on small clusters (Figure 3a and Figure 3b). The accuracy strongly depends on the size of the clusters in the training set, with larger sizes having the lowest errors. This suggests that quantum finite-size effects may be particularly pronounced for clusters with fewer than about 30 atoms, limiting the extent to which potential models trained at these sizes can be transferred to larger sizes. The training algorithm may also have a difficult time determining how the undercoordination of surface atoms in a cluster affects its energy when all clusters in the training set have approximately the same surface area. In contrast, training sets with a mixture of cluster sizes provide more information on how the energy is affected by the cluster surface area, which may improve the prediction accuracy for clusters of varying size.

Because of the particular importance of identifying the structures of low-energy clusters, we evaluated the potentials on the lowest-energy structures we found or collected from the literature with 50 to 55 atoms. The diversity of the training sets with mixed cluster sizes proved beneficial for identifying low-energy clusters, as the MTP extrapolation grade (see Methods section) for the



lowest-energy clusters in the validation set is less than one, suggesting interpolation, with respect to the training sets of odd-sized and even-sized clusters. On the other hand, the low-energy structures had an extrapolation grade above 1, suggesting extrapolation, with respect to the training sets of single-sized clusters. Accordingly, the mixed-size potentials had much lower energy errors than the single-size ones (see Figure S14 of the Supplementary Information).

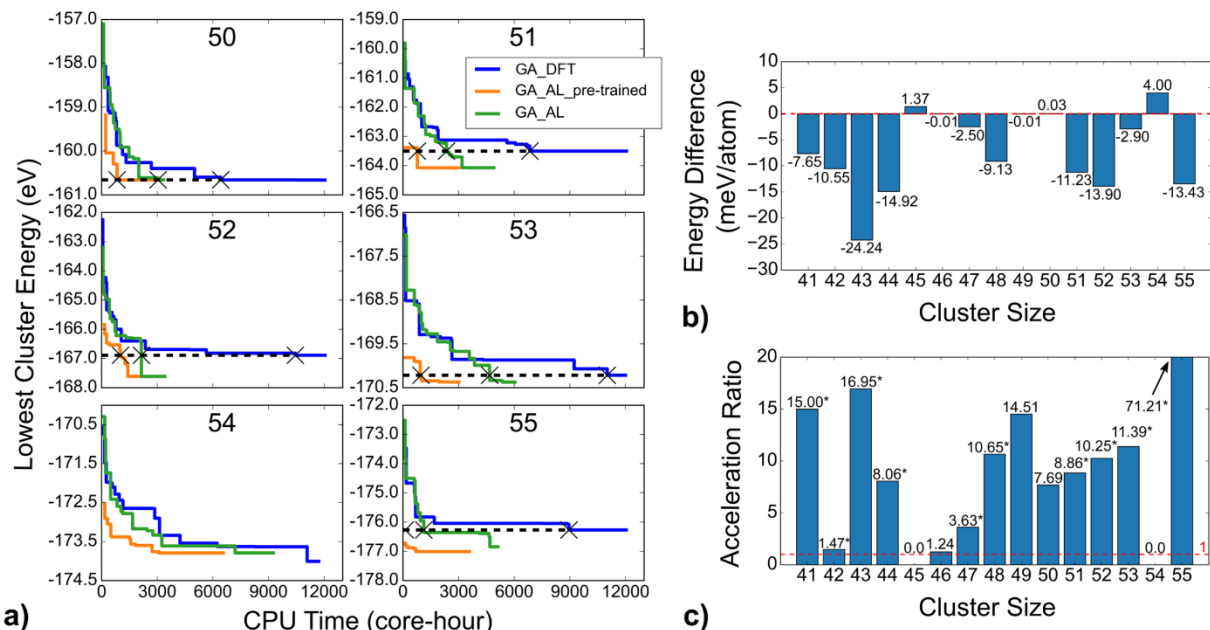


**Figure 3. Training and validation root-mean-square errors in energy and force components for different potentials.** The validation data consisted of clusters with 50-55 atoms. Plots in the left column show a) energy and c) force errors for potentials validated on odd-sized clusters, while the right column displays b) energy and d) force errors for potentials validated on even-sized structures. Potentials are labeled by the type of training data used to generate them. Numeric labels represent single-size potentials whose training sets contain exclusively clusters of the labelled sizes. The labels “odd”, “even” and “all” represent potentials whose training sets are made up by clusters with odd, even, and mixture of odd and even number of atoms respectively.

### Prediction of structures for Al clusters with 41–55 atoms

To identify low-energy structures with 41-55 atoms, we used the mixed-size potentials trained on clusters with an odd or even number of atoms to initialize GA\_AL searches for clusters with an odd or even number of atoms respectively. Initializing the GA\_AL algorithm with a pre-trained potential demonstrated significant performance advantages, with an average acceleration ratio of 12.1 compared to GA\_DFT. For sizes 45 and 54, the acceleration ratio was set to 0 since they failed to discover better or equivalent configurations as GA\_DFT.

The sizable increase of acceleration ratios for GA\_AL with pre-trained potentials can be credited to a significant reduction in the number of times DFT is called for learning on the fly in the early stages of the search. For GA\_AL runs initialized with an untrained potential, clusters in the early stages of the runs tend to have high energies (Figure 4a), so training steps in the early stages are sampling a relatively high-energy region of configuration space. For GA\_AL runs initialized with well-trained potentials, computational resources are more efficiently spent exploring the low-energy configurations.



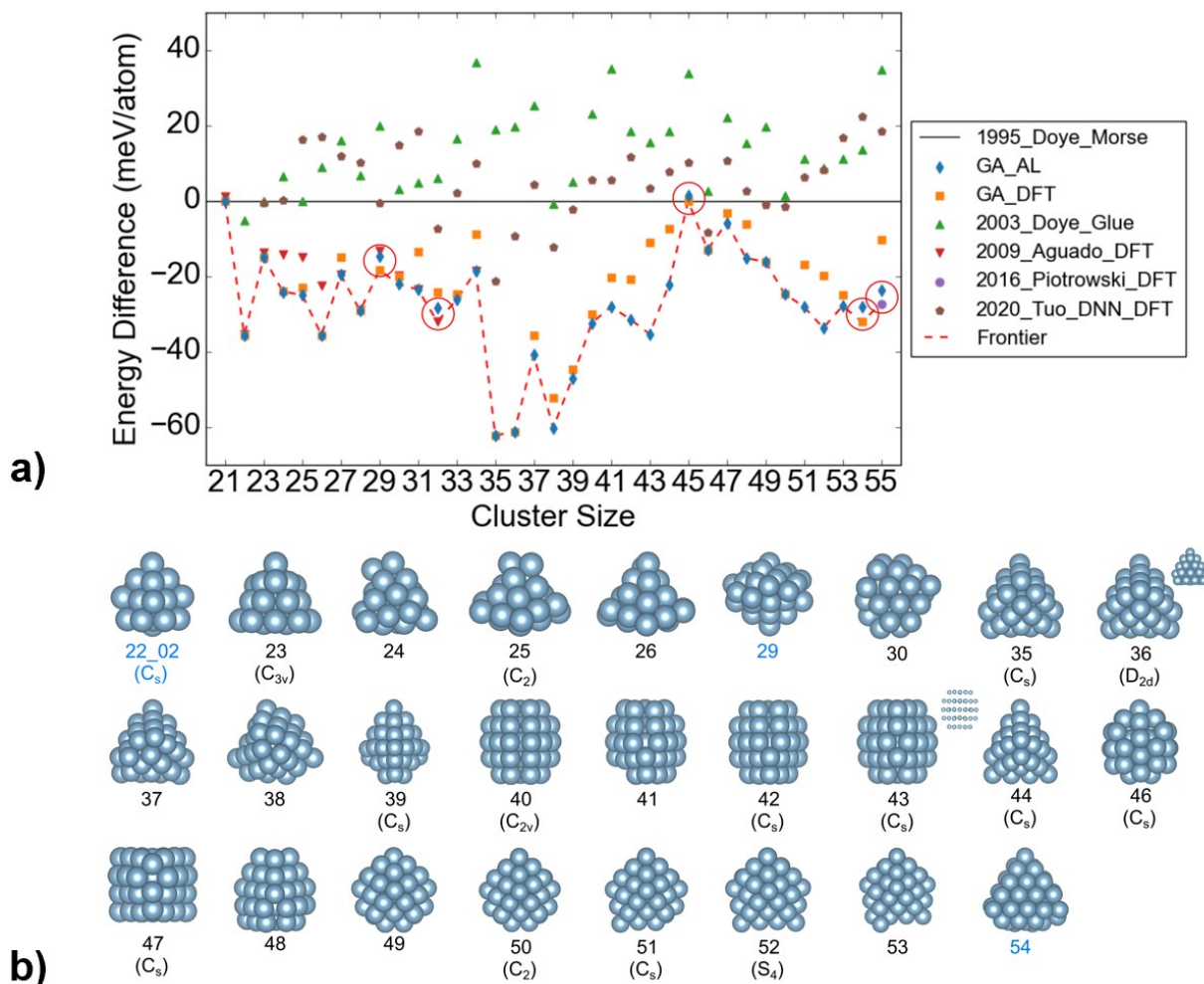
**Figure 4. Performance benchmark of GA\_AL against GA\_DFT on clusters with 41 to 55 atoms.** **a)** Energy evolution plots for GA\_AL initialized with untrained potentials (green), GA\_AL initialized with mixed-size potentials (orange) and GA\_DFT (blue), for clusters with 50 to 55 atoms. The energy levels used to calculate acceleration ratios are marked by dashed lines. The CPU times used to compute acceleration ratios are marked by crosses. **b)** Energy differences between the lowest energies of GA\_AL and GA\_DFT at the end of simulations. **c)** Acceleration ratios of GA\_AL relative to GA\_DFT. The sizes for which GA\_AL successfully discovered configurations with lower energy than the lowest of GA\_DFT are marked by \*.

## Literature comparison

We examine the quality of the ground state configurations discovered using GA\_AL for clusters of 21-55 atoms by comparing them with lowest-energy structures that have been previously reported for aluminum clusters. Here we only consider studies for which we were able to find the atomic coordinates of the discovered structures.<sup>28,29,58-60</sup> All structures collected from the literature were reoptimized by DFT using the same settings as those used in GA\_AL. For 25 of the 35 sizes, GA\_AL found structures at least 1 meV / atom lower in energy than the lowest-energy structure

in the literature, and for another 7 sizes it identified the same lowest-energy structure as was available in the literature. For clusters of 22 atoms, GA\_DFT found a structure that is structurally distinct from the lowest-energy structure reported in literature<sup>28</sup> but has only slightly lower energy (by 0.15 meV / atom). The GA\_DFT algorithm discovered structures lower in energy than those discovered by GA\_AL and the literature for 2 sizes. For clusters of 45 atoms, GA\_DFT rediscovered the best-known structure from the literature but GA\_AL did not. Detailed results are provided in Figure 5 and Table S4. A complete panel of lowest-energy clusters with 21 to 55 atoms is included in Figure S15 of the Supplementary Information and coordinates of these clusters are listed in a second Supplementary Information file.

Structures found using GA\_AL have lower energies than the lowest-energy literature structures by an average of 16.81 meV/atom, with a maximum of 51.81 meV/atom at size 36 (1.87 eV/cluster). For the five sizes for which GA\_AL did not discover the best structures (Figure 5a), the energies are no more than 4.0 meV/atom above those of the structures with the lowest known energies. The preferred morphologies of the lowest-energy clusters of aluminum alter between layered close-packed structures with FCC stacking order and tetrahedrons. The cluster with 36 atoms is particularly stable relative to its neighboring sizes (see Section 7.2 of the Supplementary Information), suggesting that it may be particularly likely to be synthesizable. Experimentally, positively-charged Al clusters with around 36 atoms have been observed to have relatively high melting temperatures.<sup>63,64</sup>



**Figure 5. Comparison between lowest-energy clusters reported in the literature and those discovered by GA\_AL and GA\_DFT. a)** Energy differences of lowest-energy clusters from various methods relative to the energies of clusters reported by Doye and coworkers.<sup>58</sup> The lowest-energy frontier is connected by a red dashed line, and the sizes for which GA\_AL did not discover the best configurations are circled. “Morse”, “Glue” and “DNN” in the legend represents the Morse potential, a glue potential and a deep neural network potential. **b)** New lowest-energy clusters discovered by GA\_AL and GA\_DFT. The labels of clusters identified by GA\_DFT and not GA\_AL are blue and corresponding symmetries are denoted in parentheses. Alternative views of representative layered close-packed structure (43) and tetrahedral clusters (36) are included above and to the right of the clusters.

## DISCUSSION

Although the GA\_AL approach presented here has clear advantages, including an order of magnitude acceleration compared to GA\_DFT, there are potential areas for improvement. One is the relatively high energy prediction errors of MTP for nanoclusters compared with bulk systems.

Benchmarks by Zuo and coworkers<sup>50</sup> showed that MTP has energy errors generally less than 5 meV/atom and sometimes even lower than 1 meV/atom for bulk elemental systems. However, for nanoclusters, as exhibited in Figure 3, validation energy errors are at the order of 10 meV/atom for small clusters with 24 atoms. They can be lowered by using smaller force weights, but the improvement comes at the expense of driving up force errors, which increases the possibility of creating artificial local minima. The relatively large energy errors increase the chance that the energy of the lowest-energy cluster is overestimated and never enters the pool. To mitigate this risk we used a relatively large pool size (25 clusters) to expand the energy window of pool clusters and raise the chance of the lowest-energy cluster being captured in the pool. A large pool has also been shown to increase the success rate of identifying the lowest-energy isomer due to the structural diversity of the pool,<sup>21</sup> although at the expense of slowing convergence speed.<sup>20,21</sup>

We found that the energy window of pool clusters became narrower as the structure search continued, implying a high density of metastable states with energies close to the global minimum, especially for large clusters. This is not unexpected since the dimension of configuration space dramatically increases as system size grows. The relatively narrow energy window increases the chance of the pool missing the lowest-energy isomer, as the window size may be comparable to the error in MTP energy predictions. A possible workaround is to run GA\_AL and then use the discovered low-energy clusters to seed a GA\_DFT search. This would consume additional computational resources but decrease the uncertainty in the proposed lowest-energy structures.

Another area for improvement is the relationship between the extrapolation grades (used to identify structures that trigger retraining) and prediction errors. A high extrapolation grade normally implies an energy evaluation with high uncertainty, but a low grade does not necessarily guarantee an accurate prediction (see Figure S17 of Supplementary Information). In practice, we addressed this challenge by starting DFT re-optimization and retraining whenever the majority of clusters in the pool had MTP-calculated energies but not DFT-calculated energies. An alternative approach would be to implement similarity-based measurements of uncertainty, which might more accurately identify structures for which the prediction errors are likely to be large.

Although we have demonstrated that potentials trained on small clusters can be used to predict the structures of clusters about twice as large, it is not clear how well these potentials will work on significantly larger particles. If transferability can be retained up to larger clusters, the methods we have presented could be used to efficiently create a comprehensive datasets of cluster structures for small particles with structures that cannot be simply described as that of a truncated crystal.

## **METHODS**

### **Similarity measurement**

We quantify geometric similarity between two cluster structures of same size by a similarity score calculated using an approach based on the spectral decomposition of extended distance matrices.<sup>65</sup> The score is non-negative and a smaller value implies higher similarity. Identical clusters have a score of 0, and visually distinguishable clusters typically have a score above about 0.3. The

similarity measure is used to prevent geometrically similar clusters from being simultaneously included in the pool, which can improve the efficiency of GA,<sup>21</sup> and to select diverse training data for the moment tensor potentials.<sup>55</sup>

## Genetic algorithm

A genetic algorithm is a global optimization method inspired by the principles of natural selection.<sup>66</sup> We developed our own code based off the pool-based Birmingham Parallel Genetic Algorithm<sup>23</sup> with some variations. A pool of low-energy clusters of fixed size is maintained during the search. Initial clusters are generated by randomly distributing atoms in space. Once the pool is filled, genetic operations, namely, crossover and mutation, are applied to parent clusters selected from the pool to generate child clusters. Child clusters that are dissimilar to all pool clusters and have a lower energy than the pool cluster with the highest energy will replace the highest-energy pool cluster. Additional details of the genetic algorithm can be found in Section 1 of the Supplementary Information.

## Genetic algorithm with actively learned interatomic potentials

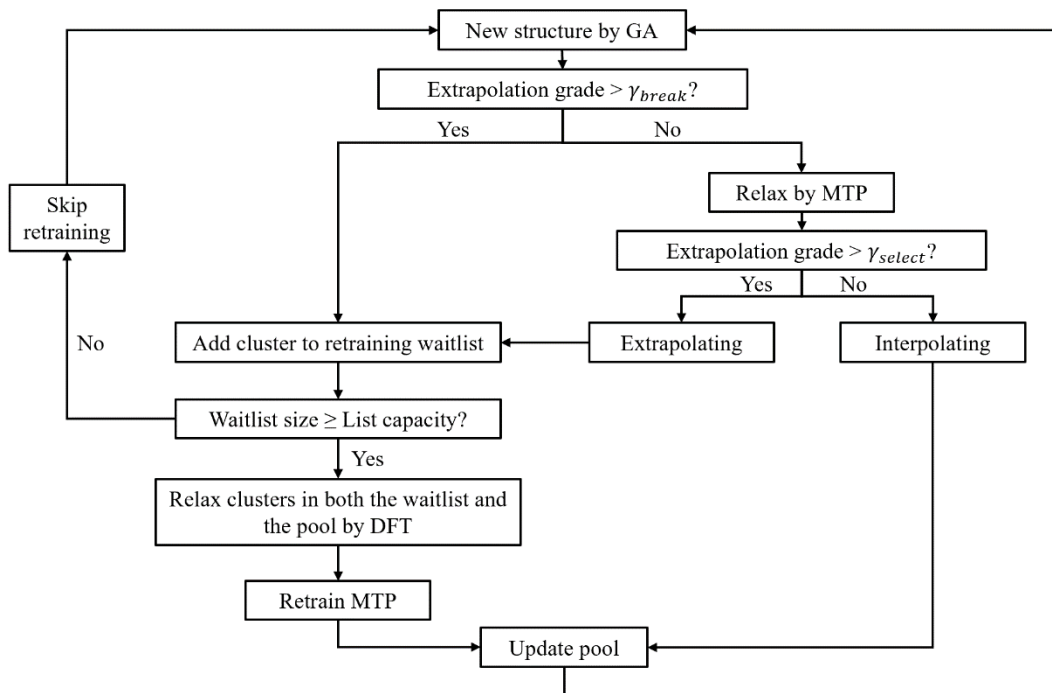
To accelerate the genetic algorithm search for new stable nanoclusters, we use machine-learned interatomic potentials (to improve speed) trained on-the-fly using active learning (to maintain accuracy). We refer to this combination of genetic algorithms and active learning as “GA\_AL”. The active-learning query strategy uses the generalized D-Optimality criterion implemented in the MLIP package,<sup>46,47</sup> which assigns unlabeled data an “extrapolation grade” based on a measure of the extent to which the unlabeled data is outside of the space spanned by the training data. An extrapolation grade above 1 implies extrapolation relative to the current training set and large errors should be expected, while a value below 1 indicates interpolation.<sup>47</sup>

The GA\_AL runs batch retraining cycles and maintains a waitlist of structures to be included in the next cycle (Figure 6). Two extrapolation grade thresholds are used when determining whether a newly-generated cluster should be added to the waitlist. The first threshold,  $\gamma_{break}$ , is used to screen clusters before relaxation using MTP. The trained potential may struggle to relax clusters with extrapolation grades above this threshold, so they are automatically added to the waitlist. Structures with extrapolation grades below  $\gamma_{break}$  are relaxed. If the extrapolation grade of the relaxed structure is greater than the second threshold,  $\gamma_{select}$ , then it too is added to the waitlist. In this work  $\gamma_{break}$  was set to 10 for GA\_AL initialized with untrained potential, as the default value recommend by MTP code.<sup>47</sup> A looser value of 1000 was used for the pre-trained potential, as it is not as important to add training data to a potential that has already been trained. As-generated clusters with extrapolation grades about 1000 typically cannot be evaluated accurately by MTP potentials, but we found they could still be relaxed by MTP to reasonable configurations. Starting DFT relaxations from configurations pre-relaxed using MTP was used to reduce computational costs. The parameter  $\gamma_{select}$  was set to 1.01 for all searches. When the waitlist reaches a user-defined value (here set to 5), the genetic algorithm is paused and a retraining cycle begins. All new clusters in the pool as well as clusters on the waitlist are relaxed using DFT and added to the training set. Before retraining the potential, a similarity screen is applied to select the most geometrically

diverse set of configurations from all relaxation steps (discussed below), which maximizes structural diversity and reduces training cost.

Because of the uncertainty in MTP-predicted energies, there is a risk that the pool over time becomes polluted with structures with erroneously low MTP-predicted energies. To mitigate this risk, a retraining cycle is also started whenever a majority of the clusters in the pool ( $>50\%$ ) have energies that were calculated using MTP and not DFT. GA\_AL is considered to be converged when no cluster with an energy lower than the lowest-energy pool cluster has been found for 4000 new clusters.

When initializing GA\_AL with pre-trained potentials, it is beneficial to switch off retraining at the beginning of the search. In this approach, extrapolating clusters are discarded and new ones are regenerated until they are interpolating, allowing the GA to more fully explore the potential energy surface of the pre-trained potential. We did this for the first 5000 clusters in GA\_AL runs initialized with mixed-size potentials when generating clusters with 41 to 55 atoms. Additional discussion and justification for this approach are provided in Section 1.4 of the Supplementary Information.



**Figure 6. Schematic workflow of genetic algorithm with on-the-fly active learning (GA\_AL).** A double-threshold scheme is employed in which a looser threshold,  $\gamma_{break}$ , and a tighter threshold,  $\gamma_{select}$ , are used to determine extrapolation of as-generated structures and MTP-relaxed structures, respectively. Retraining starts when the waitlist exceeds a user-defined capacity, which was set to 5 in this work.

## Moment tensor potentials

We used the MLIP package<sup>47</sup> to train moment tensor potentials. The hyperparameters for training include potential complexity, energy weight, force weight and stress weight. Potential complexity is characterized by the maximum level of moments,  $lev_{max}$ , of basis functions.<sup>45,47</sup> The energy weight was always set to 1, so the force weight can be seen as the weight of force components relative to the weight of the energy. The stress weight was set to 0 since it is irrelevant in the case of clusters due to the lack of lattice. We generated potentials with  $lev_{max}=14$  and a force weight of 1/1000 relative to the energy weight, to balance between accuracy and training cost, as shown in the Results section and Section 2.2 of Supplementary Information. The inner and outer cutoff radii defining the local atomic neighborhood were set to the default values of 2 Å and 5 Å, and eight radial basis functions were used.<sup>47</sup> A maximum number of 5000 training iterations were allowed for potential fitting. This limit was never reached, as the maximum number of training iterations in any GA\_AL run was 1399.

## Training data selection for pre-trained potentials

To select the training data for the pre-trained mixed-size potentials, we used a diversity-based strategy and an energy-based strategy to select structurally diverse structures from DFT relaxations and to improve accuracy in the low-lying regions of the potential energy surface. All DFT calculations were collected from GA\_DFT and GA\_AL runs on clusters with 21–40 atoms. For each of the constituent cluster sizes, relaxation trajectories were only kept if the corresponding local ground states have similarity scores larger than 0.3 with the local ground states of all other trajectories already included in the training set. Within each trajectory, only dissimilar ionic steps were selected as well. We accomplished this by including the fully relaxed structure and iterating backwards through the relaxation until encountering a structure with a similarity score, relative to the most recently-added structure, that was at least 0.3. That structure was then added to the training set, and we repeated this procedure until all relaxation steps were exhausted. This diversity-based strategy was applied throughout this work, in data preparation processes for both training and validation sets.

Following the similarity screening, we performed an energy-based selection strategy. First, all structures that passed through the diversity screening were grouped into sets based on the number of atoms in the cluster. 50% of the training data selected from each set consisted of the structures with lowest energy, 10% consisted of the structures with the highest energy, and the remaining 40% were randomly from the remaining ionic steps.

A total of 3000 structures were selected for each of the training sets. For potentials trained with clusters of multiple sizes (the ones labelled by “odd”, “even” and “all” in Figure 3), equal numbers of structures were chosen for each constituent sizes. For potentials trained with clusters of a single size, all 3000 structures were chosen from clusters of that size. The training set for the potential trained with clusters of 21 atoms only contains 2136 structures after the diversity filtering of DFT calculations, and were all included in the training set.



### Validation data selection for pre-trained potentials

We collected validation data for mixed-size potentials from GA\_DFT runs on clusters with 50–55 atoms. The diversity-based strategy discussed above was used to select a structurally diverse set of structures. The validation sets contain around 3000 structures for each size (details are provided in Section 3.1 of the Supplementary Information).

### Training data selection for on-the-fly retraining in GA\_AL

We also use similarity filtering, as described above, to select structures for retraining on-the-fly. Similarity filtering is used to select distinct clusters from each relaxation trajectory during active learning. We used a tight similarity threshold of 0.3 on small to medium clusters (21–40) and a looser threshold of 0.15 on large clusters (41–55). The looser threshold is meant to increase the fraction of available data used for retraining for large clusters, as generating this data is computationally more expensive. We do not check similarity between new training data and all existing data on-the-fly, as this is computationally costly.

### Comparison of GA\_AL against GA\_DFT

Both GA\_AL and GA\_DFT were run using the same set of genetic algorithm parameters (see Section 1.3 of the Supplementary Information) on all 24 cores of Intel E5-2680 V3 processors. The GA\_AL runs were performed until 4000 consecutive new clusters had been generated without identifying a new lowest-energy cluster. GA\_DFT runs were performed for at least the same amount of time as GA\_AL runs for fair comparison. For large clusters with 50–55 atoms, GA\_DFT searches were executed for a much longer time of 21 days, to reach comparable energy levels as GA\_AL runs (see also Section 1.2 of the Supplementary Information). The reported time spent for GA\_AL includes the time spent in the genetic algorithm search, time required to generate training data using DFT, and time spent training the interatomic potential. For GA\_AL runs initialized with pre-trained potentials, the time spent pre-training the potential was also included. The only difference between the GA\_AL and GA\_DFT algorithms was that GA\_AL used moment tensor potentials with active learning, whereas GA\_DFT used only DFT for relaxation. Low-energy structures identified in GA\_AL were re-optimized using DFT at the retraining stage and only DFT-evaluated energies were reported at the end, to ensure an *ab initio* level of accuracy.

To determine whether a new low-energy structure had been found, we considered both the DFT-calculated energy and similarity scores. A cluster that is lower in energy by at least 1 meV/atom and at the same time has a similarity score, compared to the existing lowest-energy structure, that is greater than 0.3 is identified as a new lowest-energy cluster. Clusters that have a total energy within 1 meV/atom to the existing lowest-energy cluster but are structurally dissimilar are considered as energetically similar clusters and are not counted as new lowest-energy clusters. Borderline cases were inspected manually. More details of how we determined whether the algorithm had found a new lowest-energy structure can be found in Section 6 of the Supplementary Information.

## DFT calculations

All DFT calculations were carried out using the Vienna ab-initio simulation package (VASP)<sup>62,67-69</sup> with the Perdew-Burke-Ernzerhof (PBE) exchange-correlation functional.<sup>70-72</sup> The projector augmented wave (PAW) dataset shipped with VASP with the title “PAW\_PBE Al 04Jan2001” was used.<sup>73,74</sup> Reciprocal space was sampled by a single  $k$ -point at the  $\Gamma$  point and the kinetic energy cutoff for the plane-wave basis was set to 240 eV. The electronic self-consistency loop was considered to reach convergence when subsequent steps had an energy difference below  $10^{-5}$  eV and the convergence criterion for ionic relaxation was set to a force difference below 0.01 eV/Å. Our dataset<sup>75</sup> shows that ground state Al nanoclusters with even sizes above 18 and odd sizes above 7 have net spins of 0  $\mu_B$  and 1  $\mu_B$ , respectively (see Section 4 of Supplementary Information). Therefore, all *ab initio* calculations fix the magnetic moment to 0  $\mu_B$  for even-sized clusters and to 1  $\mu_B$  for odd-sized clusters using the parameter NUPDOWN in VASP.

## DATA AVAILABILITY

Structure files for the lowest-energy clusters with 21 to 55 atoms are listed in a separate Supplementary Information file in the XYZ format.

## CODE AVAILABILITY

Our implementation of the GA\_AL procedure is open-sourced under the Apache License 2.0 at <https://gitlab.com/muellergroup/cluster-ga>. The code also supports genetic algorithm searches using only DFT or interatomic potentials through interfaces with VASP and LAMMPS. Input templates and documentation of input parameters can be found in the repository.

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## AUTHOR CONTRIBUTIONS

T.M. conceptualized and managed the project. Y.W., S.L., P.L., S.N., A.H. and T.M. developed the software. Y.W., S.L. and T.M. wrote the manuscript. Y.W. performed the benchmarks on MTP. Y.W., S.L., and S.M. carried out GA searches and analyzed data. S.L., Y.W. and S.M. collected structures from literatures. All the authors proposed, discussed, or developed ideas that improved the performance of the GA\_AL procedure and the quality of the data.

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# Supplementary Information

## for

### “Accelerated Prediction of Atomically Precise Cluster Structures Using On-the-fly Machine Learning”

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## 1 Genetic Algorithm

A genetic algorithm (GA) is a global optimization method inspired by the principle of natural selection.<sup>1</sup> Genetic algorithms have been successfully applied to discover ground state configurations for a diverse set of systems including elemental metal clusters,<sup>2-5</sup> binary and ternary alloys,<sup>6-8</sup> crystals,<sup>9,10</sup> oxides,<sup>11</sup> perovskites,<sup>12</sup> and metal organic frameworks (MOFs).<sup>13</sup> The efficiency of genetic algorithms comes from the fact that they can identify low-energy local atomic motifs and re-use them when generating candidate structures.<sup>3</sup> This section provides additional details about the genetic algorithm (GA) implementation used in this work.

### 1.1 Workflow of GA\_DFT

Our core implementation of the genetic algorithm, which we refer to as GA\_DFT, uses only density functional theory (DFT) to evaluate energies. Our approach is based off the pool-based Birmingham Parallel Genetic Algorithm (BPGA)<sup>5</sup> with some variations. In a pool-based GA, a fixed-size pool is maintained and comprises the lowest energy clusters among all structures that have been evaluated. This ensures that after each update the pool is at least as good as the old one. In this section we describe the general workflow of our code, with the specific parameters used in this work reported in later sections.

The GA has two phases: an initial phase when the pool starts off unfilled, and the genetic phase when the pool is full and child structure are generated by genetic operations. During the initial phase, structures can be generated by either a random distribution of atoms or by a seeding operation. We used the former method of random distribution of atoms in this work. With the former method, a cluster is generated by randomly distributing atoms in a cubic box with a side length of  $2r\sqrt[3]{N}$  where  $r$  is the atomic radius of corresponding element and  $N$  is the number of atoms. This approach ensures cluster volumes grow proportionately with the system size yet are not too sparsely populated.

In the **seeding** operation, new structures are generated from seed structures which are typically known low-energy structures with different numbers of atoms. Atoms are either randomly added or subtracted from the seed structure until it reaches the target size. If a seed cluster consists of a



different element, the atomic coordinates and cell size are scaled proportionately to the ratio of atomic radii. Although it is available in the open-source code, this operation was not used in this work.

Once the pool has been filled, the genetic algorithm begins to generate clusters using one of the three genetic operations: seeding (as described above), mutation, and crossover.

In a **mutation** operation, a parent cluster is randomly selected from the pool and a randomly chosen subset of atoms is either globally displaced or rotated around the center of mass to create a child configuration.

In a **crossover** operation, a roulette wheel scheme is used to pick pairs of parents from the pool, and the probability of a cluster being chosen is proportional to its selectability, which is designed to reward low-energy clusters and penalize those that have been picked many times.<sup>3,4</sup> The selectability of  $i^{\text{th}}$  cluster is defined as

$$S_i = f_i \times v_i, \quad (1)$$

where  $f_i$  represents the fitness of the  $i^{\text{th}}$  cluster and  $v_i$  is a penalty term that depends on parenting frequency. The fitness is defined using a hyperbolic function of a normalized total energy,

$$f_i = \frac{1}{2} [1 - \tanh(2\rho_i - 1)], \quad (2)$$

where the normalized energy  $\rho_i$  is given by

$$\rho_i = \frac{E_i - E_{\min}}{E_{\max} - E_{\min}}, \quad (3)$$

where  $E_{\min}$  and  $E_{\max}$  are the lowest and highest energies of the clusters in the pool.<sup>3</sup> The lower total energy one cluster has relative to the others, the higher probability it has to be selected as parent. The penalty term is formulated to avoid overusing of the same cluster and is calculated as

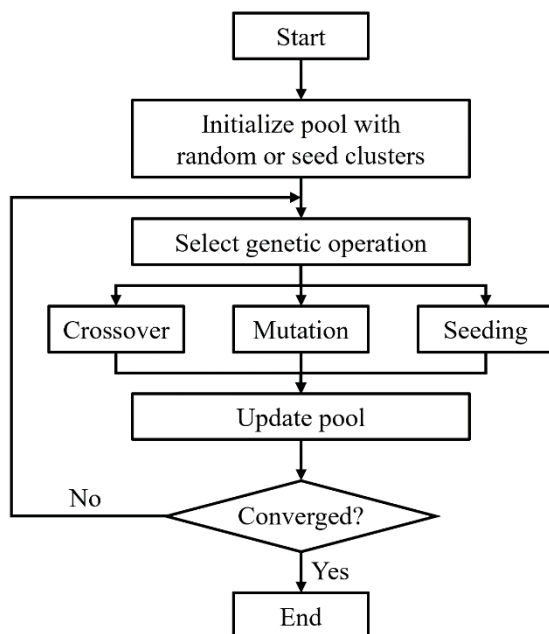
$$v_i = \frac{1}{1 + \sqrt{n_i}}, \quad (4)$$

where  $n_i$  stands for the number of times the  $i^{\text{th}}$  cluster has been picked as one of the parents in crossover during the entire GA run. It has been shown that the use of both the fitness function and the frequency penalty to determine selectability can accelerate the convergence speed and increase the success rate of GA.<sup>3,4</sup> Once a pair of parents is chosen, a child cluster is created by the cut-and-splice operation devised by Deaven and Ho.<sup>14</sup> In addition to the common practice in which a child cluster inherits equal share of total atoms from each parent (even), varying percentages are allowed. The inheritance ratio can be proportional to fitness of clusters (weighted) or be randomly decided (random).

Newly generated configurations from all methods are checked for occurrences of atom overlap. Two atoms are considered as overlapping if their pairwise distances are smaller than 80% of the

sum of atomic radii. If overlap exists, the corresponding cluster is regenerated by the same type of operation until it clears the examination.

Following successful cluster generation, the clusters are put in simulation boxes with lengths that allow at least 10 Å between adjacent periodic images, and the atomic coordinates are relaxed using either density functional theory (DFT) or interatomic potentials. Currently, our code supports VASP<sup>15-17</sup> and LAMMPS,<sup>18</sup> but any other energy-evaluation methods can be applied. Once child clusters are relaxed, they are compared with pool clusters. Child clusters which have lower energies than pool clusters and are structurally unique (defined here as having a similarity score above 0.3) are added to the pool, and the highest energy pool clusters are removed to maintain a constant pool size. If a new cluster has a similarity score of less than 0.3 with one or more pool clusters and lower energy than all of those pool structures, it will replace either the most similar pool cluster if the smallest score is less than 0.1 or the similar pool cluster with the highest energy if all scores are larger than 0.1. The similarity examination maintains the structural diversity of the pool, which improves the convergence speed and the success rate.<sup>3</sup> Following the update of pool, new parents are picked to reproduce next round of child clusters. This cycle continues until the genetic algorithm reaches a convergence criterion or the total number of clusters exceeds a maximum value. A schematic workflow summarizing this workflow is shown in Figure S1.



**Figure S1. Schematic workflow for the primitive genetic algorithm without active learning acceleration.**

## 1.2 Convergence criteria and total running time

We consider the GA to be converged when the pool becomes stagnant, which is defined here as occurring after the GA continuously generates 4000 or more clusters without discovering a cluster with lower energy than the current lowest value. For the calculations presented in the main text, GA\_ALs were initially run for 3 days (1728 CPU-hours) for sizes 21 to 40 and for 6 days (3456 CPU-hours) for sizes 41 to 55, followed with a check for stagnation. For the sizes in which the

convergence criterion was not met, the GAs were resumed until convergence. GA\_DFT, on the other hand, is computationally expensive and evaluating 4000 clusters or more can be prohibitively expensive for the sizes investigated in this work. To ensure a fair comparison with GA\_ALs, GA\_DFT calculations were run for at least the same number of CPU-hours as the corresponding GA\_AL calculations. For clusters with 50 to 55 atoms, GA\_DFT calculations were run for 21 days, much longer than GA\_DFT calculations on small to medium-sized clusters, to reach similar energy levels as GA\_AL calculations and to make comparisons meaningful. Table S1 summarizes computational time for all GA runs in this work. The computing center (MARCC) we used has a wall-time limit of 3 days for each individual job, so calculations were mostly performed for a multiple of 3 days. Some calculations that were very close to convergence (e.g., for sizes 39, 40, 47, 55) were continued for only one or two days, which was sufficient to reach convergence.

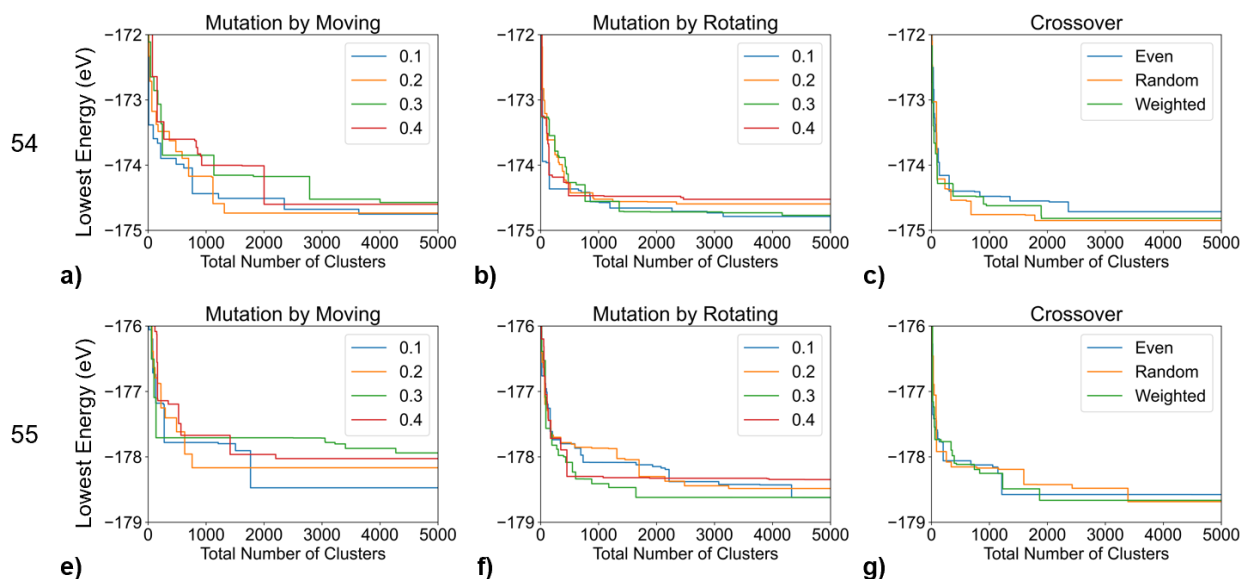
**Table S1. Computation time in days for all GA calculations in the main text.**

| Cluster Size | GA_AL with pre-trained potentials | GA_AL without pre-trained potentials | GA_DFT | Cluster Size | GA_AL with pre-trained potentials | GA_AL without pre-trained potentials | GA_DFT |
|--------------|-----------------------------------|--------------------------------------|--------|--------------|-----------------------------------|--------------------------------------|--------|
| 21           | --                                | 3                                    | 3      | 39           | --                                | 5                                    | 5      |
| 22           | --                                | 3                                    | 3      | 40           | --                                | 4                                    | 4      |
| 23           | --                                | 3                                    | 3      | 41           | 6                                 | --                                   | 6      |
| 24           | --                                | 3                                    | 3      | 42           | 6                                 | --                                   | 6      |
| 25           | --                                | 3                                    | 3      | 43           | 6                                 | --                                   | 6      |
| 26           | --                                | 3                                    | 3      | 44           | 6                                 | --                                   | 6      |
| 27           | --                                | 3                                    | 3      | 45           | 6                                 | --                                   | 6      |
| 28           | --                                | 3                                    | 3      | 46           | 6                                 | --                                   | 6      |
| 29           | --                                | 3                                    | 3      | 47           | 7                                 | --                                   | 7      |
| 30           | --                                | 3                                    | 3      | 48           | 6                                 | --                                   | 6      |
| 31           | --                                | 3                                    | 3      | 49           | 6                                 | --                                   | 6      |
| 32           | --                                | 3                                    | 3      | 50           | 6                                 | 6                                    | 21     |
| 33           | --                                | 3                                    | 3      | 51           | 6                                 | 9                                    | 21     |
| 34           | --                                | 3                                    | 3      | 52           | 6                                 | 6                                    | 21     |
| 35           | --                                | 3                                    | 3      | 53           | 6                                 | 11                                   | 21     |
| 36           | --                                | 3                                    | 3      | 54           | 12                                | 17                                   | 21     |
| 37           | --                                | 3                                    | 3      | 55           | 7                                 | 9                                    | 21     |
| 38           | --                                | 3                                    | 3      |              |                                   |                                      |        |

### 1.3 GA input parameters

Parameters controlling the GA workflow can affect the convergence speed.<sup>3,4</sup> Figure S2 presents energy evolution plots of GA runs with varying ratios of mutated atoms in mutation operations and different inheritance ratios in crossover operations. The benchmarks were performed on clusters of sizes 54 and 55 with the mixed-size potentials labelled as “even” and “odd” in the main text. Since we are primarily concerned about the efficiency of GA operations instead of prediction

accuracy, energies were only evaluated by MTP and the primitive GA workflow (Figure S1) was used (GA\_MTP). All runs were stopped once the total number of clusters exceeded 5000. For both sizes, crossover is more efficient than both types of mutation operations in terms of the rate of convergence to low-energy structures. Among the two types of mutation operations, rotation is better than random moves. Within crossover benchmarks, the random method for calculating inheritance ratios of parents outperformed the even method, while it was close to the weighted method. As for mutation, the lowest energy for both sizes and both types of mutation operations was achieved when 10% of the atoms were moved, although this approach did not have the fastest convergence speed (Figure S1a and Figure S1f). These findings suggest that adjustments to a small number of atoms are more efficient in minimizing the energy, while operations involve large number of atoms often stagnate at elevated energy levels. This can be explained from the fact that often only a small fragment of atoms is out of place compared with the lowest-energy structure in the late stages of a GA optimization. Relocating only a few atoms is more likely to put dislocated atoms into the right positions than simultaneously displacing a large chunk of atoms from a structure which is already close to the lowest-energy isomer.



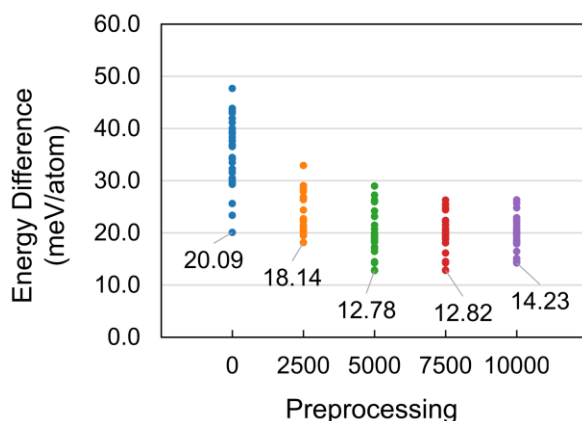
**Figure S2. Energy evolution plots of GA\_MTP for different GA operations and controlling parameters.** a) and e) mutation by randomly displacing atoms with varying ratios of mutated atoms. b) and f) mutation by randomly rotating atoms around center of mass with different ratios of mutated atoms. c) and g) crossover with various calculation methods for ratios of inherited atoms from parents. The plots on first row uses the “even” potential discussed in main text on size 54, while those on the second row uses the “odd” potential on size 55.

For all GA runs in the main text, both move and rotation were used in mutation, and in each mutation operation a maximum 10 % of atoms were allowed to be displaced. For crossover, both random and weighted methods were used with an equal probability. Moreover, since crossover is more efficient than mutation, the frequency of applying mutation in cluster generation was suppressed by capping the fraction of pool clusters created by mutation at 0.2.<sup>5</sup> In other words, if

20% or more pool clusters are produced by mutation, the subsequent GA operations will always be crossover until the percentage drops below 20%. The pool size was kept at 25, as it offers a good balance between convergence speed and success rate of identifying the lowest-energy cluster.<sup>3,4</sup>

#### 1.4 GA\_AL input parameters

When initializing GA\_AL with pre-trained potentials, it is beneficial to switch off retraining at the beginning of the search, to let GA fully explore the potential energy surfaces of pre-trained potentials. We call the parameter controlling the number of clusters that can be generated before retraining is turned on “preprocessing”. During the preprocessing period, extrapolating structures are discarded, and new clusters are generated until all MTP-relaxed configurations are interpolating. This ensures the GA search stays in interpolating region of the pre-trained potential in configuration space. As Figure S3 shows, the lowest energies in the first retraining were achieved when preprocessing was between about 5000 and 7500 clusters. Exceeding this range, artificial local minima start to populate the pool and drive up the lowest energy. To accelerate GA\_AL and minimize time spent in the preprocessing stage, we used a preprocessing value of 5000 in all GA\_AL runs in this work initialized with pre-trained potentials.



**Figure S3. DFT energy distribution of pool clusters in the first retraining after preprocessing number of clusters has been generated in GA\_AL.** Tests were performed on clusters with 55 atoms and GA\_AL runs were initialized with the mixed-size potential trained on odd number of clusters (“odd” potential in Figure 3 of the main text). Energies are reported relative to the energy of the lowest-energy cluster with 55 atoms.<sup>19</sup>

## 2 Hyperparameter selection for MTP

### 2.1 Data selection and training

To identify a good set of hyperparameters for training MTP potentials, we used Al clusters with 24 atoms as a model system and tested various combinations of hyperparameters, including potential complexity (defined by the level of moments,  $lev_{max}$ , of a MTP potential),<sup>20</sup> the amount of training data, and relative weights for force components and energies. We first performed GA\_DFT searches on Al clusters with 24 atoms and then used the same diversity-based strategy as used in the preparation of mixed-size potentials (see Methods section of the main text) to select dissimilar configurations from the GA\_DFT runs. We then randomly separated all DFT calculations into training and validation groups by keeping the ratio of the total number of dissimilar ionic steps in the training group to that of the validation group at approximately 3:1. To ensure structural dissimilarity of clusters in the training dataset from those in the validation set, we assign all ionic steps belonging to each relaxation trajectory to either the training or validation groups. This increases the likelihood that the validation results reliably reflect the accuracy of MTP potentials in areas of the configuration space that are not well-represented by the training data.

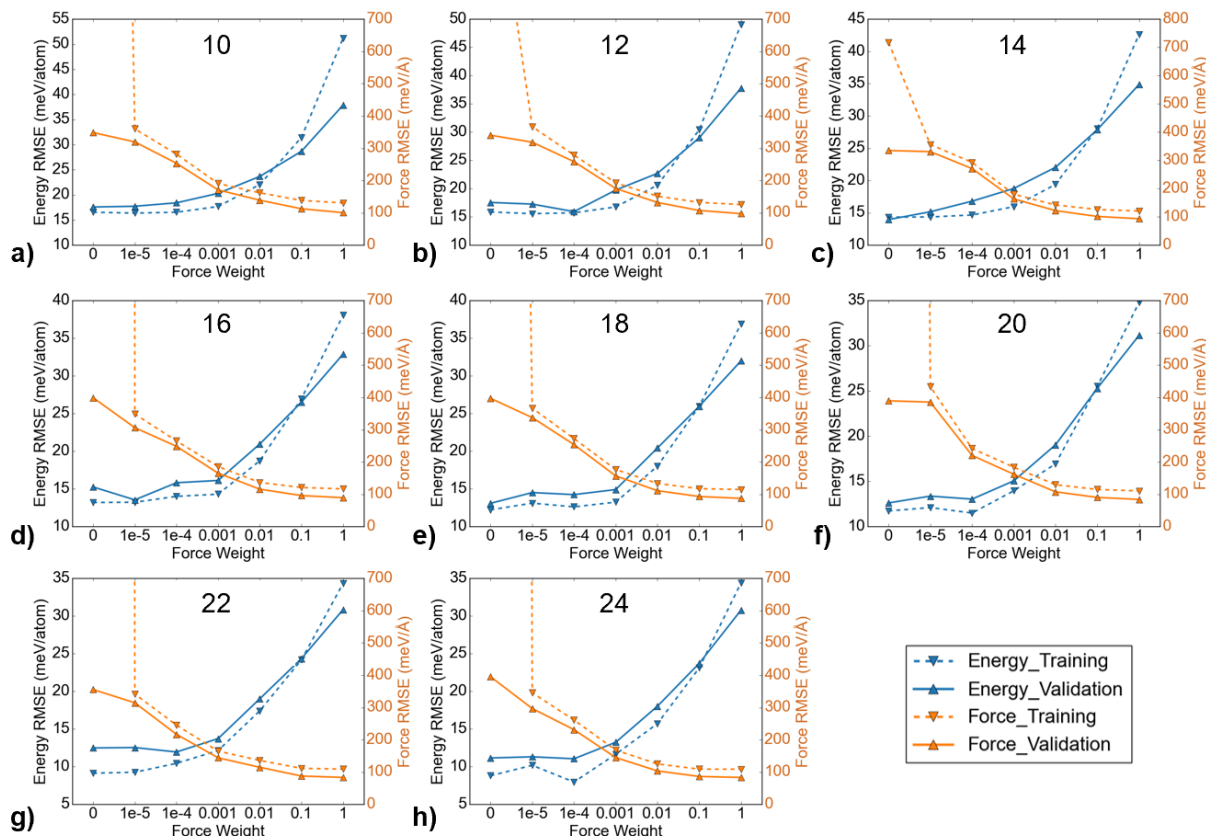
To assess the effect of training set size on potential accuracy, we used the same energy-based strategy as used for the preparation of mixed-size potentials (see Methods of the main text) to collect training datasets from the training group. We evaluated training sets containing 200, 400, 600, 800, 1000, 1500, 2000, 2500, and 3000 structures. For the validation dataset, we selected the 1000 lowest-energy ionic steps from the DFT calculations in the validation group, as the accuracy on low-lying areas of potential energy surfaces matters the most in ground state structure search.

The maximum number of iterations in Broyden-Fletcher-Goldfarb-Shanno algorithm<sup>21-24</sup> for training the potentials was set to 5000, which is 5 times larger than the default value of 1000.<sup>20</sup> Only trainings using small relative force weights less than 0.001 for potentials with levels of moments of 20, 22 and 24 stopped because of reaching this limit. We expect none of these trainings will affect the conclusions we draw from the set of benchmarks. None of the trainings in Figure 1 of the main text were terminated due to hitting the limit.

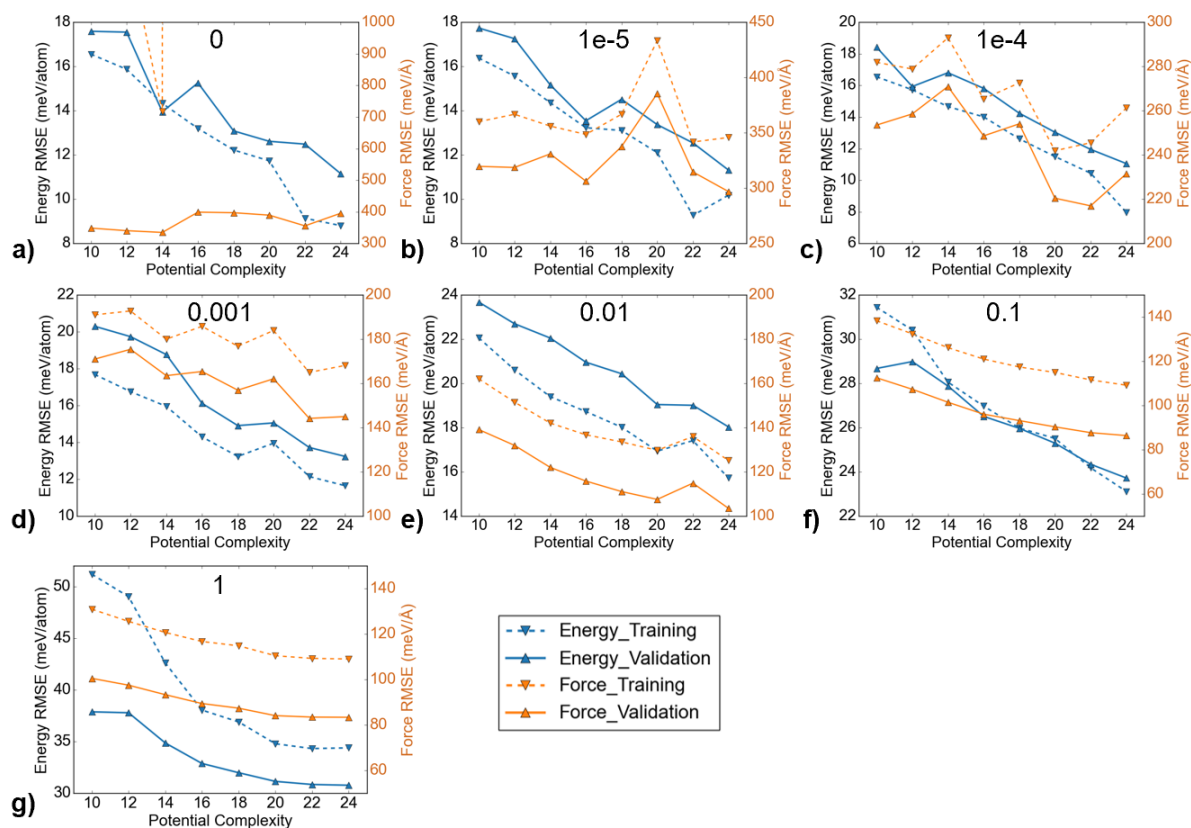
### 2.2 Analysis of various combinations of parameters

The main text presents the benchmarks that are most relevant to the hyperparameters that we used in GA\_AL runs. Here, we give a more comprehensive analysis of all parameter combinations that we tested. They all show similar trends as the ones shown in the main text. A force weight that is 1/1000 of the energy weight offers the best balance between prediction accuracies on force components and energies (Figure S4). Both force and energy root-mean-square errors (RMSEs) decrease as potential complexity rises (Figure S5). However, the training cost increases dramatically with both training data size and potential complexity (Figure S6). A somewhat surprising finding is that larger force weights took less time to train than smaller ones. A possible explanation is that the potential energy surface for atomic clusters is very rough, making it difficult to reproduce energies with a high accuracy, whereas there was less variation in the forces for nearly-relaxed structures. The potentials with  $lev_{max}$  of 10 and 12 see a steady increase of energy

validation RMSEs as the training set grows. Potentials with  $lev_{max}$  larger than 14 have relatively flat force and energy RMSEs after the training set size exceeds 2000, but there can be significant variation with respect to training set size for training sets with fewer than 1000 structures (Figure S7).

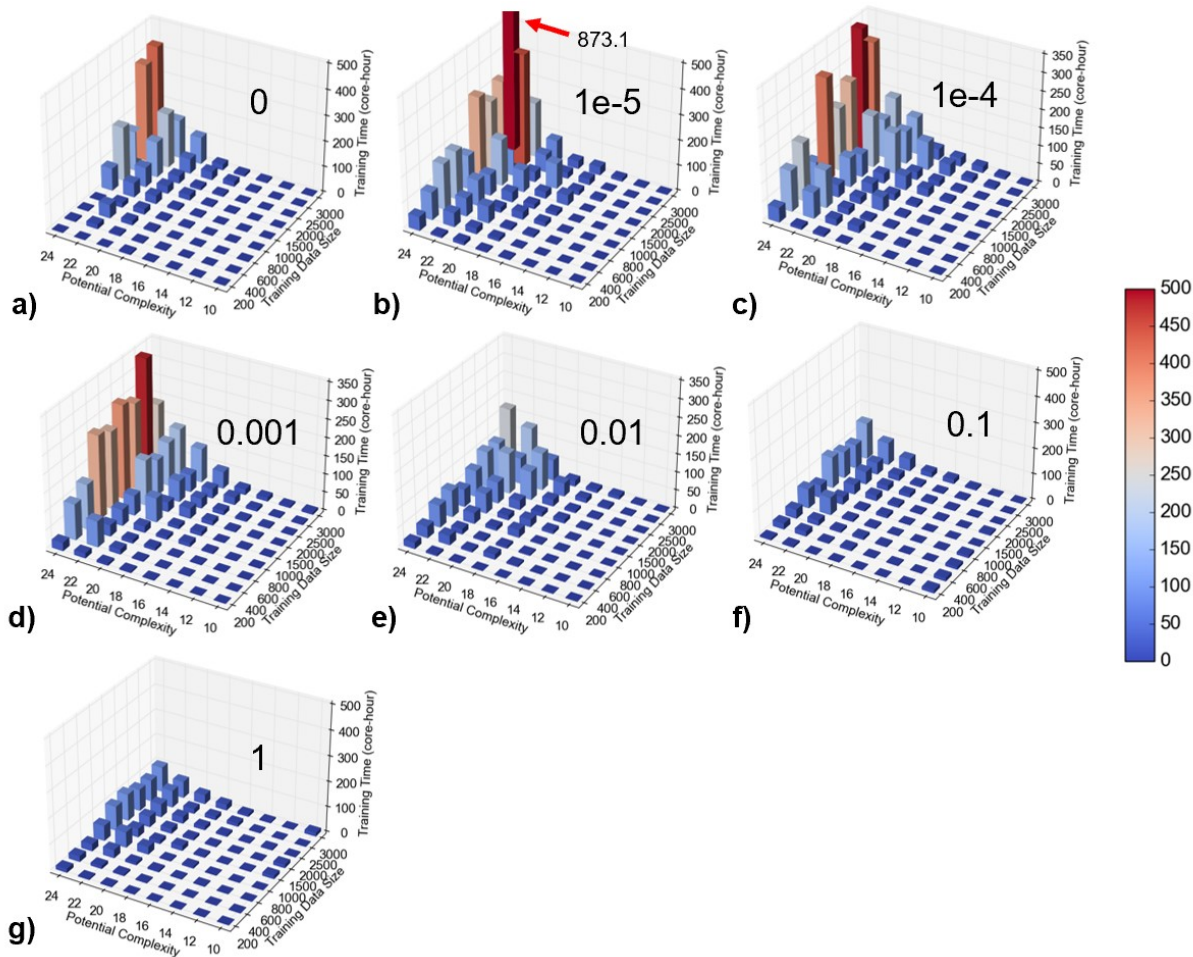


**Figure S4. Training and validation errors against relative force weights to energy across different potential complexities.** Training and validation RMSEs are marked by dashed and solid lines, respectively. In all benchmarks, the energy weight was set to 1 and a training set of 3000 structures was used.

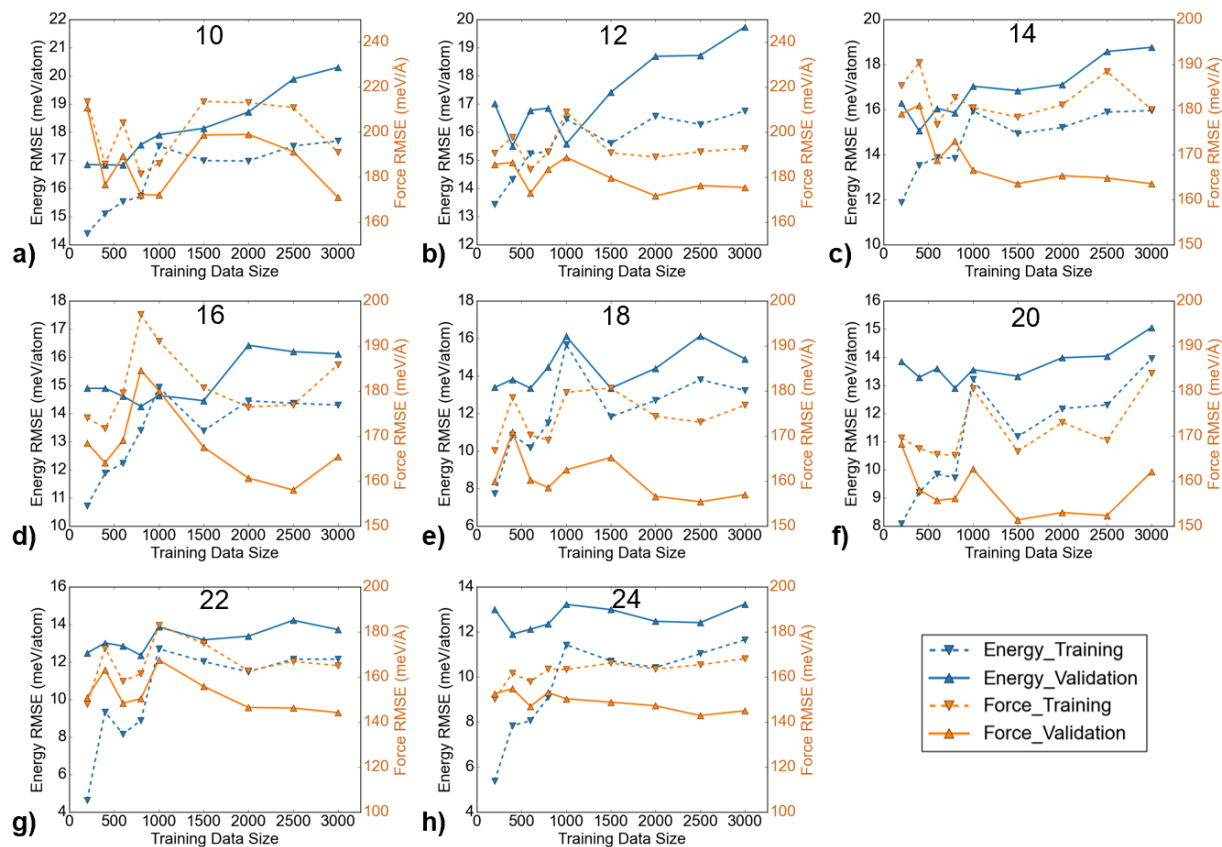


**Figure S5. Training and validation errors against increasing potential complexities across varying force weights relative to energies.** For all benchmarks, the energy weight was kept at 1 and a training set of 3000 structures was used.





**Figure S6. Training time plots in unit of core-hour against potential complexity and training set size for increasing force weights relative to energy.**



**Figure S7. Training and validation errors against size of training set across different potential complexity.** In all benchmarks, relative weight of force components to energies was kept at 1/1000.

### 3 Validation of pre-trained potentials

#### 3.1 Validation dataset for pre-trained potentials

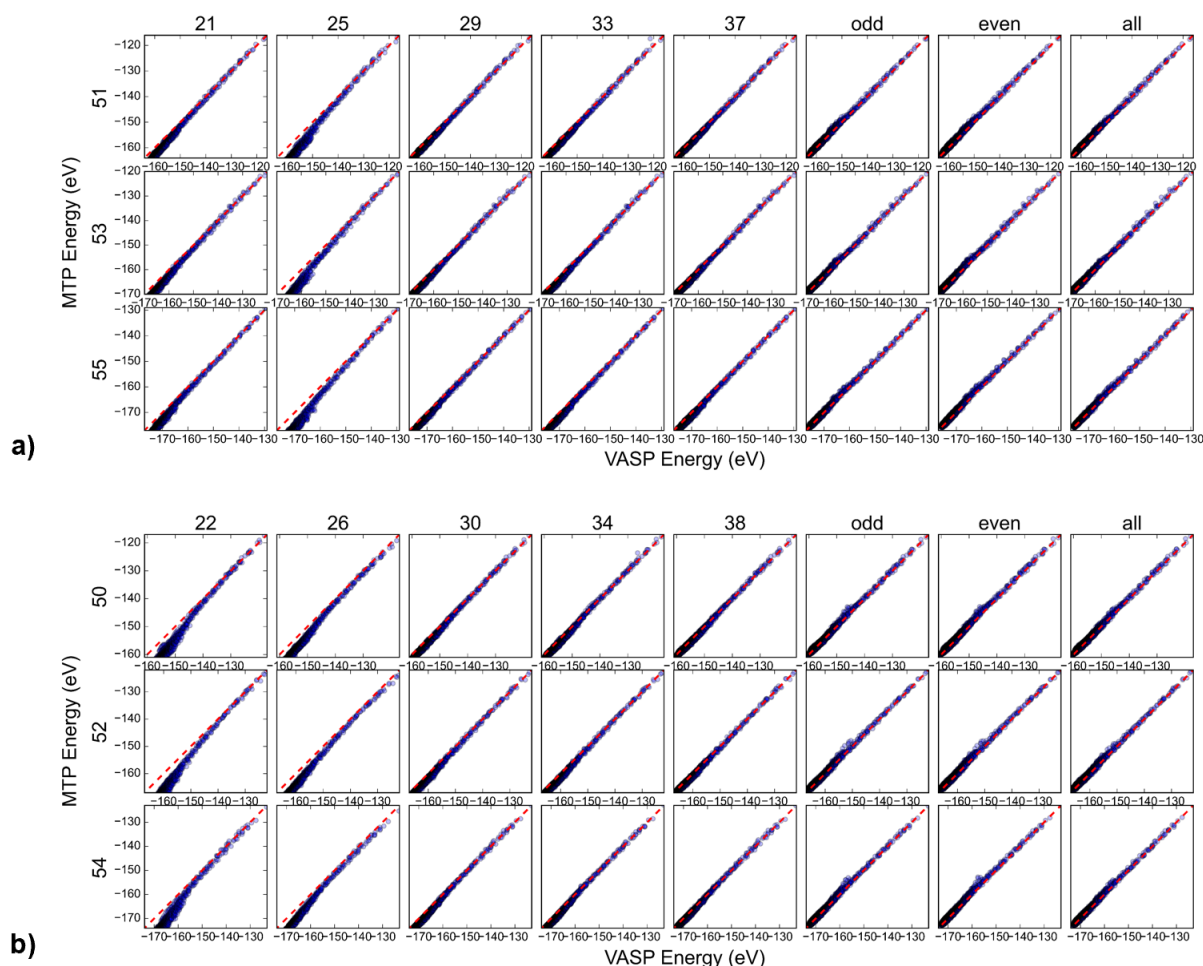
The validation data are collected from GA\_DFT runs on clusters with 50-55 atoms. The same diversity-based method as the one used in preparation of the training data of the pre-trained potentials (see Methods section of the main text) was used to select a structurally diverse set of structures. Table S2 lists the total number of validation structures for each cluster size.

**Table S2. Total number of structures in validations sets of clusters with 50 to 55 atoms.**

| Cluster Size | Number of Validation Structures |
|--------------|---------------------------------|
| 50           | 3840                            |
| 51           | 2946                            |
| 52           | 3653                            |
| 53           | 2581                            |
| 54           | 3578                            |
| 55           | 2604                            |

#### 3.2 Energy parity plots for pre-trained potentials

Figure S8 shows parity plots of MTP predicted energies against DFT calculated energies for the same set of validations presented in Figure 3 of the main text. The single-size potentials trained solely on small clusters of 21, 22, 25 and 26 atoms consistently underestimated low-lying structures of large sizes by a relatively large error, while the underestimation was smaller for potentials trained on clusters of 29 to 38 atoms. The mixed-size potentials exhibit a better fit for all validation sets, indicating enhanced transferability compared with single-size potentials.



**Figure S8. Energy scatter plots for validation of single-size and mixed-size potentials on large clusters with 50-55 atoms.** Plots are grouped by parity of number of atoms into **a)** an odd group, and **b)** an even group.

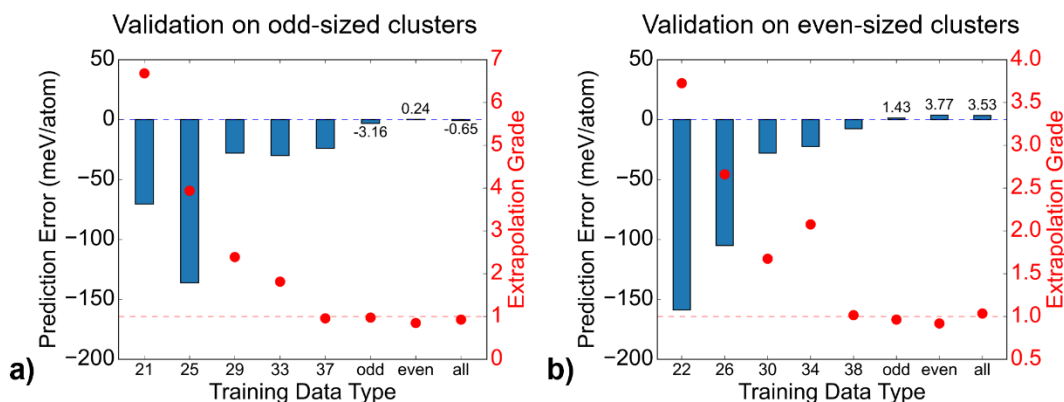
### 3.3 Validation of mixed-size potentials on lowest-energy clusters with 50 to 55 atoms

Since we are primarily concerned with potential accuracies on low-energy structures, the single-size and mixed-size potentials were also validated on the lowest-energy clusters of sizes with 50-55 atoms, which were collected from GA\_AL and GA\_DFT runs of this work, and from the literature.<sup>19</sup> The lowest-energy clusters were grouped into a set of even-sized clusters and a set of odd-sized clusters.

Figure S9 displays the average energy errors of MTP predictions, and the average extrapolation grades of the lowest-energy clusters in different potentials. The errors follow a similar trend as the energy validation RMSEs in Figure 3a and Figure 3b of the main text. Prediction errors are larger for single-size potentials trained with clusters with small number of atoms, and these errors decrease gradually as the size of the clusters in the training set increases. Mixed-size potentials have consistently lower errors than any single-size potentials. Potentials trained with clusters of both even and odd number of atoms, hence different magnetic moments, do not greatly deteriorate

accuracy. Surprisingly, the odd potential gave a better prediction than the even potential for the lowest-energy clusters with even number of atoms, which is also the case for the even potential on lowest-energy clusters with odd number of atoms. This suggests the gap between potential energy surfaces with different magnetic moments is not larger than the uncertainty of the moment tensor potentials. Mixing training structures of different magnetic moments results in an average effect: errors for the potential trained on clusters with both even and odd number of atoms are between the errors of the potentials trained on clusters with only even or odd numbers of atoms.

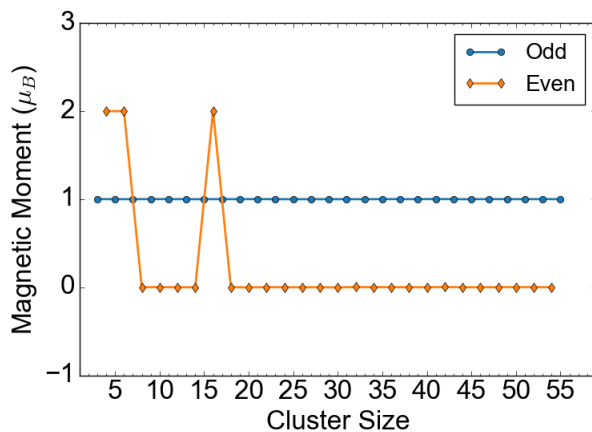
The trend in extrapolation grades provides additional evidence showing that mixing training data across a range of number of atoms can systematically enhance transferability to clusters of sizes that were not in the training set. For all six of the lowest-energy clusters, extrapolation grades of single-size potentials get smaller as the number of atoms in the training structures grows. Nearly all of them are extrapolating (i.e., the extrapolation grade is above 1), with the exception of the potential trained on clusters with 37 atoms, which has an extrapolation grade of 0.95. However, for the even and odd potentials, these lowest-energy clusters are interpolating, and energy predictions are in good agreement of DFT calculations, with errors well within potential uncertainties as shown in Figure 3 of the main text. Since the goal of the structure search is to discover unknown low energy configurations, high prediction accuracy on low-lying regions of potential energy surface is vital. Thus, validations on the lowest-energy clusters demonstrate that our strategy of constructing mixed-size potentials transferrable to clusters with number of atoms not present in the training data can be very helpful in improving the efficiency and success rate of the genetic algorithm.



**Figure S9. Validations of different potentials on the lowest-energy clusters with a) 50, 52, 54 atoms, and b) 51, 53, 55 atoms.** Labels on x-axis share the same meaning as those in Figure 3 of the main text. The primary y-axis on the left and blue bars indicate average energy prediction errors for the lowest-energy clusters. The secondary y-axis on the right and the red dots represent the average extrapolation grades of the lowest-energy clusters in corresponding potentials.

## 4 Magnetic moments of low-energy clusters

We assembled in another work a database comprising low-energy clusters of 55 elements in the first five periods of the periodic table.<sup>25</sup> For aluminum clusters, the even-sized structures having the lowest energies are always non-magnetic when they have more than 16 atoms, while the low-lying odd-sized structures always have a net magnetic moment within  $0.002 \mu_B$  of  $1 \mu_B$ . Figure S10 displays the magnetic moments of the lowest-energy clusters in the database at the time we submitted this manuscript.

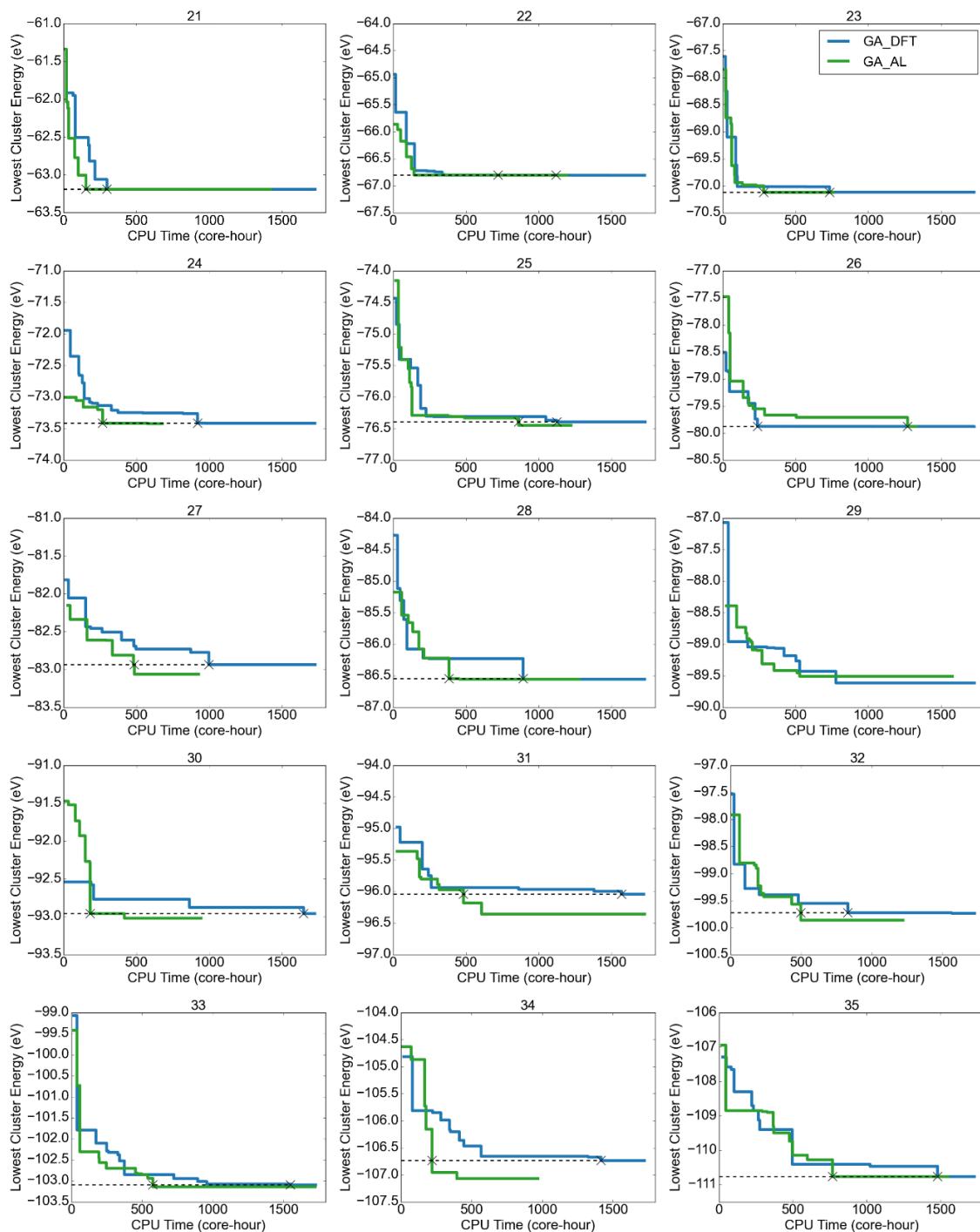


**Figure S10. Magnetic moments of the lowest-energy aluminum clusters in the database with 3 to 55 atoms.**

## 5 Performance analysis of GA\_AL and GA\_DFT

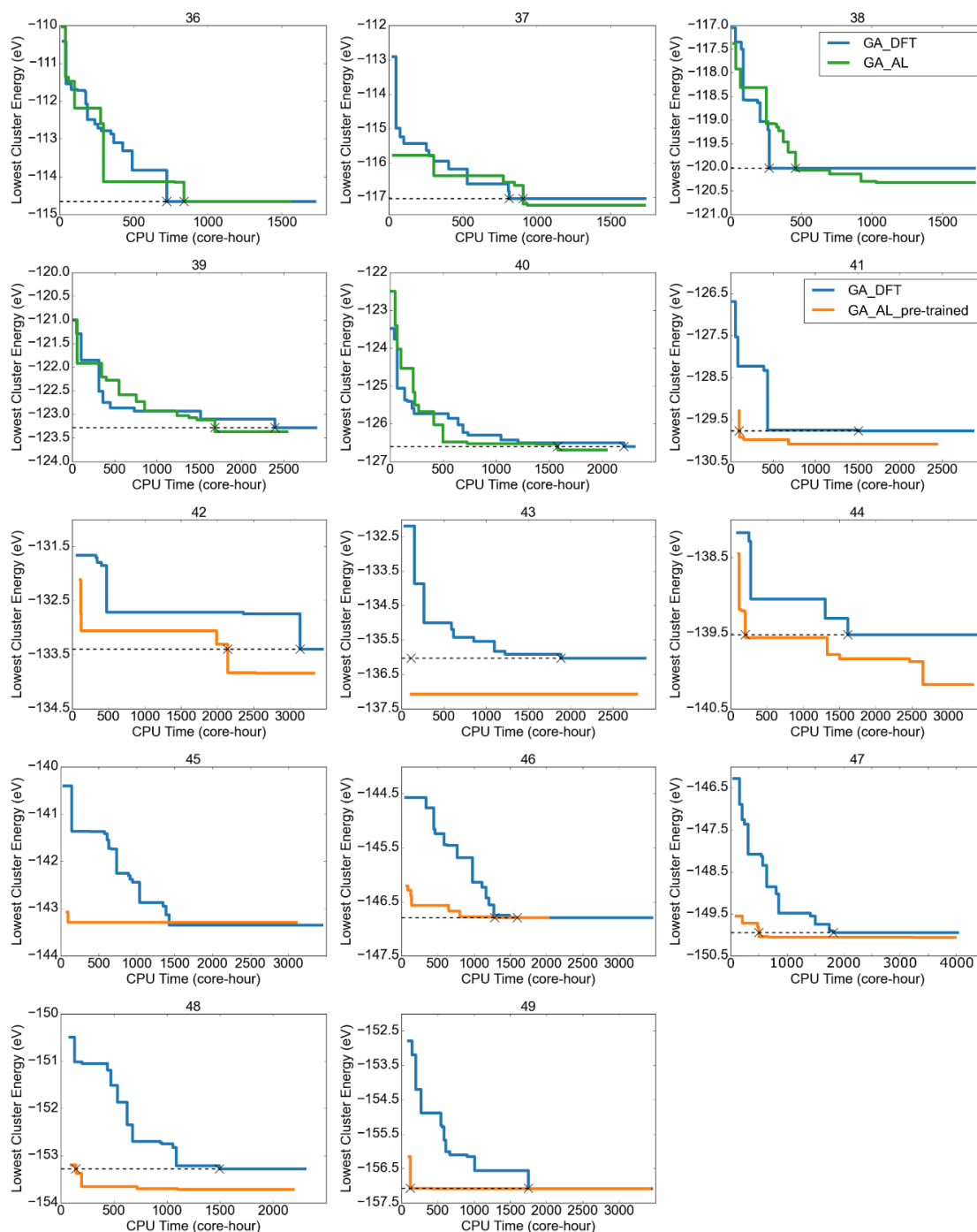
### 5.1 Energy evolution plots for GA\_AL and GA\_DFT on clusters with 21 to 49 atoms

Figure S11 and Figure S12 present the energies of the lowest-energy clusters as GA\_AL and GA\_DFT proceeded, for clusters with 21 to 49 atoms. Acceleration ratios are computed by dividing the earliest time that GA\_AL discovers clusters with equal or lower energy than the energy of the lowest-energy cluster found by GA\_DFT over the earliest time that GA\_DFT finds that lowest energy cluster. These energy values are marked by black dashed lines in the energy evolution plots. The time points at which the step lines cross the dashed lines indicate the times used in acceleration ratio measurement and are marked by black crosses. In cases where GA\_AL did not discover equivalent or better clusters (size 29 and 45), the acceleration ratios are 0, and the dashed lines and crosses are not drawn.



**Figure S11. Energy evolution plots of GA runs for clusters with 21–35 atoms.** The blue, and green step lines represent GA\_DFT and GA\_AL initialized with untrained potentials. The black dashed lines mark the energy levels that are used in determining acceleration ratios. The black crosses mark the earliest time at which each method reached the corresponding energy levels. These times are used in computing the acceleration ratios. For size 29 in which GA\_AL did not find clusters with energies as low as the lowest-energy cluster in GA\_DFT, the ratio is set to 0 and no crosses were marked.

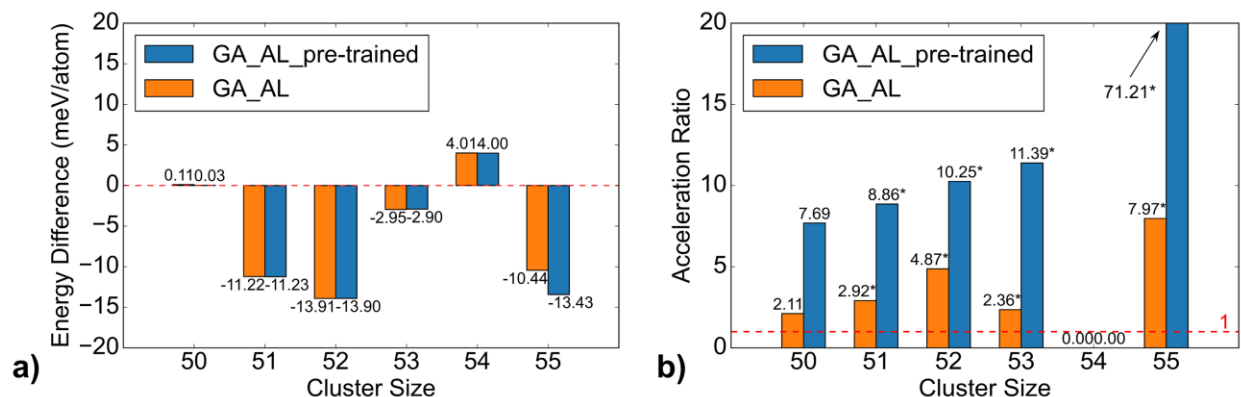




**Figure S12. Energy evolution plots of GA runs for clusters with 36 to 49 atoms.** The blue lines represent GA\_DFT, green lines represent GA\_AL initialized with untrained potentials, and orange lines represent GA\_AL initialized with pre-trained transferable potentials. The black dashed lines mark the energy levels that are used in determining acceleration ratios. The black crosses mark the earliest time at which each method reached the corresponding energy levels. These times are used in computing the acceleration ratios. For size 45 in which GA\_AL did not find clusters with energies as low as the lowest-energy cluster in GA\_DFT, the ratio is set to 0 and no crosses were marked.

## 5.2 Performance comparison of GA\_AL with and without pre-trained potentials

We performed both GA\_AL initialized with pre-trained potentials and GA\_AL initialized with untrained potentials on clusters of 50 to 55 atoms. The latter approach took longer time than the former to find the same lowest-energy clusters (see Figure 4a of the main text). Figure S13 shows the acceleration ratios and differences of lowest energies compared to GA\_DFT for both approaches. Averaged over cluster sizes of 50 to 55, GA\_AL initialized with untrained potentials had an acceleration ratio of 3.37, compared with the much larger ratio of 18.23 for GA\_AL initialized with the mixed-size potentials.



**Figure S13. Comparison of performances between GA\_AL initialized with pre-trained mixed-size potentials and with untrained potentials on clusters with 50 to 55 atoms.** Acceleration ratios and energy differences of lowest-energy clusters are calculated in the same way as Figure 2b and Figure 4c of the main text.

## 6 Identification of new lowest-energy clusters

In the main text, low-energy clusters were collected from three sources: GA\_AL runs, GA\_DFT runs and literature. To determine whether we have identified a new lowest-energy structure for a cluster size and to compare performance between the GA\_AL and GA\_DFT algorithms, the lowest-energy clusters found from different sources are compared both in total energies and similarity. A cluster is said to be a new lower-energy cluster than another if it is lower in total energy by at least 1 meV/atom and the similarity score between the two clusters is larger than 0.3. If a cluster has an energy difference smaller than 1 meV/atom but is structurally dissimilar to another cluster, we count it as an “energetically similar” cluster. For the borderline cases where a cluster is lower in energy by more than 1 meV/atom, but the similarity score between the two clusters is slightly less than 0.3, we manually checked the structures to determine whether they are equivalent or not. These scenarios are summarized in Table S3.

**Table S3. Summary of comparison results between clusters by total energy and structural similarity.**

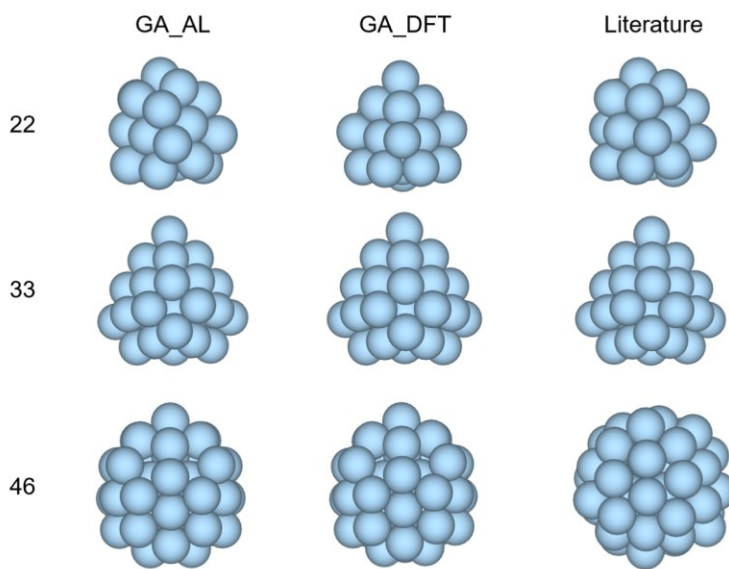
|                   |                            | Similarity Comparison  |                             |
|-------------------|----------------------------|------------------------|-----------------------------|
|                   |                            | Similarity Score > 0.3 | Similarity Score $\leq$ 0.3 |
| Energy Comparison | $\Delta E > 1$ meV/atom    | New, lower-energy      | Visually check              |
|                   | $\Delta E \leq 1$ meV/atom | Energetically similar  | Equivalent                  |

The same comparison methods are used in deciding whether a GA search (GA\_DFT or GA\_AL) has identified a new lowest-energy cluster compared with those reported in the literature<sup>19,26-28</sup> and in comparing between clusters found by the GA\_AL and GA\_DFT algorithms. Cluster sizes for which there is a cluster that is energetically similar to the current lowest-energy cluster are considered to have multiple lowest-energy isomers.

Table S4 listed the total energies and the similarity scores for lowest-energy clusters identified in GA\_AL and GA\_DFT runs of this work, and the ones reported in literature. The total energies are reported relative to the energies of the clusters found in GA\_AL runs.

Three cluster sizes, 22, 33 and 46, requires additional examination. GA\_DFT identified an isomer for clusters with 22 atoms that is energetically similar to the lowest-energy cluster reported in literature<sup>27</sup> and found by GA\_AL. For size 33, although the two low-energy clusters from GA\_DFT and GA\_AL have an energy difference larger than 1 meV/atom, the similarity score and visual inspection (Figure S14) show that they are equivalent. For size 46, the lowest-energy cluster reported in literature has a similarity score with the ones found in GA\_AL and GA\_DFT that is less than 0.3, but the energy difference and visual inspection show the structures are not equivalent. Therefore, the genetic algorithm is considered to have identified new lowest-energy clusters for sizes 22 and 46, but only confirmed the literature-reported cluster for size 33.

GA\_AL identified new lowest-energy clusters for 23 sizes out of the total 35 sizes studied in this work. GA\_AL runs and GA\_DFT runs together identified 25 new lowest-energy clusters (26 if counting the energetically similar cluster for size 22) and confirmed the literature-reported lowest-energy clusters for 8 sizes. Overall, the GA runs of this work only missed the lowest-energy literature structures for clusters with 32 and 55 atoms. It is likely that these clusters could have been found more quickly if as seeing operation was used, given the similarity between the lowest-energy clusters of these two sizes to the clusters with close number of atoms (see Figure S15).



**Figure S14.** Lowest-energy clusters from GA\_AL, GA\_DFT and the literature<sup>27,28</sup> for clusters with 22, 33, and 46 atoms.

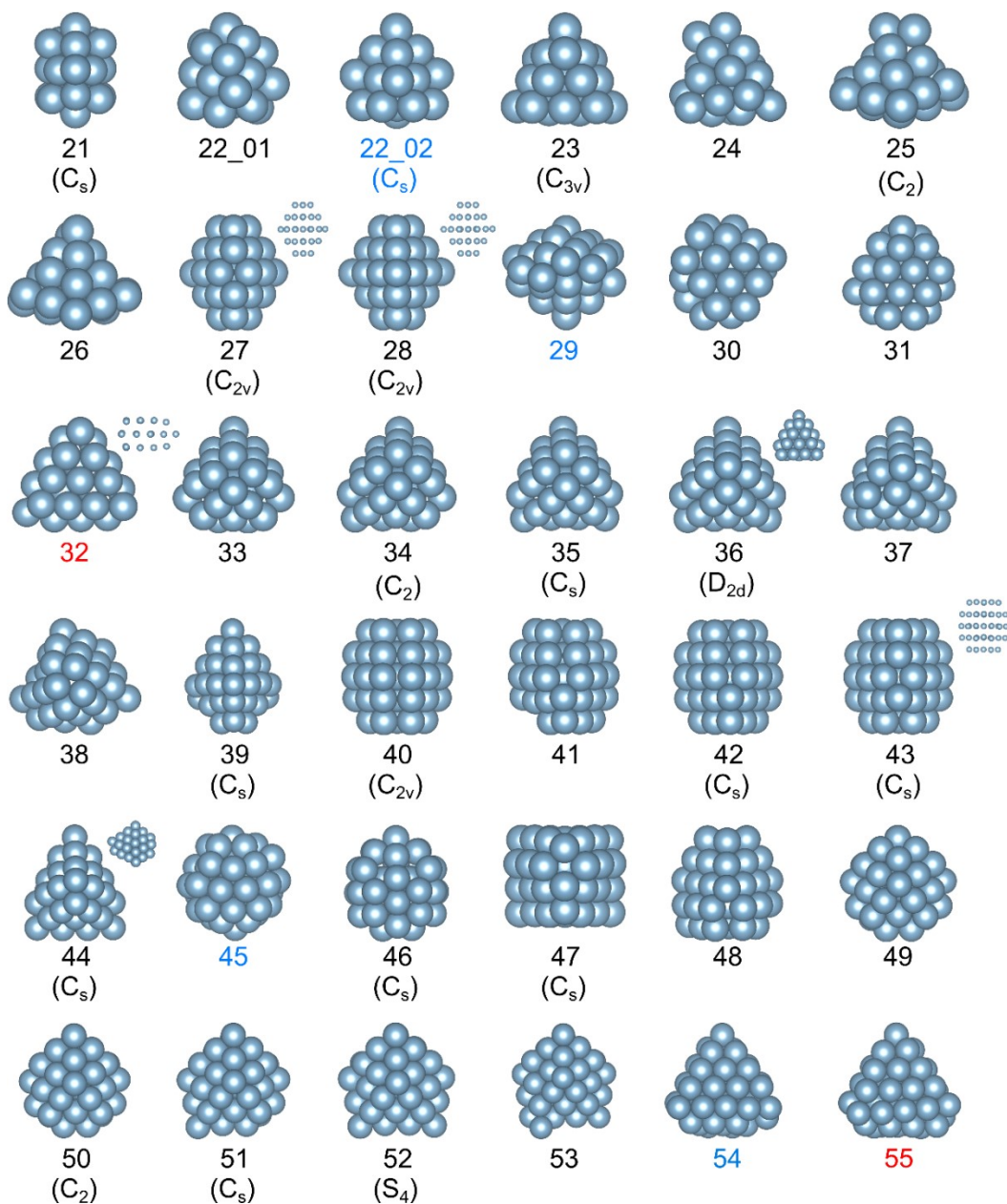
**Table S4. Total energy differences and similarity scores between lowest-energy clusters found in GA\_AL and GA\_DFT runs of this work, and the ones reported in literature. “Lit.” is the abbreviation for “Literature”. Energy differences are reported in meV/atom relative to the total energies of lowest-energy clusters from GA\_AL (include GA\_AL initialized with untrained potentials or pre-trained potentials). Similarity scores are calculated using the same method reported in Methods section of the main text. The origins of the final lowest-energy clusters are listed in the last column. Three cluster sizes that display large energy differences but small similarity scores or small energy differences but large similarity scores are marked with asterisks and are discussed in detail in Section 6.**

| size | $\Delta E$ relative to GA_AL<br>(meV/atom) |        |       | Similarity Score |                |                 | Origin of<br>lowest-energy cluster |
|------|--|--------|-------|------------------|----------------|-----------------|------------------------------------|
|      | GA_AL                                      | GA_DFT | Lit.  | GA_AL-<br>GA_DFT | GA_AL-<br>Lit. | GA_DFT<br>-Lit. |                                    |
| 21   | 0.00                                       | 0.11   | 0.14  | 0.010            | 0.008          | 0.015           | GA_AL / GA_DFT / Lit.              |
| 22*  | 0.00                                       | -0.08  | 0.15  | 0.556            | 0.013          | 0.554           | GA_AL / GA_DFT / Lit.              |
| 23   | 0.00                                       | 0.06   | 1.31  | 0.009            | 0.766          | 0.767           | GA_AL / GA_DFT                     |
| 24   | 0.00                                       | 0.03   | 9.98  | 0.009            | 1.611          | 1.614           | GA_AL / GA_DFT                     |
| 25   | 0.00                                       | 2.03   | 10.11 | 0.560            | 1.672          | 1.460           | GA_AL                              |
| 26   | 0.00                                       | 0.02   | 13.10 | 0.009            | 1.411          | 1.411           | GA_AL / GA_DFT                     |
| 27   | 0.00                                       | 4.61   | 0.02  | 1.191            | 0.010          | 1.193           | GA_AL / Lit.                       |
| 28   | 0.00                                       | 0.00   | 0.04  | 0.018            | 0.019          | 0.006           | GA_AL / GA_DFT / Lit.              |
| 29   | 0.00                                       | -3.62  | 1.63  | 1.118            | 1.083          | 1.373           | GA_DFT                             |
| 30   | 0.00                                       | 2.08   | 2.21  | 0.820            | 0.754          | 1.070           | GA_AL                              |
| 31   | 0.00                                       | 10.10  | -0.07 | 0.663            | 0.011          | 0.662           | GA_AL / Lit.                       |
| 32   | 0.00                                       | 4.13   | -3.78 | 0.694            | 1.111          | 0.992           | Lit.                               |
| 33*  | 0.00                                       | 1.46   | 0.06  | 0.240            | 0.008          | 0.241           | GA_AL / GA_DFT / Lit.              |
| 34   | 0.00                                       | 9.73   | 0.03  | 0.600            | 0.010          | 0.603           | GA_AL / Lit.                       |
| 35   | 0.00                                       | 0.08   | 40.99 | 0.008            | 0.953          | 0.953           | GA_AL / GA_DFT                     |
| 36   | 0.00                                       | 0.02   | 51.81 | 0.008            | 1.090          | 1.089           | GA_AL / GA_DFT                     |
| 37   | 0.00                                       | 5.26   | 40.87 | 0.611            | 1.525          | 1.337           | GA_AL                              |
| 38   | 0.00                                       | 8.10   | 48.28 | 0.661            | 1.445          | 1.355           | GA_AL                              |
| 39   | 0.00                                       | 2.28   | 44.85 | 0.879            | 1.296          | 1.229           | GA_AL                              |
| 40   | 0.00                                       | 2.29   | 32.45 | 1.022            | 1.011          | 1.213           | GA_AL                              |
| 41   | 0.00                                       | 7.65   | 28.09 | 0.941            | 1.159          | 0.993           | GA_AL                              |
| 42   | 0.00                                       | 10.55  | 31.44 | 0.609            | 0.986          | 1.086           | GA_AL                              |
| 43   | 0.00                                       | 24.24  | 35.38 | 1.138            | 1.100          | 1.047           | GA_AL                              |
| 44   | 0.00                                       | 14.92  | 22.25 | 0.901            | 1.257          | 1.293           | GA_AL                              |
| 45   | 0.00                                       | -1.37  | -1.38 | 1.600            | 1.602          | 0.027           | GA_DFT / Lit.                      |
| 46*  | 0.00                                       | 0.02   | 4.65  | 0.017            | 0.269          | 0.272           | GA_AL / GA_DFT                     |
| 47   | 0.00                                       | 2.50   | 5.80  | 1.423            | 1.465          | 0.350           | GA_AL                              |
| 48   | 0.00                                       | 9.13   | 15.21 | 1.025            | 1.052          | 0.453           | GA_AL                              |
| 49   | 0.00                                       | 0.01   | 15.21 | 0.026            | 0.603          | 0.602           | GA_AL / GA_DFT                     |
| 50   | 0.00                                       | -0.03  | 23.21 | 0.010            | 0.815          | 0.816           | GA_AL / GA_DFT                     |
| 51   | 0.00                                       | 11.23  | 28.12 | 0.909            | 1.117          | 1.100           | GA_AL                              |
| 52   | 0.00                                       | 13.91  | 33.77 | 1.118            | 1.462          | 1.238           | GA_AL                              |
| 53   | 0.00                                       | 2.95   | 27.97 | 1.021            | 1.254          | 0.991           | GA_AL                              |
| 54   | 0.00                                       | -4.00  | 28.11 | 1.009            | 1.117          | 1.383           | GA_DFT                             |
| 55   | 0.00                                       | 13.43  | -3.71 | 0.853            | 1.084          | 1.307           | Lit.                               |

## 7 Analysis of lowest-energy clusters with 21 to 55 atoms

### 7.1 Visualization of lowest-energy clusters

Visualizations of the lowest-energy aluminum clusters with 21–55 atoms were generated using VESTA<sup>29</sup> and assembled into a panel in Figure S15. Symmetries were determined using Materials Studio<sup>30</sup> with a spatial tolerance of 0.1 Å. The lowest-energy cluster with 21 atoms has a dumbbell shape, which packs a 13-atom icosahedron at one end and a 14-atom variant resembling an icosahedron on the other end. From size 22 to 28, the lowest-energy clusters gradually transform from irregular shapes to layered close-packed structures, with 27 and 28 having ordered layers in triangles stacked in the FCC stacking order. Lowest-energy configurations then start to alter the stacking order, with the cluster of 32 atoms forming a structure with closed-packed triangular layers stacking in HCP order. Starting from size 33, the lowest-energy isomers begin to favor tetrahedral shapes, and the 36-atom cluster has the most ordered shape and highest order of symmetry ( $D_{2d}$ ) among all lowest-energy clusters. After 36, the preference of lowest-energy morphology changes back to layered close-packed structure. Lowest-energy clusters with 37, 38 and 39 atoms form transitional shapes between tetrahedron and layered configuration. Lowest-energy clusters with 40 to 43 atoms regain the FCC stacking order with close-packed layers, some of which have trapezoidal shapes instead of triangular. The 44-atom lowest-energy isomer loses the layered morphology but mostly retracts close-packed outer surfaces. Lowest-energy clusters of 45 and 46 have spherical shapes, while the cluster with 47 atoms have a layered structure with HCP stacking order. From size 49 to 53, lowest-energy clusters are neither layered nor tetrahedral, but still favor close-packed outer surfaces. Finally, the 54-atom and 55-atom lowest-energy structure transform back to tetrahedral configurations with close-packed surfaces. The preferred morphologies adopted by lowest-energy aluminum clusters periodically transform between layered close-packed structures and tetrahedrons with close-packed outer surfaces. This observation reflects the facts that Al in its bulk phase has an FCC crystal structure, and (111) surfaces possess the lowest free surface energies among low-index surfaces.<sup>31</sup> The prevalence of preferred morphologies for lowest-energy isomers across a wide range of sizes also suggests that the seeding operation, which generates new clusters from seed structures of other sizes, can potentially accelerate ground state search by a large extent, when low-energy structures of neighboring sizes are available.



**Figure S15. Lowest-energy clusters assembled from GA\_AL, GA\_DFT and the literature.** Labels of the lowest-lowest energy clusters discovered by GA\_AL and GA\_DFT are colored in black and blue, while the ones reported in the literature<sup>19,27</sup> are labelled in red. Proposed symmetries are included below corresponding clusters. Informative alternative views are put at the top right corner.

## 7.2 Stability Analysis

To measure relative stabilities, cohesive energies, binding energies, and the second energy differences were calculated and plotted in Figure S16. The cohesive energy is defined as the energy required to break the cluster into isolated neutral atoms at 0K and is computed by

$$E_c(n) = [n \times E_1 - E_n] / n, \quad (5)$$

where  $E_c(n)$ ,  $E_1$ , and  $E_n$  represent the cohesive energy per atom for an  $n$ -atom cluster, the total energy of an isolated aluminum atom, and the total energy of a cluster with  $n$  atoms respectively. The binding energy measures the energy necessary to break one atom from the cluster and is calculated as

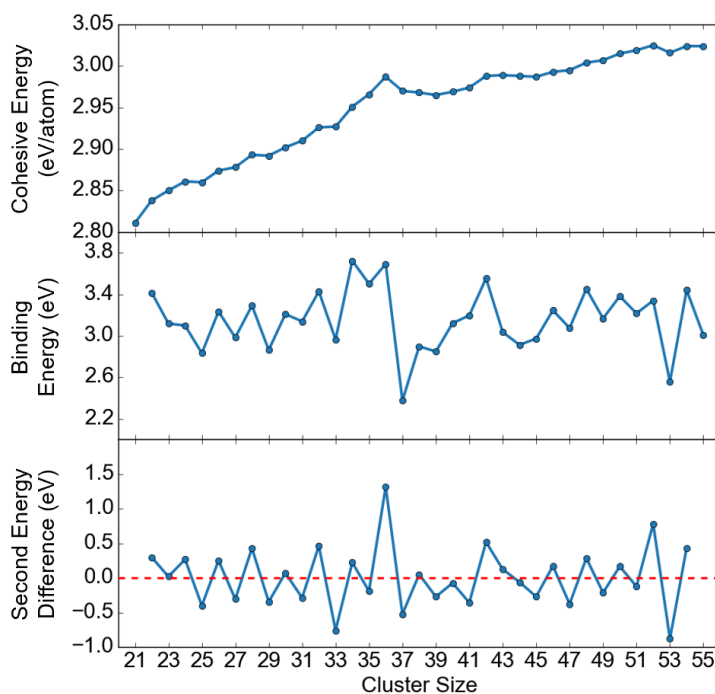
$$E_b(n) = E_{n-1} + E_1 - E_n, \quad (6)$$

where  $E_b(n)$  represents the binding energy and the other symbols carry the same meaning as those in equation (5). The second energy difference describes the stability of the lowest-energy cluster of one size relative to the adjacent sizes and is calculated as

$$\Delta_2 E(n) = E_{n+1} + E_{n-1} - 2E_n, \quad (7)$$

of which a positive value implies higher stability.

Cohesive energies steadily increase as clusters grow, and values of all sizes are still well below the bulk value of 3.39 eV/atom,<sup>32</sup> suggesting clusters in this size are significantly less stable than the bulk crystal. The binding energies and the second energy differences display an odd-even oscillation, which can be attributed to the existence of one unpaired electron in odd-sized clusters. The lowest-energy cluster with 36 atoms exhibits peaks in both the second energy differences and cohesive energies, meaning it is more stable than lowest-energy clusters of neighboring sizes. This suggests it could exist in relatively high abundance in gas-phase experiments and agrees with the experimental observation of surprisingly high melting temperatures of Al cation clusters with around 36 atoms.<sup>33,34</sup>

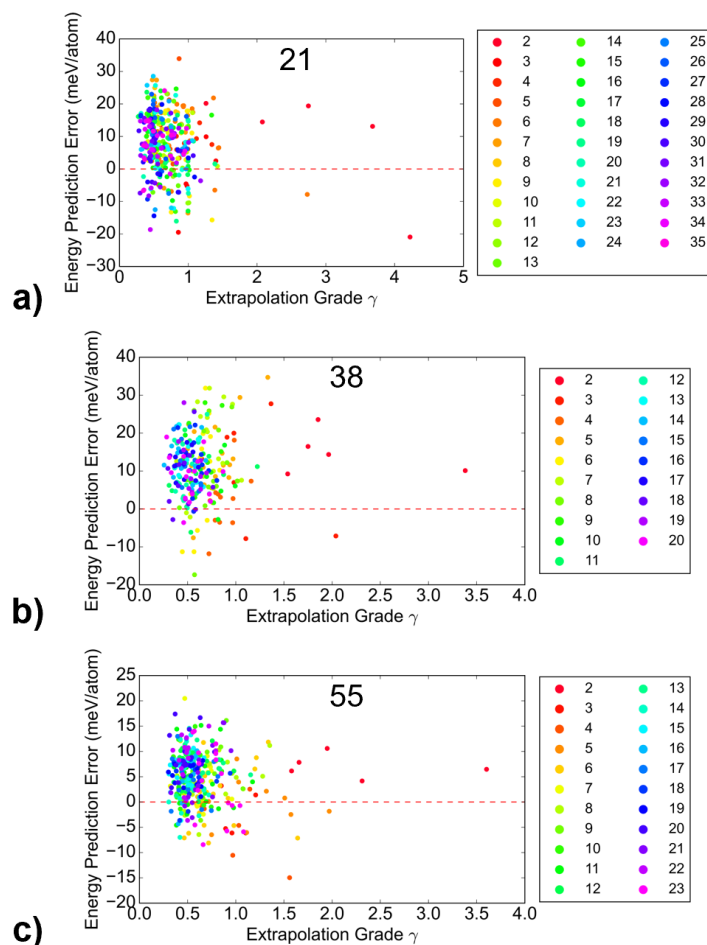


**Figure S16. Cohesive energy, binding energy and second energy difference for clusters with 21 to 55 atoms.**



## 8 Relationship between extrapolation grade and energy prediction error

Although high extrapolation grades generally indicate a large prediction error, lower extrapolation grades below the extrapolation threshold (1.0) do not necessarily guarantee smaller prediction errors. Figure S17 shows the extrapolation grades of DFT-relaxed clusters at each retraining step for GA\_AL runs on clusters with 21, 38 and 55 atoms. We focus on the local ground states of DFT relaxations because we are primarily interested in low-energy structures for the purpose of the structure search. Extrapolation grades decrease as more retraining steps were performed. However, the prediction errors of those having extrapolation grades smaller than 1.0 scatter randomly with a tendency of over-estimating the total energy (i.e., MTP predicted an energy higher than corresponding DFT energy).



**Figure S17.** Energy prediction errors against extrapolation grades for ground states of DFT calculations at retraining steps of GA\_AL for clusters with a) 21, b) 38 and c) 55 atoms. The legends represent the retraining step of a GA\_AL run.

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# Supplementary File: Atomic Coordinates for Clusters

for

## “Accelerated Prediction of Atomically Precise Cluster Structures Using On-the-fly Machine Learning”

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## 1 Format description

The following sections list the atomic coordinates of the lowest-energy structures discovered in this work and reported in literature for aluminum nanoclusters with 21 to 55 atoms. For clusters with 22 atoms, a cluster which has total energies close to the lowest-energy structure (difference below 0.1 meV/atom) but is structurally dissimilar is also listed.

Clusters are provided in the XYZ format with the center of mass positioned at the origin. The first line provides the number of atoms of the cluster, with a trailing blank comment line. The following lines list the element name and atomic coordinates in the Cartesian coordinate system.

## 2 Atomic coordinates for clusters

### 2.1 Al<sub>21</sub>

21

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.596503690031571 | 0.159271003097976  | -3.637482070185255 |
| Al | 2.286585294345931  | 0.830951886913486  | -2.958567160431452 |
| Al | 1.033664713140647  | -1.711617999236393 | -2.828193465536144 |
| Al | -0.042429815500894 | 2.514381907574577  | -2.676371340719141 |
| Al | -1.994125385900171 | -0.685373895053791 | -1.472730847744709 |
| Al | 0.612813037104070  | 0.428119171309620  | -1.022610715335517 |
| Al | 1.833938631530993  | 3.051752633436998  | -0.952245932007926 |
| Al | 3.291204481516125  | 0.981503395903907  | -0.306840864540883 |
| Al | 2.789670119756137  | -1.552478690758089 | -0.867246860234246 |
| Al | -1.664565852575888 | 2.027852564097188  | -0.560851053283410 |
| Al | 0.009875492165804  | -2.207470007291898 | -0.318426825750153 |
| Al | -3.604814527091738 | 0.276220671025232  | 0.598594008319584  |
| Al | -2.522434942937107 | -2.342980190580137 | 0.716628552719046  |
| Al | -0.985441439688779 | -0.079146205215778 | 0.984734309665225  |
| Al | 0.542902588090966  | 2.164276391641030  | 1.265649863144239  |
| Al | 1.644517023945994  | -0.349686734470989 | 1.467023095161313  |
| Al | 2.126418896541706  | -3.156685964118831 | 1.350095917319861  |
| Al | -2.058244515402470 | 2.002878305992782  | 2.268744418261399  |
| Al | -0.197081901030137 | -2.178984135548673 | 2.442399216653241  |
| Al | -2.523049637430812 | -0.694121355932231 | 3.032404506840242  |
| Al | 0.017101429451174  | 0.521337247214026  | 3.475293247684695  |

## 2.2 AI\_22

22

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | 0.849728635095090  | 0.305009796490726  | -3.641055973590975 |
| AI | -0.454824354095036 | 2.443823170741856  | -2.806899464901289 |
| AI | 0.200621843188900  | -2.141129322536001 | -2.552127948594587 |
| AI | 2.782544148441950  | -0.774659877732478 | -2.197050280925546 |
| AI | -2.394278740440318 | -2.130111060577140 | -1.676982107194558 |
| AI | -0.854519914911067 | 0.085643404371087  | -1.434072955075608 |
| AI | -2.644206095113693 | 2.085007797081076  | -1.389093002230260 |
| AI | 1.625569506058016  | 1.598954010301494  | -1.345107336727165 |
| AI | -3.752206017392702 | -0.164218898227926 | -0.568191139334245 |
| AI | -0.689465884416474 | -3.752523983064122 | -0.344462829080266 |
| AI | 1.259316911884746  | -1.841946607729521 | -0.183061918330887 |
| AI | -0.411116837144375 | 2.845396321557978  | -0.042262882594020 |
| AI | 3.220950536848502  | 0.151081614879093  | 0.366981695883849  |
| AI | -1.627935327892790 | -1.409536831307928 | 0.753515558904583  |
| AI | 0.461794221361892  | 0.421670607211276  | 0.952522431393229  |
| AI | 2.077040383287498  | 2.576923997608393  | 1.110497092794137  |
| AI | -2.366524639077721 | 1.077835212236092  | 1.359454454187127  |
| AI | 0.093044330099149  | -2.968377169100195 | 2.070980544702035  |
| AI | -0.467048104567413 | 2.399369999028631  | 2.677518521706466  |
| AI | 2.144295536701726  | -1.318854489262010 | 2.370380580588957  |
| AI | -0.747427286109438 | -0.477667969318798 | 3.149598326185417  |
| AI | 1.694647148193528  | 0.988310277348424  | 3.368918632233607  |

## 2.3 AI\_22 alternative

22

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | -0.273801078607146 | -1.340367421326714 | -3.539314417717854 |
| AI | 0.631529234117727  | 1.279665740298393  | -3.239023148413977 |
| AI | -2.444039637864109 | -1.927089695781956 | -2.177553151957000 |
| AI | 1.828964825895078  | -0.828965378053382 | -2.064753233220965 |
| AI | -1.297437102997620 | 0.356793147320385  | -1.596541602866242 |
| AI | 2.543496086157049  | 1.773103591436458  | -1.359611348464474 |
| AI | -0.143376031862703 | 2.799614407693678  | -1.124717451371499 |
| AI | -3.680389476081229 | -0.199129038131561 | -0.414782086796510 |
| AI | -0.180165326010709 | -2.204536632151846 | -0.636085018634361 |
| AI | 2.487504027315685  | -2.792688091329786 | -0.529716013261412 |
| AI | -2.333658271352760 | 1.995225721572432  | 0.329205761692021  |
| AI | 0.691084087782505  | 0.415975152092672  | 0.177439435331653  |
| AI | 3.365751811688295  | -0.344826198924196 | 0.239165595951697  |
| AI | -2.690379192424030 | -2.644302807473455 | 0.389749848834397  |
| AI | -0.784357279261744 | 4.114047510257121  | 1.071995641485788  |
| AI | 1.602456637337488  | 2.883314037886288  | 0.954819139639666  |
| AI | 2.154889262873093  | 0.695381808328641  | 2.507738222190516  |
| AI | 1.454132773891089  | -1.857863952216194 | 1.639150680508083  |
| AI | -1.572787833253606 | -0.449174592458125 | 1.286605771852454  |
| AI | -0.847037290909182 | -2.953433510123781 | 2.237982687877929  |
| AI | -0.476532277352966 | 1.827788708617755  | 2.351101666908820  |
| AI | -0.035847949080184 | -0.598532507532829 | 3.497143020431277  |



## 2.4 AI\_23

23

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | -1.056193001906759 | 1.777458535036345  | -4.036878242070214 |
| AI | -1.662845417163185 | -0.872606028202219 | -3.488247009877295 |
| AI | 0.653104718818746  | 0.286429079172084  | -2.656690502518004 |
| AI | 0.089152507202098  | -2.255358054416752 | -1.927324851136791 |
| AI | 0.156320098678565  | 2.904917665603040  | -1.876940119742804 |
| AI | -1.932600585443816 | 1.091473573747804  | -1.626174040440808 |
| AI | 2.464785313255726  | 1.716796485701495  | -1.298765167302959 |
| AI | 2.318705617130146  | -0.951961493967749 | -1.129356311861211 |
| AI | -2.395356756729070 | -1.481344920235703 | -0.936044132261309 |
| AI | -0.589089941602092 | -2.806354368148527 | 0.714849590896245  |
| AI | 1.856699371701371  | -3.483994907661041 | -0.259740100527672 |
| AI | 0.191299236238606  | 0.430005959652693  | 0.145207098402887  |
| AI | -1.433579596687037 | 2.930858856737441  | 0.255628372498501  |
| AI | 3.275887497917266  | 0.592648304426140  | 0.971365198075032  |
| AI | 1.279634692163729  | 2.904809352806383  | 0.939964029986895  |
| AI | -2.562934555885653 | 0.568457707329271  | 0.817290499861969  |
| AI | 1.615349035816470  | -1.534377231181534 | 1.625564926792130  |
| AI | -2.992212418616184 | -1.973084496731546 | 1.674942847302498  |
| AI | 4.027255095758347  | -1.997848032972129 | 0.613465689831806  |
| AI | -0.684719772516469 | -0.817934721112872 | 2.544071910146068  |
| AI | -2.985833973130648 | 0.187957288239364  | 3.410903288896693  |
| AI | 1.395881851760645  | 0.849361539831863  | 2.835401906179976  |
| AI | -1.028709016760793 | 1.933689906346157  | 2.687505118868362  |

## 2.5 Al\_24

24

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.433675090612311 | -3.258981745336208 | -3.378502173492810 |
| Al | 0.975993444053721  | -1.141796800024916 | -2.714739942358611 |
| Al | -0.406944439062864 | 1.112899725983780  | -2.548662634934395 |
| Al | -1.795507514705186 | -1.229779723794793 | -2.355963455822907 |
| Al | 1.990309770384018  | -3.674831856940134 | -2.175416678832782 |
| Al | 1.154584705385499  | 3.257141858896498  | -1.762800392526183 |
| Al | 2.270405035823764  | 0.766150852993876  | -1.672058591354554 |
| Al | -2.746882840456522 | 0.781275028755681  | -1.059218370740322 |
| Al | -1.261365489727298 | 3.011745934298293  | -0.717479950215399 |
| Al | -0.298114585330627 | -2.991831670807974 | -0.768205229815478 |
| Al | 3.381195691765104  | 2.953934334762456  | -0.400716225806839 |
| Al | 2.085592426481522  | -1.670240935735511 | -0.210946215893742 |
| Al | -0.083385372808253 | 0.028157967689429  | -0.059502118294979 |
| Al | -2.254062459338492 | -1.600119730350939 | 0.305774694150244  |
| Al | -3.659666402918445 | 2.940422847274382  | 0.541119587657548  |
| Al | 0.968435432854637  | 2.632472455922597  | 0.860314543922312  |
| Al | 2.899900445572504  | 0.717613116960974  | 0.964069819991568  |
| Al | -3.662661194791258 | 0.445080110257445  | 1.469965630185792  |
| Al | -1.382412018774617 | 1.909540911259665  | 1.747158357600874  |
| Al | 0.113949285254231  | -2.363662275207487 | 1.655713522091370  |
| Al | -1.577724554743486 | -0.582082277879082 | 2.789060302438751  |
| Al | 0.781898802339503  | 0.773829264452365  | 2.691423146855286  |
| Al | 0.460320630913570  | -1.474161478643536 | 4.240447385427561  |
| Al | 2.479816292441303  | -1.342775914786874 | 2.559164989767694  |

## 2.6 Al\_25

25

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 1.628563383619499  | -2.624387910024781 | -2.111071705976232 |
| Al | -0.821696822560490 | -1.446565006798433 | -2.336395021395763 |
| Al | 1.492074868923996  | 0.074386302774872  | -2.453532584112666 |
| Al | 3.826591781869309  | -1.227357935876961 | -2.637803243939576 |
| Al | -0.832161932449985 | 1.092825957232115  | -2.748175570065385 |
| Al | -2.690695100695137 | 0.105368653977994  | -0.921306739788555 |
| Al | -3.043567168900930 | 2.497247908695186  | -2.203179095981397 |
| Al | 3.840650452350468  | 1.032439502737924  | -1.237340847512977 |
| Al | 1.219726208090211  | 2.366301839941899  | -0.949605723206790 |
| Al | -0.472394767765564 | -4.116944395630505 | -1.606064581936121 |
| Al | -0.619433214951366 | 3.720962222388939  | -2.460165055431859 |
| Al | -1.421225692215415 | 2.715891969257697  | -0.116056125714143 |
| Al | -2.028719624739835 | -2.523330114241594 | -0.207692972251067 |
| Al | 2.705179925893926  | -1.111959412037868 | -0.087554927040285 |
| Al | -0.012458736166073 | -0.016658414538014 | -0.014930779166127 |
| Al | 0.669178464717639  | -2.732531845864647 | 0.504292626839518  |
| Al | 2.792092707439046  | 1.317997849142156  | 1.140935307668466  |
| Al | -3.643879173553937 | -1.364357759733855 | 1.489394563634551  |
| Al | 0.338576296641282  | 2.229903243699887  | 1.697248483423444  |
| Al | -2.124608813550182 | 0.989880163624363  | 1.630294576112416  |
| Al | 1.482257833396979  | -0.894685770593627 | 2.239271296685054  |
| Al | -1.066137956879192 | -1.659377157889812 | 2.186988800428161  |
| Al | 2.089700542531453  | 1.374332744449115  | 3.674168122345407  |
| Al | -0.405227625559105 | 0.546057464111200  | 3.700958047238545  |
| Al | -2.902385835486625 | -0.345440098803257 | 3.827323149143382  |

## 2.7 Al\_26

26

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 3.782683677001378  | -0.418392815828973 | -2.858410831689532 |
| Al | -0.486763736986296 | -1.521568889561746 | -2.132062776534922 |
| Al | -3.061260669639445 | -2.233174507576222 | -2.778958895199447 |
| Al | 1.278781780032805  | 0.386113413412104  | -2.724295337231490 |
| Al | -3.242467759016775 | 0.379278399713737  | -3.266240833033504 |
| Al | -0.942711248110358 | 1.422049810851082  | -2.086586893481969 |
| Al | 3.319608308251988  | 2.247295420261866  | -2.552728267623671 |
| Al | 2.130190478968743  | -1.950748659655162 | -1.403189018998420 |
| Al | -3.389396423456468 | 2.198437020405265  | -1.274109547490195 |
| Al | 3.126251146198289  | 0.463867834189687  | -0.470417668738532 |
| Al | -2.685925525552402 | -0.422228871048732 | -0.760175795720453 |
| Al | 1.223725954615784  | 2.458445439292221  | -0.710893364332282 |
| Al | -2.213486995810824 | -3.159129960584931 | -0.466013162577058 |
| Al | 0.193643523973424  | -0.039498752344796 | 0.064307418526331  |
| Al | -1.320413924249091 | 3.496414485451142  | -0.310319825099564 |
| Al | 0.361861763515163  | -2.884885236303719 | 0.253447002852095  |
| Al | 2.321575244991342  | -1.324110547363793 | 1.289829694318291  |
| Al | -2.139019341650411 | 1.338761436057421  | 1.123230054181722  |
| Al | 2.048569723587674  | 1.382816798016947  | 1.698110458513286  |
| Al | -1.868982742292058 | -1.244912965060674 | 1.580288612473559  |
| Al | -1.529732042616033 | -3.930359961363529 | 1.990232815865880  |
| Al | -0.077177051550805 | 3.097212082846887  | 1.921012754288045  |
| Al | 0.321795830027478  | -2.207514015756094 | 2.896444179387792  |
| Al | -0.402108715198768 | 0.492871207204681  | 2.911142031819303  |
| Al | 1.968936137725521  | -0.310944794414389 | 3.977115038993714  |
| Al | 1.281822607240141  | 2.283906629159727  | 4.089242156530998  |

## 2.8 AI\_27

27

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | -0.729972903416934 | 0.051191731730603  | -3.652518869609882 |
| AI | -1.833601492497438 | -3.963868559194810 | -1.552264963279044 |
| AI | 0.787429330195128  | 2.140411459437814  | -3.924882817576236 |
| AI | -0.030504062560370 | -2.028324198600061 | -2.002115867469147 |
| AI | -2.476518902489318 | -1.937387226980463 | -3.322412346090994 |
| AI | 1.681421598559602  | -0.012004254169101 | -2.353872188603862 |
| AI | -0.755645024854486 | 2.153156256313910  | -1.768960625648887 |
| AI | 2.027147246239759  | 2.624991267270104  | -1.695692719389744 |
| AI | -2.263607004118568 | -0.071087674714011 | -1.378576862882770 |
| AI | -3.886850889775681 | -2.197415245355524 | -0.991950423476760 |
| AI | 0.482761122505865  | -4.040072456756921 | -0.157754474510627 |
| AI | 0.099545610586196  | -0.141363561940334 | 0.045166100906236  |
| AI | 2.252224045382750  | -2.008229998623601 | -0.532430764401912 |
| AI | 2.786889915606045  | 0.600497183894170  | -0.004183387493443 |
| AI | -1.573981316842049 | -2.313423569485248 | 0.549085497730886  |
| AI | -2.117787957227561 | 1.903061143883610  | 0.479457118969760  |
| AI | 0.521640748601779  | 2.529963038826029  | 0.597133853963797  |
| AI | 3.116606806898345  | 3.284654801795167  | 0.647416622644364  |
| AI | -3.554612629425213 | -0.468323062521366 | 1.011500568715098  |
| AI | 0.785393625130418  | -2.278326830322540 | 1.892975137038432  |
| AI | -1.279138156041713 | -0.556623444035065 | 2.535226852789702  |
| AI | 3.347333410204250  | -1.365698376039774 | 1.874424837265808  |
| AI | 1.241728062335412  | 0.317301387771710  | 2.546253771123414  |
| AI | 3.753614172717208  | 1.253111236453163  | 2.413741856753340  |
| AI | -3.288426491786304 | 1.391662622628694  | 2.802959446646666  |
| AI | -0.798148947708869 | 2.106080821911476  | 2.966368038424658  |
| AI | 1.705060083781749  | 3.026065506822356  | 2.975906607461102  |

## 2.9 Al\_28

28

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 1.968660359780511  | 0.395161660166940  | -2.396916656190634 |
| Al | -2.954572153901250 | -1.495581647192072 | -3.221871477312488 |
| Al | -0.433525647363364 | -0.531407799583340 | -2.844129639158764 |
| Al | -2.713899064996837 | 0.992726404146676  | -2.437915021807872 |
| Al | -0.181782086499354 | 1.981203974506052  | -1.899460570026360 |
| Al | 1.265451687816853  | -2.019817778686984 | -1.372503130719915 |
| Al | -1.266876843967653 | -3.008971608051977 | -1.908254708331541 |
| Al | 4.295850936139832  | 1.337906003198112  | -1.612791141799505 |
| Al | -2.436230265589599 | 3.521076524194370  | -1.611430743411053 |
| Al | 2.239105088180612  | 2.961600074288233  | -1.085470351873552 |
| Al | 0.061769325135892  | 4.494514562508583  | -0.914310833041249 |
| Al | -2.655857772501116 | -1.033494023786936 | -0.553407953205239 |
| Al | 0.471388104615940  | -4.515931117105588 | -0.549865843842074 |
| Al | 3.684809854346396  | -1.039907804075224 | -0.554962939708741 |
| Al | -0.010726291876393 | -0.008674902489338 | -0.034959245393072 |
| Al | -2.276345578509611 | 1.512157671502719  | 0.247466692055621  |
| Al | 2.553002524879203  | 1.019753125895440  | 0.747248003852553  |
| Al | 0.393434299945612  | 2.564562711339734  | 0.961506579026395  |
| Al | -0.828773783030927 | -2.497617579770858 | 0.778149121464663  |
| Al | -1.859556067763900 | 4.061099479752142  | 1.049109123844209  |
| Al | 2.951105221028033  | -3.507111460348034 | 0.143742143410890  |
| Al | 1.836660614205726  | -1.437173530523022 | 1.491368833355313  |
| Al | -2.268855888248574 | -0.553474424936402 | 1.991534085598262  |
| Al | 1.027728302510035  | -3.945126503166433 | 2.108175259147171  |
| Al | 0.599299210747565  | 0.577177654361337  | 2.730167798101720  |
| Al | -1.612164969173708 | 2.109390586091989  | 2.851397541328126  |
| Al | -0.158291194239187 | -1.922231758967897 | 3.384485443322319  |
| Al | -1.690807921670738 | -0.011808493268211 | 4.513899631314819  |

## 2.10 Al\_29

29

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -2.128378260979966 | -0.440914962218839 | -3.786902108828111 |
| Al | 0.011462219120478  | 1.306890759017893  | -3.457402527940071 |
| Al | 2.195255852912073  | 2.825977567879489  | -3.004038395670577 |
| Al | -1.647739474319252 | -3.147029679954036 | -2.889292076345215 |
| Al | -4.422768355422162 | 0.301244684831222  | -2.408559885355727 |
| Al | 0.148466000302117  | -1.257135309050163 | -2.460794410761399 |
| Al | 0.840550503591222  | -4.005305806100899 | -2.096471344518847 |
| Al | 2.185636303886357  | 0.420825432261076  | -1.935132951653242 |
| Al | -2.083860347824498 | 1.405591862735605  | -1.698408207483228 |
| Al | -2.715151934304025 | -1.365676190247702 | -1.199219589380829 |
| Al | 0.162921585375157  | 2.652613694759740  | -1.109298450632182 |
| Al | 2.191281728597335  | -1.927489628931908 | -0.806307577389628 |
| Al | 2.856726453102976  | 2.710983552238535  | -0.402383279143240 |
| Al | -0.791219733002364 | -2.840138338693492 | -0.250628328853510 |
| Al | -0.132762927851731 | 0.001772033336593  | -0.055546454529715 |
| Al | -3.854185832073336 | 0.597185873293027  | 0.254000936655494  |
| Al | 1.473044474694188  | -4.115567230094621 | 0.522557710794771  |
| Al | 2.593294872389547  | 0.166704114507509  | 0.728433562437376  |
| Al | -1.684051544105290 | 2.198013923652795  | 0.822593438906102  |
| Al | 0.856778192414717  | 2.333137173666717  | 1.428302806103323  |
| Al | -2.117759275879060 | -1.320910653889927 | 1.473678771386941  |
| Al | 3.238134869448590  | -2.454519124645142 | 1.807538029786185  |
| Al | 0.559601763084926  | -1.905146123693266 | 1.850073110829110  |
| Al | 3.441634108578743  | 2.251463545971056  | 2.220090717695797  |
| Al | -3.125735659541575 | 0.892392378387145  | 2.835257451565498  |
| Al | -0.439320481032574 | 0.269792670819207  | 2.827002079519024  |
| Al | 2.284500748329343  | -0.344544834953746 | 3.318784503934596  |
| Al | 1.313371797066505  | 2.034531554864287  | 4.019636655539315  |
| Al | -1.209727646558421 | 2.755257060251825  | 3.452435813331979  |

## 2.11 Al\_30

30

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 0.881568511723474  | 0.070655135789744  | -3.878262665466415 |
| Al | -0.664233206619651 | 2.582378216448971  | -3.089607168828661 |
| Al | -1.726085868102536 | 0.456632912649159  | -4.224197547773762 |
| Al | 2.063630029699462  | 2.339981503311222  | -2.865073706024676 |
| Al | -3.449238064584084 | -0.955008445458171 | -2.770984526881142 |
| Al | 0.715014681373487  | -2.241953991938430 | -2.514019543321493 |
| Al | -1.954630327002677 | -2.922965910002024 | -1.844797005641716 |
| Al | -3.006333038042778 | 1.432297120808242  | -1.808450213615561 |
| Al | -0.860679098370364 | -0.205586380834726 | -1.600726806255257 |
| Al | 1.882236165291692  | -0.002109583912432 | -1.423159295472177 |
| Al | 0.059573934647661  | -4.477739716359872 | -1.065300335939657 |
| Al | -1.975214858761119 | 3.728708876114695  | -0.932527496238777 |
| Al | 0.348544692002207  | 2.298444715689380  | -0.604685658902058 |
| Al | 3.088351345345545  | 2.206710157208528  | -0.425509402143000 |
| Al | 1.795026720366222  | -2.326061081623907 | -0.056677888457273 |
| Al | -3.116218433053360 | -0.909465578988204 | -0.116092545201562 |
| Al | -0.910471254147824 | -2.438195660522726 | 0.570204382018700  |
| Al | -1.923030545791171 | 1.391772928860657  | 0.612507081286854  |
| Al | 0.360598506954162  | -0.008113381291775 | 0.956715791458738  |
| Al | 3.150513553010123  | -0.206266353997650 | 1.028421235931710  |
| Al | 0.729143610783305  | -4.313286513564929 | 1.592451821555532  |
| Al | -0.941875077700219 | 3.796197731116282  | 1.492319242764871  |
| Al | 1.524413818407353  | 2.908134957459119  | 1.796897225639610  |
| Al | 4.032759059921796  | 2.142867448813222  | 2.153662912362179  |
| Al | -2.034609296088661 | -0.806146029958185 | 2.378184310263860  |
| Al | 1.585495581309026  | -1.902456971998199 | 2.535947828651981  |
| Al | -0.720958047171461 | 1.487157283788720  | 3.038449318041092  |
| Al | 1.990500101290333  | 0.671749155328778  | 3.304389070030894  |
| Al | -0.865565554634687 | -3.070524807697399 | 3.338867307415706  |
| Al | -0.058227642055279 | -0.727807735238073 | 4.421054278741432  |



## 2.12 Al\_31

31

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 0.708787611284013  | -1.230551523064696 | -2.469245962379290 |
| Al | 2.575595354226735  | 0.433053351486643  | -3.334671682065337 |
| Al | 0.254238203805901  | 1.439720984785383  | -2.409884436070056 |
| Al | -1.329142538480056 | -2.937068578361913 | -1.946102388845816 |
| Al | 2.533099984600616  | 2.805164816255616  | -1.882789494483561 |
| Al | -1.689204600512022 | -0.234828698819417 | -1.697259789173422 |
| Al | 4.672519509228687  | 0.760307917828008  | -1.655134385539768 |
| Al | -2.081434162899650 | 2.455744799755889  | -1.530112106692224 |
| Al | 0.052574940094798  | 4.029845046852921  | -1.496816686415030 |
| Al | -3.902262802818930 | -1.758724493743015 | -1.547480564658239 |
| Al | 3.345504506611737  | -1.667535994834702 | -1.484826363262331 |
| Al | -4.230072236160512 | 0.862878333559438  | -1.228197515993820 |
| Al | 1.276272458473375  | -3.371776568945472 | -0.901927855959258 |
| Al | 1.920344921660744  | 0.403298517792779  | -0.506558450891474 |
| Al | -0.707888100708479 | -4.669648407359550 | 0.260364098284301  |
| Al | -0.529565641339952 | -1.656478078946630 | 0.311318583247488  |
| Al | 3.780541265815447  | 2.248843958757385  | 0.493972932886445  |
| Al | -0.793904520054939 | 1.087751031378575  | 0.531745804831779  |
| Al | -2.895153198157868 | -3.219867206282081 | 0.502320325233757  |
| Al | 1.228006827729304  | 2.955612954780394  | 0.695298790046010  |
| Al | -3.042397432170222 | -0.515007610329318 | 0.771247714159035  |
| Al | -1.354810298784599 | 3.855541346576864  | 0.903815606759490  |
| Al | 4.271365927596479  | -0.371104381487978 | 0.880002858067568  |
| Al | -3.481408401754265 | 2.175589383570328  | 1.139671099200285  |
| Al | 2.093472484437257  | -1.748896068368079 | 1.395746888006864  |
| Al | 2.102374040236345  | 0.916061459010731  | 2.164125897222496  |
| Al | 0.188679231733689  | -3.306915209771034 | 2.415610233139345  |
| Al | -0.024557290004356 | -0.564981837756369 | 2.699573837429719  |
| Al | -2.243383093815300 | -2.073424514091212 | 2.855896696121919  |
| Al | -0.235745890555968 | 2.213264630917990  | 2.928966171992744  |
| Al | -2.462447059318009 | 0.684130638852494  | 3.141330145800362  |

## 2.13 Al\_32

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.091651783915736 | -3.488049905533137 | 2.080477719728476  |
| Al | -0.266510392201306 | -0.399717553765274 | 4.144445545897872  |
| Al | -1.877113473541417 | -1.996393878753715 | -4.574300448644244 |
| Al | -2.178337953248363 | 0.355515233285114  | -2.966586571131538 |
| Al | 2.199902756664265  | 2.762636292246338  | -1.236587248606588 |
| Al | -0.275460593007566 | 4.200840085463836  | 1.659386422288318  |
| Al | 0.208321330028213  | -3.035139311044789 | -3.088786581403158 |
| Al | 2.368057685505292  | -1.897895755303637 | -4.344576076656026 |
| Al | -0.203181961333707 | 4.182728799204483  | -1.221454231001324 |
| Al | -2.470693205831557 | 0.309339515492848  | 2.523109954577887  |
| Al | 2.197006682175518  | -2.185002187249372 | 1.043235584654418  |
| Al | -2.399743404915659 | -2.401369907424343 | 0.755701490436779  |
| Al | 2.064387349840056  | 0.302988955206734  | 2.730605735928131  |
| Al | 0.192606983211656  | -0.234179147713515 | -4.237385809466016 |
| Al | 0.071556008280700  | 2.034921283434564  | -2.846779434069552 |
| Al | 2.101983373314260  | 2.686197555039582  | 1.561964600305874  |
| Al | -2.255384218940092 | -2.175923786107688 | -1.868024094644724 |
| Al | 2.384334912712429  | -2.021863219562281 | -1.585733999502546 |
| Al | -2.449571914859204 | 2.659784234710573  | 1.240984190738317  |
| Al | 0.025559093804755  | -3.294864320295940 | -0.504900177485460 |
| Al | -0.237411962197962 | 1.967966630953981  | 3.043590166178552  |
| Al | 2.385184987531686  | 0.468281476851869  | -2.708053451547300 |
| Al | -2.318589877831055 | 2.610516551334575  | -1.492544044104644 |
| Al | -2.315859353631772 | 0.168134021288532  | -0.208861382680045 |
| Al | 2.202922839283959  | 0.345960065600883  | 0.017886943177253  |
| Al | -0.114796998452910 | 1.754721079937203  | 0.129855089698482  |
| Al | 0.027651739781433  | -0.593060548668300 | -1.470562282558872 |
| Al | -0.165391789055729 | -0.765156905362067 | 1.374498464595041  |
| Al | 1.818471430318890  | -2.177843621086925 | 3.634524193549007  |
| Al | -2.244788236096563 | -2.191673078965222 | 3.343068113093068  |
| Al | -0.297824209136740 | -3.117557355597104 | 4.787904223580187  |
| Al | 1.914364155744252  | 5.165158702382218  | 0.283897395074348  |

## 2.14 Al\_33

33

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 0.650617869267137  | -0.252916929973676 | -4.240450873506258 |
| Al | -0.503980320029875 | -2.667965498359179 | -3.776107139505211 |
| Al | -0.880627735606124 | 1.740014396608689  | -3.262634579216585 |
| Al | 2.682080014665997  | -0.896998546966751 | -2.561528216962873 |
| Al | -2.258448759384247 | -0.857392800215715 | -2.900885039160990 |
| Al | 1.557252641905301  | 1.514619018849791  | -2.109244871634606 |
| Al | 1.432675754907310  | -3.242096638865539 | -1.996751197050518 |
| Al | -2.510780789975382 | 3.429270046828128  | -2.008712955646382 |
| Al | 0.146677220130405  | -0.885311978647454 | -1.543457646359300 |
| Al | -1.436026794305230 | -3.183805202057383 | -1.316166925146238 |
| Al | -3.820688662849577 | 1.055625774756663  | -1.910403277527191 |
| Al | 0.017050231747145  | 3.498107679568834  | -1.127845260657473 |
| Al | 3.637110622334049  | 0.767898159193777  | -0.577798151138072 |
| Al | 2.421844993582495  | 3.221565576426201  | -0.016151787703857 |
| Al | -1.390290622713624 | 1.062922977579509  | -0.615264312424788 |
| Al | 2.415389156231830  | -1.577591600061815 | 0.044007408919585  |
| Al | -3.117774263160652 | -1.288028804803579 | -0.363944368672358 |
| Al | 0.818904876395138  | -3.728875974343579 | 0.547768730221916  |
| Al | 1.114168773871500  | 0.860327339049492  | 0.653673084567346  |
| Al | -2.256182366657323 | 3.299131832384074  | 0.657502611711042  |
| Al | -2.246283400045062 | -3.543979971428013 | 1.166638954902140  |
| Al | -0.546155494469399 | -1.366574541690029 | 1.089602805951936  |
| Al | 4.764966798947498  | 2.374475585045991  | 1.253384746794977  |
| Al | -3.765761457767328 | 1.041067692788474  | 0.795462416855665  |
| Al | 2.513733294677431  | 2.634651347472573  | 2.683735404673561  |
| Al | -3.007717487857729 | -1.225634831506701 | 2.329453496673363  |
| Al | -1.496701637982474 | 1.041725762937613  | 2.216062044091220  |
| Al | 0.128514423229447  | 3.187730631156370  | 1.790093041555529  |
| Al | 3.386466539676645  | 0.091510748802319  | 2.111453269650431  |
| Al | 1.756588296049950  | -1.939961839596496 | 2.640383742616658  |
| Al | -0.261899401608501 | -3.656554593563964 | 2.971787411921857  |
| Al | 0.892396694392090  | 0.717539045303500  | 3.438178448557107  |
| Al | -0.837119007598830 | -1.224493862672119 | 3.938158982648362  |

## 2.15 Al\_34

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.208909008009144 | 0.618185771775696  | -3.644036212665020 |
| Al | 2.398911892151920  | 1.106880562712949  | -3.867445022702866 |
| Al | -0.639977770573052 | -2.116173017653392 | -2.937458109104309 |
| Al | -2.715316169714209 | -0.013559461461334 | -3.138184800786533 |
| Al | 1.916852639748829  | -1.519072833484232 | -3.462418171970583 |
| Al | 3.892208014630352  | 1.262822467799028  | -1.632141094473233 |
| Al | -3.109716903546364 | -2.561274135082980 | -2.179400105529874 |
| Al | 1.069353995105372  | 2.832901726798966  | -2.303283168898118 |
| Al | -1.537300684342097 | 2.229574505464495  | -1.898375627883601 |
| Al | -5.129803670049934 | -0.821183496819854 | -1.943720984488730 |
| Al | 1.067612062059260  | -2.489768156264651 | -0.853080672919907 |
| Al | 3.480909304344374  | -1.359194882670282 | -1.232284377820326 |
| Al | 1.305485484118682  | 0.262256397661233  | -1.307109896607296 |
| Al | -0.181222679044643 | 4.370495307115613  | -0.520772210106852 |
| Al | -3.657909255133466 | 1.075336175450865  | -0.726342485508255 |
| Al | -1.331602703596987 | -0.471034610582974 | -0.823950367652809 |
| Al | 2.612123717268494  | 2.762098922848590  | 0.163016129360226  |
| Al | 2.776269349673115  | -2.411131085749831 | 1.237812916640372  |
| Al | -1.451746659329038 | -3.247381313153071 | -0.279269081891620 |
| Al | 0.046680608657896  | 1.789195849503178  | 0.522205074042452  |
| Al | -2.310027471566589 | 3.137050976041213  | 0.551287244992292  |
| Al | -3.672791665497447 | -1.507927242155453 | 0.312481725912248  |
| Al | 1.073670481719182  | 4.164663443529951  | 1.853629931428063  |
| Al | 3.155283636770033  | 0.231067121371717  | 0.954275225741668  |
| Al | 0.301265739404364  | -3.616268100292602 | 1.649936425149383  |
| Al | 0.467367170827432  | -0.838895505357543 | 1.322401535279754  |
| Al | -2.284888325035650 | 0.639772017821270  | 1.559058449064354  |
| Al | 2.249920963298626  | -3.551478464254196 | 3.619933544885225  |
| Al | -1.962745126711837 | -1.938526287209921 | 2.265014083170207  |
| Al | 2.213065602484894  | -0.748706273450843 | 3.425001320475536  |
| Al | -1.028929343288308 | 2.869000203320775  | 2.945357295084650  |
| Al | 1.772290061072392  | 1.753509234966506  | 2.736795932585570  |
| Al | -0.539967888725929 | 0.325424752255644  | 3.620525514872110  |
| Al | -0.036415399170535 | -2.218660570794492 | 4.010540042325822  |

## 2.16 Al\_35

35

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.538233276045817 | 0.971734771859058  | -3.518290807343323 |
| Al | 1.910592060712272  | 0.029393907380268  | -3.727666691275817 |
| Al | -2.658556368460377 | -0.587062517234557 | -2.490405752474440 |
| Al | -1.101408318013016 | 3.655134346409216  | -2.055483021882206 |
| Al | -0.162195938834316 | -1.649071433347116 | -2.834367260867475 |
| Al | -2.956830651331162 | 2.003255513014961  | -3.059567353531734 |
| Al | 4.519450674584887  | -0.885747919975602 | -3.136785511796102 |
| Al | 1.336305911592508  | 2.627194891784582  | -2.221677303442179 |
| Al | 2.404565877916733  | -2.214499404861276 | -2.106104634194828 |
| Al | -2.301189446946436 | -3.067116863890454 | -1.684150894310275 |
| Al | 3.712688037181611  | 1.501322702791988  | -2.247214157835963 |
| Al | -1.255870268407383 | 1.173332551449560  | -0.812284726546087 |
| Al | 0.380283145666175  | -3.702714030461355 | -1.124115415604317 |
| Al | 1.283535076147189  | 0.138597267109402  | -1.044943039851786 |
| Al | 3.956055867433257  | -0.570079756828472 | -0.429328510030071 |
| Al | -1.548965947116013 | -5.348902899243620 | -0.193377584735321 |
| Al | -3.557647607536847 | -0.972731987624602 | 0.151203534504488  |
| Al | -0.847325917772210 | -1.427289813950232 | -0.010472064008770 |
| Al | 2.893771627734658  | 2.088507222160624  | 0.176913758017506  |
| Al | 0.424806602722976  | 3.180016264545801  | 0.284483131645347  |
| Al | -2.313946653579444 | 3.412872243467142  | 0.404721216931568  |
| Al | 1.800954632518170  | -2.086430191316388 | 0.549007986447611  |
| Al | -4.010000124532089 | 1.514743467952041  | -0.644989469167498 |
| Al | -2.938729584489874 | -3.391173152111334 | 0.978643330486225  |
| Al | 0.288779837371607  | 0.611792507467333  | 1.520822837959621  |
| Al | -2.530466859013276 | 1.004657150487432  | 1.717005925050561  |
| Al | -0.261529731435482 | -3.670824458602731 | 1.615812642969532  |
| Al | 2.967596612040520  | -0.086955280442996 | 1.976042822171440  |
| Al | 1.919507352956119  | 2.570351093690748  | 2.557902707970038  |
| Al | -1.999637079122836 | -1.509779298889576 | 2.550094819561183  |
| Al | -0.883943313010812 | 2.951058983532166  | 2.652861278305817  |
| Al | 0.775993953316663  | -1.697207446485848 | 3.037650556955755  |
| Al | -0.973656391885891 | 0.442201606438058  | 3.960428479211942  |
| Al | 1.784311070419434  | 0.389077499603683  | 4.269144598750845  |
| Al | 0.480935137218552  | 2.602342464122041  | 4.938484571958774  |

## 2.17 Al\_36

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 1.303807163274493  | 1.450344834560378  | -3.646114239940347 |
| Al | 1.459504743658870  | 4.239303428488414  | -3.397987634687738 |
| Al | 0.787080591996421  | -1.099992826376580 | -3.319737024825664 |
| Al | -0.921798964782944 | 3.074796154788009  | -3.356593957120663 |
| Al | -1.482877600585736 | 0.506889240626307  | -3.226703572066469 |
| Al | -2.007207450164802 | -2.050570066362720 | -2.976854258701393 |
| Al | 0.259940506441929  | -3.638929070686460 | -2.871895731806346 |
| Al | -2.055767891868165 | -4.747676705496916 | -2.209352957983109 |
| Al | 2.693935416261315  | 0.014149990720879  | -1.719154908167373 |
| Al | 2.592064431252284  | 2.757627942281479  | -1.470709813702607 |
| Al | 2.092468103154268  | -2.611732950843209 | -1.287144984727581 |
| Al | -2.363663231459701 | 2.380675180513609  | -1.267201644577314 |
| Al | 0.192036771246094  | 1.261321146082452  | -1.122147852868622 |
| Al | -2.989524335612063 | -0.267993299906173 | -1.088842005448093 |
| Al | 0.093520614214453  | 4.019318982752749  | -0.958661667900763 |
| Al | -0.398336866053093 | -1.439501770596832 | -0.812962433572313 |
| Al | -2.790206844526590 | -2.925665643468113 | -0.383633576564549 |
| Al | -0.215989779094635 | -4.125460735361177 | -0.186418327382096 |
| Al | 3.844983810979846  | -1.491279975103964 | 0.310373154747328  |
| Al | 3.806189339182444  | 1.353209823575153  | 0.418436391270795  |
| Al | 1.459234986866681  | -0.203033845383413 | 0.848726301844847  |
| Al | -3.720986076398541 | 1.594972365842391  | 0.834610929265176  |
| Al | 1.451620832661689  | 2.647678872301631  | 1.036219163690395  |
| Al | -1.251023764204939 | 0.378018516867440  | 1.085937960201269  |
| Al | -1.222989097855477 | 3.171013579346775  | 1.144776010525309  |
| Al | 1.493231545684962  | -2.939075148008362 | 1.410247710163736  |
| Al | -3.608346626030078 | -1.184082038053667 | 1.436114511982943  |
| Al | -1.155251190547848 | -2.395502135380662 | 1.771966659111534  |
| Al | 5.127913231801390  | -0.007049745608876 | 2.314414021387838  |
| Al | 3.151889005835837  | -1.641777786500340 | 2.988455428376264  |
| Al | -4.532082597523872 | 0.516073061990141  | 3.292831037004868  |
| Al | 2.633743846685162  | 1.079203360702888  | 2.997771807734338  |
| Al | -2.491085102382153 | 2.207006621430088  | 3.240129910634360  |
| Al | 0.612199179353871  | -1.064443588943105 | 3.367488790253537  |
| Al | 0.082728689640613  | 1.660944612370349  | 3.178932747555665  |
| Al | -1.930955391102045 | -0.478780383160490 | 3.624684056292825  |

## 2.18 Al\_37

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 1.291467587870505  | -1.762430639158910 | -4.251668456253457 |
| Al | 0.065827186230139  | 0.468478512221811  | -3.638620198300393 |
| Al | 3.714931006947596  | -2.485513423250991 | -3.508581880708847 |
| Al | 2.884740321916066  | 0.134650726564411  | -2.967487460769646 |
| Al | -1.201932445006975 | 2.624056153539014  | -2.840933907974335 |
| Al | -0.960401630249494 | -2.221563364366165 | -2.959662921589810 |
| Al | 1.585170166421609  | 2.260436065104095  | -2.094676307487558 |
| Al | -2.170235320498493 | 0.081461546873109  | -2.166390150113759 |
| Al | 1.512522855051238  | -3.371544567406096 | -1.995686401063604 |
| Al | -2.397532252730887 | 4.798996519021300  | -1.708717153086521 |
| Al | 0.170951084839961  | 4.292533804336996  | -1.078480601171355 |
| Al | 0.848040945138179  | -0.767186740272866 | -1.321393949799825 |
| Al | -3.150610803370349 | -2.394899536617409 | -1.489563534466285 |
| Al | 3.562418954767988  | -1.777134039199479 | -0.947942294574494 |
| Al | -3.200661743017180 | 2.316606403452264  | -0.891586219575602 |
| Al | 3.115501242428769  | 0.811771816025487  | -0.300886998422563 |
| Al | -0.492832435119674 | 1.489548671457269  | -0.400562793145959 |
| Al | -0.686674859437518 | -3.622905977451593 | -0.566120397941921 |
| Al | -4.024309986543248 | -0.067069405144546 | -0.118710856718275 |
| Al | 1.760662433661247  | 2.884854908285508  | 0.572243145732182  |
| Al | -1.385849118981140 | -1.052628927225761 | 0.251397337268282  |
| Al | -1.655538635406407 | 3.708860647539209  | 0.841803457034361  |
| Al | -5.117541597955740 | -2.405308281154284 | 0.537338979517944  |
| Al | 1.505313551278560  | -2.669175746271323 | 0.663846301157188  |
| Al | 0.970720548633128  | -0.004686273359514 | 1.374984961986986  |
| Al | -2.850871314434176 | -3.665371574775271 | 0.956024253149044  |
| Al | 3.586236213266021  | -1.171426860038359 | 1.648239268439527  |
| Al | 0.544060183645847  | 4.921165285267364  | 1.858171583996960  |
| Al | -2.416260797283754 | 1.272736038372424  | 1.652920465844170  |
| Al | -0.717644039844323 | -2.863313439621681 | 2.232027014221568  |
| Al | 1.481212738858947  | -1.954102713672244 | 3.351935479993184  |
| Al | -0.993768635720219 | -0.288484281863141 | 3.360157058169435  |
| Al | -0.020012038383467 | 2.269441140365279  | 2.608805260646617  |
| Al | 1.300846334027581  | 0.620839843015583  | 4.282631297975000  |
| Al | 3.075852221696016  | 1.463468203448807  | 2.435815909498006  |
| Al | 3.692764275228754  | -0.601726291236176 | 4.302563573443244  |
| Al | -3.226562197925146 | -1.273434202804102 | 2.316767135090416  |

## 2.19 Al\_38

38

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 1.971522221702559  | 1.261204940437686  | -3.541348054873320 |
| Al | -2.670097641729762 | 0.455430818923874  | -3.775636298489649 |
| Al | -0.694168376173190 | 2.146866741558634  | -3.459895497154189 |
| Al | 1.214726539651430  | 3.840622363377786  | -3.038877427980128 |
| Al | -0.105583422825454 | -0.481245747587389 | -3.244403541431219 |
| Al | 3.849960174420533  | 3.395051879480169  | -2.870521709711163 |
| Al | -2.350979298597703 | -2.094706561081195 | -2.937823109513090 |
| Al | 2.303832787574386  | -1.122736862751662 | -2.233675350521561 |
| Al | -4.608896606206741 | -3.012430449051241 | -1.572915047794443 |
| Al | 0.198733675387947  | -2.688199434353888 | -1.874703371709781 |
| Al | 3.634554294277457  | 1.148545975199896  | -1.420788336017967 |
| Al | -3.919244966188391 | -0.342291489278971 | -1.466956954485677 |
| Al | -2.165773867935512 | 1.712323156040998  | -1.217562401414812 |
| Al | 0.579998428607889  | 1.178583713034907  | -1.025682957256473 |
| Al | -0.422371541674540 | 3.758745302242804  | -0.897734401165781 |
| Al | -2.057554114814183 | -3.750619090370882 | -0.796638803054559 |
| Al | -1.316285724546615 | -0.735462592542445 | -0.569081996680866 |
| Al | 2.259754362616018  | 3.355273531940917  | -0.578063046254380 |
| Al | 2.602783696803526  | -3.336104557607677 | -0.699175948269911 |
| Al | 0.334235424187989  | -4.715343460329046 | -0.016176626326546 |
| Al | 3.942271671768715  | -0.992631549787198 | 0.060055668998130  |
| Al | 1.157355029536795  | -1.156423358015045 | 0.420641507670998  |
| Al | -3.531879100680072 | -1.941301660934175 | 0.718961923950632  |
| Al | -3.412670445985589 | 0.799224319336984  | 0.961874257747210  |
| Al | 2.355034470793113  | 1.165866846850060  | 1.049535566721326  |
| Al | -1.945414231081504 | 3.181078400461523  | 1.223346978549268  |
| Al | -0.910276726504138 | -2.599889440905611 | 1.400164936507029  |
| Al | 0.747425922487672  | 3.270402221147213  | 1.615321577548713  |
| Al | -0.568839854483425 | 0.899584022508083  | 1.728795093511250  |
| Al | 4.229667375038487  | -3.222793860888467 | 1.515956928089613  |
| Al | 1.585952819401610  | -3.319410242953187 | 2.055536648944482  |
| Al | 2.924768442532789  | -0.950650604448338 | 2.657555977472100  |
| Al | -2.312943472331325 | -0.805620870126148 | 2.878648898351432  |
| Al | 0.348243568074940  | -1.277962053733159 | 3.355344866869762  |
| Al | -3.028254823738735 | 1.855130294678517  | 3.390789730876010  |
| Al | 1.432582123015584  | 1.265219928575659  | 3.582230566472639  |
| Al | -0.716503831536246 | 3.198195272670654  | 3.867251090098227  |
| Al | -0.935664980846335 | 0.658474158279338  | 4.755648661726676  |



## 2.20 AI\_39

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | 0.769185795776082  | -2.436949627720938 | -3.715233198288293 |
| AI | 0.562365813929331  | 0.146926749796366  | -3.450914997573470 |
| AI | 0.643248010713240  | 2.692477138664451  | -2.885606623633043 |
| AI | -1.628073725303466 | -1.500481679520792 | -2.889293993410174 |
| AI | -1.761421919849509 | 1.168847573880502  | -2.585864975340890 |
| AI | 2.122335186339868  | -4.418387037875296 | -2.576290476607337 |
| AI | -3.913342616384819 | -0.466341011293498 | -2.081657760830405 |
| AI | 2.938181349555569  | -1.794086406386130 | -2.074668441041549 |
| AI | 2.552162903823822  | 0.898445052842343  | -1.664505835909798 |
| AI | 0.560659245976680  | 5.226402121689068  | -1.779205512350982 |
| AI | -0.447806144532604 | -3.777826861642934 | -1.598367038009821 |
| AI | -1.554874496808978 | 3.621709036922855  | -1.373921050208356 |
| AI | 0.424791291355314  | -1.225727562634903 | -1.004542865326867 |
| AI | 2.530480692410402  | 3.604423632298900  | -1.073074598905455 |
| AI | 4.992318998751617  | -0.316723677173243 | -1.048719710459787 |
| AI | -3.774327665599226 | 2.201019397437260  | -1.039503351792050 |
| AI | -2.751980143538001 | -2.848716936582888 | -0.705986858119765 |
| AI | -5.992256690819710 | 0.632166929725912  | -0.651457046928334 |
| AI | 0.118095829841973  | 1.457366817294233  | -0.497046800254711 |
| AI | 4.741858270058747  | 2.352972380194885  | -0.473260448221499 |
| AI | -2.012346902642344 | -0.200708282316436 | -0.074365510281636 |
| AI | 1.846786544721137  | -3.507690202165733 | -0.047727421088618 |
| AI | -4.986839962046621 | -1.675301528356135 | 0.224718587322169  |
| AI | 2.780345800460218  | -1.010652222836788 | 0.552678149996543  |
| AI | 0.387736588439658  | 3.927074618835308  | 0.656407132213266  |
| AI | -0.637427840808844 | -2.963590256443212 | 0.984445648257365  |
| AI | -2.025999591563062 | 2.440228265586830  | 1.058188642769142  |
| AI | 2.356842193504370  | 1.849711217369730  | 1.064345057351730  |
| AI | -4.165371999736382 | 0.748960889713517  | 1.401894464655907  |
| AI | 4.796081406915514  | 0.621298467732354  | 1.651615406297559  |
| AI | 0.237271973362985  | -0.325039204862932 | 1.590479673593398  |
| AI | -2.943432157901888 | -1.925034013574107 | 1.955107374014286  |
| AI | 1.667279508450672  | -2.641885101895272 | 2.441065430152506  |
| AI | 0.121822633362614  | 2.271037103561680  | 2.655518879667515  |
| AI | -2.050207464566105 | 0.534205651986737  | 2.956837043654431  |
| AI | 2.562749380729741  | 0.005011595862085  | 3.095044946596079  |
| AI | -0.816937858275296 | -1.999388189318214 | 3.516004516351201  |
| AI | 0.175090162058591  | 0.416207764974587  | 4.486150470731562  |
| AI | 1.574957599838797  | -1.781962603770186 | 5.000713090958191  |

## 2.21 Al\_40

40

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 2.779760244010900  | 1.810312376512039  | -3.168246320466817 |
| Al | 0.856452628679085  | 3.647808092758849  | -3.792136898090773 |
| Al | 0.182768601801994  | 1.031213852987015  | -3.129275414999467 |
| Al | -1.740992738761616 | 2.870497356747606  | -3.771992945851408 |
| Al | 2.134977352907550  | -0.711736174983121 | -2.409678975419629 |
| Al | -0.448845116907357 | -1.580037721985109 | -2.625829011071579 |
| Al | -2.426134842938548 | 0.267285249815760  | -3.245631772373326 |
| Al | 4.782133513269347  | 0.143051645971447  | -2.344149393315284 |
| Al | -4.721930908181282 | 0.364160404675776  | -1.746305031879164 |
| Al | 1.564520922037506  | -3.295429432726158 | -1.783924024278667 |
| Al | 4.147662996982632  | -2.387179477924892 | -1.609601325211175 |
| Al | -3.090258256848147 | 2.532815319042284  | -1.418190766295842 |
| Al | -2.744720693557891 | -1.429426517596605 | -1.145673934387824 |
| Al | -0.463083817784925 | 3.354441850922829  | -1.302651942334609 |
| Al | 2.256638010283908  | 4.064881628826795  | -1.393600563058298 |
| Al | -1.110055555830222 | 0.721006201273347  | -0.743335116355055 |
| Al | -0.793090600783486 | -3.175901592332344 | -0.432537326691126 |
| Al | 1.535793752348145  | 1.449812753015776  | -0.653165280768849 |
| Al | 4.206792604358883  | 2.319851857758865  | -0.674350701610534 |
| Al | -5.138617086001998 | -1.390578725782760 | 0.278894132827505  |
| Al | 0.985046765045894  | -1.154079999658503 | 0.027712622771583  |
| Al | 1.164063323391998  | -4.896774136279277 | 0.406485425266318  |
| Al | 3.609079707041031  | -0.287403473543240 | 0.198394409222885  |
| Al | -3.175800408276068 | -3.199540522927824 | 0.862721837398196  |
| Al | -3.451459492047508 | 0.842414500460107  | 0.726933087676903  |
| Al | 2.986146629892723  | -2.899541935510702 | 0.810232869711390  |
| Al | -1.853902316147620 | 3.022216503965060  | 1.008243301117307  |
| Al | -1.415687949540178 | -1.033548495245503 | 1.345028509284012  |
| Al | 0.918789363419947  | 3.656437234434717  | 1.051732952673307  |
| Al | -1.213844892084301 | -4.904923062798523 | 1.706442359425619  |
| Al | 0.293136700653736  | 1.037876235892941  | 1.719544184712245  |
| Al | 2.945297264007591  | 1.843707169816497  | 1.802045600832436  |
| Al | 0.621224972546573  | -2.804224568666316 | 2.220061674376410  |
| Al | -3.847257521224689 | -1.071735708608673 | 2.748350831167015  |
| Al | 2.471302589824484  | -0.848481677779866 | 2.598295463101714  |
| Al | -2.103130207350411 | 1.114935083328156  | 3.106228479958876  |
| Al | -1.900835894720746 | -2.823780920682037 | 3.455850107680314  |
| Al | -0.403870058915967 | 3.190633258576060  | 3.350729190566126  |
| Al | -0.081020425130353 | -0.703481915488146 | 3.845177053344639  |
| Al | 1.682950840529390  | 1.312447483737708  | 4.121172651344658  |

## 2.22 Al\_41

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -1.058794427570684 | -1.020294426625927 | -3.278050712099232 |
| Al | -0.329113028953811 | 1.707779064209960  | -3.895370954607547 |
| Al | -0.542724141473277 | 4.321538346879098  | -2.959395666017612 |
| Al | 0.908117579863818  | -2.536008625950129 | -2.050497259517187 |
| Al | -1.615826124189292 | -3.549794721751199 | -2.434693644984424 |
| Al | -2.815988811896961 | 0.780003678233296  | -4.298515249502582 |
| Al | 0.336491321494302  | -5.151320557074119 | -1.342865064704416 |
| Al | 1.540750664617224  | -0.001180977683276 | -2.892514479966295 |
| Al | 1.320730921098804  | 2.620301785326880  | -1.982441518695735 |
| Al | 1.060103709168301  | 5.290480394263469  | -1.024461387118926 |
| Al | -3.288799328472281 | -0.165997718093312 | -1.749034484486206 |
| Al | 2.848251463628957  | -4.158428527400914 | -0.927492649944513 |
| Al | -0.733062385997469 | 0.891746089087544  | -1.242971954745567 |
| Al | -0.996138932161289 | 3.510849743851853  | -0.331176818081849 |
| Al | -1.742659170182439 | -4.440384072239021 | 0.316899128814010  |
| Al | 3.460252649116379  | -1.596838339310530 | -1.667555251824188 |
| Al | -3.778131373289575 | -2.732471848253920 | -0.952748467559701 |
| Al | 3.141321034774572  | 1.009625825932510  | -0.904021820664440 |
| Al | -1.229526155705624 | -1.672098424152386 | -0.423921177173538 |
| Al | 4.797888402437797  | 2.048284794337048  | 1.073653432651977  |
| Al | -3.064015920028441 | 1.842912707175449  | 0.238455066281022  |
| Al | 3.312155067776571  | -2.408060232744046 | 1.140037211361966  |
| Al | 0.927335586661608  | 1.793980055039427  | 0.746888064175852  |
| Al | -0.998736065707625 | 0.088532848516969  | 1.627699429473704  |
| Al | 5.089757327372745  | -0.530696744520679 | 0.257283539987210  |
| Al | 2.953028074746543  | 3.672526178196408  | -0.032305022084669 |
| Al | -3.505669602509023 | -0.755577529063308 | 0.925770067081906  |
| Al | 1.281109881406254  | -0.772952263050302 | 0.006027791909737  |
| Al | -3.846234170062171 | -3.416507150985084 | 1.702764973001084  |
| Al | 2.601456556713062  | 2.853951595434005  | 2.686286171607009  |
| Al | 0.709276925720511  | 4.561325093276182  | 1.620301103386057  |
| Al | -1.227537260759284 | 2.805108476546261  | 2.337052619246514  |
| Al | 0.792647625673425  | -3.376646732556365 | 0.744181408400292  |
| Al | -3.224035869845046 | 1.151989406816838  | 2.823451848166032  |
| Al | 2.943157075391843  | 0.220757107210494  | 1.943406211126810  |
| Al | -2.605309735694560 | 2.590239913781335  | -2.316016358010636 |
| Al | 0.760669287764600  | 1.098431136392412  | 3.499229945914575  |
| Al | -3.399437868588243 | -1.435411822474325 | 3.635757871817574  |
| Al | -1.257488938950106 | -2.571614540496094 | 2.252497671271158  |
| Al | -0.828430198898220 | -0.863899551591164 | 4.261736456295989  |
| Al | 1.303158355508156  | -1.704179434491326 | 2.866669929818761  |

## 2.23 AI\_42

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | 1.288398893562944  | 2.592648670819079  | -4.198846286137164 |
| AI | 1.921561393695340  | -0.040359743926663 | -3.729622616447640 |
| AI | -1.220129979512958 | 3.575548706198660  | -3.688759688511284 |
| AI | -0.579439678219911 | 0.874152657325247  | -3.159723713706938 |
| AI | 4.445711012291271  | -0.138153628398884 | -2.707528336794345 |
| AI | -0.005582929719898 | -1.770615368838017 | -2.738859866271747 |
| AI | 3.024816527311433  | 2.128406590135910  | -2.097460909440416 |
| AI | -3.082409482521229 | 1.880091802110142  | -2.636972264195935 |
| AI | 2.520198725281015  | -1.851818505198406 | -1.769736133745569 |
| AI | -2.465457049919564 | -0.743750263883555 | -2.108727295661118 |
| AI | 0.514058338385730  | 3.079337792456862  | -1.563940060048909 |
| AI | -1.954721856884078 | -3.363091678335216 | -1.575474210219395 |
| AI | 1.097441749451017  | 0.433124214804124  | -1.051985172655803 |
| AI | 5.091973001113722  | -1.878953987355663 | -0.709927178026026 |
| AI | -5.030798007684562 | 0.303757325623410  | -1.558360329180930 |
| AI | -2.074505318970127 | 4.094069453585281  | -1.110524523814439 |
| AI | 0.648787713190135  | -3.461710470287516 | -0.731844426081318 |
| AI | -4.431978414591816 | -2.314346791374370 | -1.013017993672388 |
| AI | 3.659633414121885  | 0.400976442137823  | -0.044940098254390 |
| AI | -1.432471845258521 | 1.357885173657127  | -0.478601368610677 |
| AI | 3.203771347021972  | -3.579664903868397 | 0.236693299917992  |
| AI | -0.907587623044535 | -1.292907784071296 | 0.064464127716933  |
| AI | 2.318206141683810  | 2.677676013518825  | 0.497768595785763  |
| AI | -4.085348431419762 | 2.367597534039311  | 0.014540424338115  |
| AI | -1.258875348606125 | -5.029664842255768 | 0.463165048148229  |
| AI | 1.650430014257651  | -1.310996963332983 | 0.938691221696459  |
| AI | -0.313377492949801 | 3.498603575081624  | 0.956165949929110  |
| AI | -3.416799048879047 | -0.301099764234266 | 0.630784662938435  |
| AI | 1.305478413251910  | -5.134302793088906 | 1.391074440341526  |
| AI | -2.855360489151562 | -2.919841596087765 | 1.178524095694257  |
| AI | 4.320524782040005  | -1.453291744294749 | 1.920485698405649  |
| AI | 0.176600290025233  | 0.867231138734331  | 1.658108848815003  |
| AI | -2.972735582482513 | 4.422464822431429  | 1.478040326741281  |
| AI | -0.271683464517050 | -2.979257972856122 | 2.144701370487272  |
| AI | -2.422899195503280 | 1.685518733085278  | 2.135864552902781  |
| AI | -1.916168093847169 | -0.965633787698920 | 2.861272914638096  |
| AI | 2.855576476692358  | 0.735811467541419  | 2.615036401100163  |
| AI | -1.185002875834508 | 3.687827602871097  | 3.566384095877780  |
| AI | 1.370332210810739  | 2.913086652119761  | 2.999625621401467  |
| AI | 2.364267456134833  | -3.139972118613164 | 3.004669017223137  |
| AI | -0.777045881847418 | 1.061735205529837  | 4.200795412060705  |
| AI | 0.882610191042394  | -0.968116865805985 | 3.717996345316200  |

## 2.24 Al\_43

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -1.610274542508295 | 0.048282533709465  | -3.447237721096299 |
| Al | -3.496337682687113 | 1.874079670315009  | -2.827385487700468 |
| Al | 0.379858867374196  | 0.415105119036982  | -5.300625317200580 |
| Al | 2.443833264379908  | 0.214033487340355  | -3.596316254148393 |
| Al | 0.401254384796006  | -1.878406459132842 | -3.907385188624088 |
| Al | 0.405431091729627  | 2.108611555494415  | -3.141940894256632 |
| Al | 3.077866636194173  | -2.480556112583783 | -3.516199079633680 |
| Al | -1.558573394912305 | 3.854585307481337  | -2.535927600398002 |
| Al | -1.525585389754539 | -2.396163158857656 | -2.044106695124850 |
| Al | 0.543353475566871  | -0.286532185486449 | -1.648487597172295 |
| Al | 1.171620397955174  | -2.982110597143964 | -1.576277478786158 |
| Al | -3.329531307446007 | -0.470032238318462 | -1.367476126561240 |
| Al | 3.902299579131876  | -3.627432665507710 | -1.202978762391286 |
| Al | 2.616402566996076  | 1.843097137384067  | -1.412134550124435 |
| Al | 3.220816600824975  | -0.883000416448988 | -1.263064763399512 |
| Al | -1.392952845009196 | 1.500191517454422  | -0.944764138743070 |
| Al | -5.231384514395317 | 1.411904521011367  | -0.734290292424326 |
| Al | 0.551476985366115  | 3.502199758619993  | -0.779275958688482 |
| Al | -3.402190680789429 | 3.373650453625723  | -0.331328811538976 |
| Al | -1.465360688798560 | 5.265360697272451  | -0.167118307834446 |
| Al | -3.243678352004157 | -2.768362514478348 | 0.116365563825722  |
| Al | -0.601309795057789 | -3.412384912371658 | 0.367872255799773  |
| Al | -1.343481691809181 | -0.772781759744509 | 0.522730573683853  |
| Al | -5.104870715964935 | -0.851621068085597 | 0.770205054119348  |
| Al | 2.116005496489826  | -4.172064213373657 | 0.761947522526990  |
| Al | 1.305448294750354  | -1.389544966483658 | 0.765894739178009  |
| Al | 0.640236402568542  | 1.258244025497179  | 0.823665545703008  |
| Al | 4.085337965378365  | -2.154051689343354 | 1.064747218545124  |
| Al | 3.297871245628134  | 0.580056823319248  | 0.964456378797061  |
| Al | 2.606809402834086  | 3.221459524243874  | 1.007323490617063  |
| Al | -3.250946158260596 | 1.134293543802153  | 1.220809188877871  |
| Al | -1.410744690445231 | 3.017661360771720  | 1.499228728022311  |
| Al | 0.554094178230518  | 4.941493275442040  | 1.628228257175570  |
| Al | -2.338435317661048 | -3.821582470661784 | 2.510396171369369  |
| Al | -3.141930932533098 | -1.187003167518858 | 2.650441142580094  |
| Al | 0.285047850835076  | -4.545538728513559 | 2.754041927212606  |
| Al | -0.498882858840526 | -1.840307318230899 | 2.936223953642076  |
| Al | -1.293522851034648 | 0.799947730872615  | 3.120349027531798  |
| Al | 2.166410382556247  | -2.577404370855043 | 3.094636443742298  |
| Al | 1.369227259611019  | 0.072772098977278  | 3.220360904974093  |
| Al | 0.676053875787440  | 2.721710782624809  | 3.230000796476551  |
| Al | 4.096110584091898  | -0.641732943910558 | 3.339541592473241  |
| Al | 3.327127620835418  | 1.979873032754897  | 3.374854548973277  |

## 2.25 Al\_44

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 0.979638404963167  | 0.778785655607951  | -4.159801839175485 |
| Al | -1.754260860955373 | 0.148098127596057  | -3.972202482991848 |
| Al | 3.733372186249419  | 1.358963324046426  | -4.282955124860052 |
| Al | -2.471620217294497 | -2.224787538299033 | -2.946659646587122 |
| Al | 2.786919582135075  | -1.013114322151681 | -3.311037599651656 |
| Al | 0.145153931116425  | -1.519850507329439 | -2.976227245861579 |
| Al | -0.621697642667852 | 1.991334714188998  | -2.355680185720557 |
| Al | -0.605257537018469 | -4.120032471506920 | -2.489080562354324 |
| Al | -4.474955428541820 | -0.532855009747417 | -3.717169094689660 |
| Al | 2.099876041885077  | 2.642074040563786  | -2.575020842670470 |
| Al | -3.356235365624672 | 1.383569084699390  | -2.198551476580469 |
| Al | 2.011706334703689  | -3.516496221009179 | -2.668722263820335 |
| Al | -2.431392305684351 | 3.492776067133626  | -0.962192170245284 |
| Al | 0.517141603562122  | 4.172474434443423  | -1.165789779716068 |
| Al | 4.018794164810121  | 0.894512077434985  | -1.559496297074288 |
| Al | 1.292686491553924  | 0.306690248917892  | -1.185371168828008 |
| Al | -1.439156214121768 | -0.323644227785813 | -0.996359344789047 |
| Al | -4.172779494012408 | -0.995331726050313 | -0.995043258866808 |
| Al | 3.484079099187143  | -1.698090162631997 | -0.838229223693553 |
| Al | -2.470916644937628 | -3.072888957873801 | -0.427414329458344 |
| Al | -0.901874573595489 | -5.119141947071699 | 0.002470684087571  |
| Al | 0.587835871684810  | -2.680800622845537 | -0.449560988603475 |
| Al | 3.038362217419168  | -4.210404644530770 | -0.268824158334539 |
| Al | -1.235851377701582 | 5.450803069666136  | 0.306835971073163  |
| Al | -0.446789940557279 | 2.058934655936968  | 0.408303923806638  |
| Al | 2.582412080478145  | 2.768369311767884  | 0.137196880814265  |
| Al | -3.487223594565118 | 1.367988038541450  | 0.555242131906169  |
| Al | 3.076029174606399  | 0.243770023209148  | 1.009874706108228  |
| Al | -2.539644511228778 | 3.524422892347596  | 1.783867222815408  |
| Al | 0.957124992832727  | 4.330735664201834  | 1.542392853645577  |
| Al | -2.714300135392881 | -1.091759624741155 | 1.408878833416292  |
| Al | 0.217743728712572  | -0.561341600360857 | 1.284261263675422  |
| Al | -0.982742236823697 | -3.103703029546143 | 1.764372360447663  |
| Al | 1.218330874725423  | -4.790254569011045 | 1.629041644290606  |
| Al | 2.458981361403881  | -2.309660354826573 | 1.527667519663611  |
| Al | -1.771699664099536 | 1.033569265505649  | 2.629355362362340  |
| Al | 1.468600539454684  | 1.780376763944034  | 2.405788640899692  |
| Al | -1.091197324849947 | 5.609922740448699  | 2.938515627620530  |
| Al | 0.967962315943080  | -3.094268995196819 | 3.691992153448684  |
| Al | 2.090829423007706  | -0.598307261093170 | 3.542281018524806  |
| Al | -1.141421261922217 | -1.344210120810159 | 3.764310025853581  |
| Al | -0.482312325289444 | 3.176237353958197  | 3.627010885737649  |
| Al | 0.122391027282278  | 0.867136925637174  | 4.693475262012111  |

A1    0.737357209167746    -1.460600565377815    5.848254112362975

## 2.26 Al\_45

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -2.250555778190532 | -0.715219243784238 | -4.080387017295015 |
| Al | 0.183052194619018  | 0.920730435491574  | -4.239303168184971 |
| Al | 2.587018132303134  | -0.489623102959847 | -4.038310943456543 |
| Al | 0.235045520479462  | -1.815589557535087 | -4.302078672386088 |
| Al | -2.363581089418928 | 1.959121418760869  | -3.956204008080005 |
| Al | 1.711041090077662  | 4.208315596362636  | -1.354934071948167 |
| Al | 2.310689174607930  | 1.997703095498029  | -2.886753598405523 |
| Al | -3.711324830645768 | -1.353939937318291 | -1.819433795873218 |
| Al | -2.850263573489304 | 3.032006736627318  | -1.507494688353491 |
| Al | 0.898631846565362  | -3.779864722621939 | -2.547308317806882 |
| Al | 3.248006094932155  | -2.505352748561187 | -2.281219257644935 |
| Al | -1.480352810548572 | 0.581395803009421  | -1.810706173775055 |
| Al | 3.930536112533515  | 0.132342820166173  | -1.733572431520802 |
| Al | -1.807794918181379 | -3.190423729204567 | -2.788620440007318 |
| Al | -0.313396274340509 | 3.258683292382443  | -2.800819264070413 |
| Al | 1.034120409387905  | -0.638880324264456 | -1.810051296070815 |
| Al | -3.106392385545186 | -3.653214184975233 | -0.454727109901576 |
| Al | -1.109207798332744 | -1.785506112201188 | -0.464781850451921 |
| Al | 0.601243795386980  | 1.784838897833675  | -0.576370021920994 |
| Al | -4.316581886918838 | 0.973724922566758  | -0.471280074479075 |
| Al | 3.470921365191044  | 2.430880650617228  | -0.399962907986637 |
| Al | 4.234101283618145  | -2.059063099338476 | 0.196394118252348  |
| Al | 1.599152172501046  | -2.842597328199934 | -0.068135657907931 |
| Al | -0.848494770265864 | 4.310623689525675  | -0.216464320738769 |
| Al | -0.549933258737374 | -4.564291141462033 | -0.384077446496905 |
| Al | 2.246950483879154  | -0.136114732876736 | 0.616625661132721  |
| Al | -3.848640381213069 | -1.500625620075068 | 1.010498150332882  |
| Al | -1.802788082115672 | 0.593106050702071  | 0.843997765263497  |
| Al | 4.864461958916161  | 0.472750412138067  | 0.813523977637436  |
| Al | -3.079652626881389 | 3.207690523004111  | 1.203892340363296  |
| Al | 1.696136029388761  | 3.846524116168109  | 1.342426783439244  |
| Al | -1.995090962044629 | -3.266048100085281 | 1.987646084660250  |
| Al | 0.106233819402130  | -1.321907325041771 | 1.880806667144142  |
| Al | 0.429047089690856  | 1.340630416891901  | 2.005905230205212  |
| Al | -4.148530683257718 | 0.859067611360251  | 2.241928664933194  |
| Al | 0.565215748652086  | -4.151641167945160 | 2.083363256998977  |
| Al | 2.757293688303090  | -2.616030440365062 | 2.403834773324679  |
| Al | 3.261586368267446  | 1.911172640519936  | 2.429821973460175  |
| Al | -0.721994971409773 | 3.925763523843209  | 2.467712540994967  |
| Al | -2.471511852858684 | -0.986312495477783 | 3.326955437496681  |
| Al | -2.008949595192425 | 1.836310307205457  | 3.578442089931864  |
| Al | 2.274793194673684  | -0.266245521011792 | 3.695546239084704  |
| Al | -0.370291249257539 | -2.604798420168708 | 4.141624164796623  |



|    |                    |                   |                   |
|----|--------------------|-------------------|-------------------|
| A1 | 1.143932002598152  | 2.586499234898120 | 4.176894030197543 |
| A1 | -0.233879797128854 | 0.073406859900853 | 4.545156585112538 |

## 2.27 Al\_46

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.210768845157771 | -4.256117333551958 | 0.617954167411254  |
| Al | 2.313425607740385  | -0.127124463825430 | -3.772936917639191 |
| Al | -2.356346270195679 | -0.221835931040829 | -4.445258110331390 |
| Al | 0.075205860490657  | 1.298268183864307  | -4.211139966180753 |
| Al | -2.574375905075281 | 2.290909597784891  | -3.673940491892535 |
| Al | 2.053251600350890  | -2.885914454026133 | -3.234194768188364 |
| Al | -0.089309448163688 | -1.585519668046018 | -4.098561500881339 |
| Al | -0.536715432775196 | 3.560286107262543  | -2.518748204991903 |
| Al | 3.800427088454493  | -1.600790513505604 | -1.656974598322391 |
| Al | -4.022694614106545 | 0.332594905552524  | -2.202947549726371 |
| Al | 2.078857411438289  | -3.709517051377015 | -0.653500003939319 |
| Al | 2.080006696202904  | 2.568441340299801  | -2.946230837356803 |
| Al | -2.568143077406280 | -2.209515750501053 | -2.658223393164537 |
| Al | -3.010349816113492 | 2.861191027014486  | -1.090112130904837 |
| Al | -1.299874894507134 | 0.455103111006922  | -1.875504263697623 |
| Al | 2.445467301966124  | 0.169951944953850  | 0.163935184548516  |
| Al | 3.825446942946877  | -2.256776525356632 | 0.955919479795002  |
| Al | 0.571553311417770  | 1.836739900989931  | -0.547416890750078 |
| Al | -0.281620378570356 | -3.679874835802828 | -2.116646662717552 |
| Al | 4.143553348397839  | 1.010064540373728  | -2.146591174523815 |
| Al | 0.835026511581152  | -1.103234637471141 | -1.446845098169755 |
| Al | -4.468926311781731 | 1.075639040916485  | 0.462655024109157  |
| Al | 1.405181506338934  | 4.479606887696113  | -0.971924558473614 |
| Al | -4.205411883152169 | -1.838480086221330 | -0.447325211019100 |
| Al | -0.966748646036679 | 4.134686211647363  | 0.283173147796999  |
| Al | -1.710185401709086 | 1.014932125562762  | 0.861980013950941  |
| Al | -1.471100546920674 | -1.589391422938503 | 0.064502550072556  |
| Al | 2.125265459219852  | -4.009081486566014 | 2.065729923419578  |
| Al | 1.425366620432506  | 3.719977208335667  | 1.627029562494279  |
| Al | -3.205051788436822 | 3.238785655032048  | 1.565841027151674  |
| Al | 0.640284487719164  | 0.815481079987142  | 2.120802244445319  |
| Al | 3.225570960835395  | -0.593793781767248 | 2.954574251813163  |
| Al | 3.416018556476400  | 2.834216086132280  | -0.326653143263034 |
| Al | 5.022491687928758  | 0.285537766997187  | 0.505902257267197  |
| Al | 0.879308920990225  | -1.747110095496176 | 1.344157605548636  |
| Al | -2.570954571931389 | -4.022028789144343 | -0.667274083097215 |
| Al | -3.999749329798337 | -1.025943332228961 | 2.150019934798632  |
| Al | 1.567129630451708  | 2.426320037513669  | 4.047373150452627  |
| Al | -2.591343363120085 | -3.324527977529081 | 1.970184225128163  |
| Al | -0.853771157593561 | 3.096633793583452  | 2.878252259172941  |
| Al | -3.008832241217779 | 1.187631920040223  | 3.347814836994914  |
| Al | 3.470557282134827  | 1.949362046303667  | 2.212462231295913  |
| Al | -0.201603683014531 | -3.451133746096237 | 3.295148453792857  |

|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| Al | -1.551927951695307 | -1.141342938854784 | 3.273231341165229 |
| Al | -0.610110375573141 | 0.956462273297721  | 4.638718215426254 |
| Al | 0.966519140537677  | -1.219767970801353 | 4.301588471179610 |

## 2.28 AI\_47

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | 3.620623126506262  | 3.553611262250820  | -1.941561348898237 |
| AI | 1.046189484953246  | 2.682832457514971  | -2.514604698508089 |
| AI | -1.591578720461746 | -0.793661899877870 | -4.145354153233464 |
| AI | 0.818769383675976  | -2.328991987942253 | -3.814702748361261 |
| AI | -1.611688581411601 | -3.466862315194031 | -3.950985405854373 |
| AI | 3.126024480061442  | -1.098054741393824 | -3.538022424087788 |
| AI | 5.263521346432732  | 0.493706624699836  | -3.011411580664004 |
| AI | 0.775393363084049  | 0.449804078382028  | -4.169624288768762 |
| AI | -1.285409531622332 | 1.452922131747121  | -2.590809594337205 |
| AI | -3.770501172399071 | 0.282318694314096  | -2.698028405497102 |
| AI | 3.100628814110076  | 1.748379314473432  | -3.971506371527921 |
| AI | -0.054312803727118 | -0.810218570682261 | -1.689439745715369 |
| AI | 2.554461317735623  | 0.605132772752702  | -1.446230682724391 |
| AI | -2.656434633143631 | -2.178768648198742 | -1.774644196095529 |
| AI | -0.306591717633149 | -3.581192692156244 | -1.543268055233378 |
| AI | 2.160090405562054  | -2.336206732670323 | -1.339181579179435 |
| AI | -1.000471135741781 | 3.815674914053682  | -0.936186927433230 |
| AI | -5.914942420287647 | 1.482955564561781  | -1.424344770527856 |
| AI | -3.421676089769724 | 2.604304724988395  | -1.169552508725777 |
| AI | 4.588794649450506  | -1.201363295753383 | -0.998890388884485 |
| AI | 1.549118803148970  | 4.691355730188750  | -0.701846460152920 |
| AI | 5.049926397098121  | 1.611762027774825  | -0.586528439987896 |
| AI | 0.313592018562757  | 1.408882949679537  | -0.122393696442664 |
| AI | -2.233736095399829 | 0.205984447552527  | -0.291454693235208 |
| AI | -4.850074202855096 | -0.888439136965751 | -0.444594671171245 |
| AI | 2.792078445909894  | 2.560551495744797  | 0.516917761521869  |
| AI | 1.401928756113715  | -0.915006191611676 | 0.842800231716275  |
| AI | -1.272499953479546 | -2.213476389703755 | 0.642084179094653  |
| AI | 1.052043013377610  | -3.679270898371462 | 0.815990930263739  |
| AI | -3.865371240212734 | -3.220557833115215 | 0.558941482861364  |
| AI | 3.623711895964714  | -2.527227097789766 | 1.056975630391012  |
| AI | -4.501838681791082 | 1.430822919491639  | 1.038477535582917  |
| AI | -2.037896641098964 | 2.545807495557868  | 1.279636762467003  |
| AI | 4.006256560221424  | 0.221604746404417  | 1.441237312316840  |
| AI | 0.441116544717175  | 3.714877901915910  | 1.566974224054171  |
| AI | -0.863386592123973 | 0.107708765384883  | 2.089043344538254  |
| AI | -3.457202544413455 | -0.924047454762198 | 1.953515593473215  |
| AI | 1.643190992637590  | 1.245117196557135  | 2.494149623091133  |
| AI | 0.120062337411131  | -2.379434487122207 | 3.048196217564010  |
| AI | 2.481154400018399  | -3.756855205764256 | 3.155261607401071  |
| AI | -2.540586421800000 | -3.317782074915248 | 2.859168966562477  |
| AI | 2.754291872381256  | -1.101392417986293 | 3.399007353216231  |
| AI | -3.106963787272701 | 1.329736522123758  | 3.460375685674832  |

|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| Al | -0.615365832470395 | 2.403527946893654  | 3.702087274119979 |
| Al | -2.083978566819741 | -1.088608773427511 | 4.357469216407797 |
| Al | 0.446389151068480  | -0.020454592618856 | 4.622437504091748 |
| Al | -1.686850194267889 | 1.178490753014568  | 5.914419398837010 |

## 2.29 AI\_48

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | 0.301899832630578  | -0.694184775728370 | -5.360218486047732 |
| AI | -2.094437008155129 | -1.829088175388051 | -4.588863826173792 |
| AI | -0.578651490473435 | 1.863611833032760  | -4.956402485978772 |
| AI | -3.863445483974847 | -0.345717386334329 | -3.218208415916992 |
| AI | 2.491633976449856  | -1.379824357375840 | -3.754173558960298 |
| AI | -2.793877927716262 | 2.279973626571991  | -3.480492268388654 |
| AI | -0.209412358092742 | 3.306541080867484  | -2.690865602889600 |
| AI | 1.307754661438928  | 1.081818524075352  | -3.270708134928171 |
| AI | -0.058514519092414 | -2.294213478183410 | -2.969105961469340 |
| AI | -1.249715104313260 | 0.173960879043564  | -2.819161576546527 |
| AI | 4.731568536093752  | -2.129742776658348 | -2.225111204823458 |
| AI | 3.193107352378883  | 0.192290758473327  | -1.604775778282923 |
| AI | -2.678956964526054 | -2.740341469893638 | -2.125593985396371 |
| AI | 2.049572158693778  | -2.751967442391063 | -1.408302222027313 |
| AI | 1.684463525324643  | 2.399983768561237  | -0.951845241095977 |
| AI | -2.529996900672481 | 3.801616575824474  | -1.268461103119257 |
| AI | 0.562048759480209  | -0.402244328119609 | -0.897452368873646 |
| AI | -3.486776561325799 | 1.253884561316120  | -1.024200993796777 |
| AI | 5.221846460310607  | -0.456995965630879 | -0.073152704427452 |
| AI | 0.042626626076498  | 4.549808200715621  | -0.356988982165928 |
| AI | -0.523159214542098 | -3.129060404219906 | -0.490703075769344 |
| AI | -4.573248068197347 | -1.262194871944065 | -0.758117500462946 |
| AI | -0.946455623504871 | 1.852820832879008  | -0.238544984430760 |
| AI | 4.046372687018346  | -3.423361456755083 | 0.160615273687521  |
| AI | -2.021043783987814 | -0.793821542314726 | 0.022414908429804  |
| AI | 3.854791809470623  | 1.788988795099138  | 0.582056160159288  |
| AI | 2.626132722580813  | -1.113571505364106 | 0.738037243724600  |
| AI | -3.152016962793120 | -3.397487695882241 | 0.429274297325854  |
| AI | 1.566539512873112  | -4.115946648913416 | 0.986588533209769  |
| AI | 2.203676401580720  | 3.808023671014853  | 1.333863158221279  |
| AI | -2.198604064565086 | 5.279523447081830  | 1.005733535885360  |
| AI | 1.034208192610960  | 1.191359299905240  | 1.461860731249324  |
| AI | -3.247092358620990 | 2.811191150837731  | 1.258869967677303  |
| AI | -0.024872370322786 | -1.479007375481856 | 1.695572799301985  |
| AI | -4.224386914382967 | 0.359296695621262  | 1.514279576221352  |
| AI | -0.966779366295901 | -4.094223854084825 | 1.991607416429266  |
| AI | 4.485030876094704  | -0.268498804273571 | 2.504939413349883  |
| AI | -0.611063682833608 | 3.390853616303119  | 2.209122679042956  |
| AI | -5.172435292173324 | -2.147386263777646 | 1.768447113465744  |
| AI | 3.348638452618717  | -2.829905494267975 | 2.711341647021396  |
| AI | -1.612357975841414 | 0.809924163494934  | 2.398059213454331  |
| AI | 2.969992661698914  | 1.870479778093440  | 3.226638602006483  |
| AI | -2.637476771964597 | -1.716679980874742 | 2.705715112075650  |

|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| A1 | 1.876477262380041  | -0.601652960277930 | 3.424360930175862 |
| A1 | 1.402773291642795  | 3.980094490172380  | 3.915230958905374 |
| A1 | 0.823679492305212  | -3.138597980018146 | 3.747911070820013 |
| A1 | 0.325284374553394  | 1.506506543017700  | 4.235149608304999 |
| A1 | -0.695342857937764 | -1.016835297848745 | 4.503760511826590 |

## 2.30 Al\_49

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 1.292534131588038  | -4.569740489149821 | 0.858005198332940  |
| Al | -2.166279724438157 | -1.543489843898820 | -4.288139326829584 |
| Al | 0.353998472683346  | -0.669409049146827 | -4.829739136312480 |
| Al | 1.370263010346392  | 1.540294845437771  | -3.875861688184359 |
| Al | -1.399603722304636 | 1.035644975544361  | -3.667316847197740 |
| Al | -3.986971450442901 | 0.053047784669436  | -3.101291708208019 |
| Al | -0.462863868995134 | 3.280650305142974  | -2.781415953413322 |
| Al | -3.216982647474322 | 2.495144996888477  | -2.255838675107464 |
| Al | 0.018051981884323  | -2.747718620606441 | -3.097381558807429 |
| Al | -2.665463794261791 | -3.466865308804201 | -2.450644540408364 |
| Al | 2.471966207477228  | -1.398875422706848 | -3.417428258123216 |
| Al | 3.161384461383356  | 0.781834352486413  | -1.932869430733758 |
| Al | -4.601172679379957 | -1.891164707506983 | -1.269457314552088 |
| Al | 0.548013330673527  | -0.202497934125130 | -1.817265687953929 |
| Al | -1.948740497098017 | -0.935680991201076 | -1.448090471083948 |
| Al | 2.353969217435742  | 3.580980005495029  | -2.451830716031960 |
| Al | 0.623154251073515  | 4.928846148882592  | -0.846441708092842 |
| Al | 4.287055995572041  | -1.736641090853965 | -1.440173010988117 |
| Al | -4.006543237938067 | 0.682656998803376  | -0.361700275531998 |
| Al | -1.324915441708729 | 1.582295422360719  | -0.479257845875814 |
| Al | -2.084876668533745 | 4.441735060988133  | -0.885259839995879 |
| Al | 1.885166489449650  | -3.045573051921080 | -1.315008775674649 |
| Al | -0.598745046795051 | -4.272035169504821 | -0.924311066544401 |
| Al | -0.048727604778408 | -1.861811485352460 | 0.458042831004223  |
| Al | -2.590183356662320 | -2.962655379477723 | 0.420434533514969  |
| Al | 3.653672128605587  | -3.278497935433442 | 0.771943542734421  |
| Al | 2.325994649704779  | -0.720421377104129 | 0.328519679825394  |
| Al | 3.971599398227005  | 2.936598211595665  | -0.332875487246868 |
| Al | 2.152881589500108  | 4.199895274922744  | 1.273574602520812  |
| Al | 1.253733990539743  | 2.105652236392995  | -0.385373339611368 |
| Al | 4.855097982667038  | 0.440498793433767  | 0.181696351875615  |
| Al | -4.738296528245702 | -1.403359155295274 | 1.375475917451775  |
| Al | -2.143281684263579 | -0.288418764850933 | 1.457714284769944  |
| Al | 6.095196555065225  | -2.020466437672194 | 0.694529205065809  |
| Al | -3.299397258078974 | 2.958614220748254  | 0.894920521991448  |
| Al | -0.647090470655732 | 3.839283584977412  | 1.361922296560275  |
| Al | -0.753530715800236 | -3.799876727663764 | 2.333483726800951  |
| Al | 1.782564913104677  | -2.540740166348531 | 2.620737786185355  |
| Al | 0.270525625239840  | 0.698538480808415  | 1.768926678251383  |
| Al | 4.125469673551649  | -1.059730513785333 | 2.418047397930312  |
| Al | 2.952398397913873  | 1.517755089572473  | 1.835352053274525  |
| Al | 1.010966402895674  | 2.870058437283490  | 3.321413382506293  |
| Al | -2.931113776140881 | -2.429386886197893 | 3.188530847531911  |



|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| Al | -2.625938990236324 | -0.010706313150841 | 4.330543096068372 |
| Al | -1.820124502559329 | 2.326760903260793  | 3.096175052501172 |
| Al | -0.349043478199157 | -1.433800214350192 | 3.606088336976267 |
| Al | 2.047580863349786  | -0.076102866695168 | 3.793920457067172 |
| Al | -4.316138302600486 | 1.065773622296375  | 2.489356909759753 |
| Al | -0.137214272340529 | 1.003106150812231  | 4.775617972008520 |

## 2.31 Al\_50

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.082844433460918 | -0.155634593848422 | -6.429235475905539 |
| Al | -0.026507432458194 | 1.876017854006001  | -4.522909667643668 |
| Al | 1.869834107182733  | -0.180601760263160 | -4.425082839921967 |
| Al | -2.055853622834928 | -0.044972343971271 | -4.509123610318191 |
| Al | -0.174009695008172 | -2.055787210895982 | -4.343415350002210 |
| Al | 0.111138096936301  | 3.499686778520575  | -2.407331104858631 |
| Al | -2.259372540405078 | 2.146369864734554  | -2.861155935517250 |
| Al | 2.114663021712163  | 1.701339447974345  | -2.456459471322296 |
| Al | 3.970818150271093  | -0.273697175395302 | -2.695227539999180 |
| Al | -4.087197874723653 | 0.075879457553063  | -2.721590470637221 |
| Al | -0.081295355220712 | 0.045148311929676  | -2.428964402977744 |
| Al | 1.879297555740775  | -2.013677662635965 | -2.413774159519313 |
| Al | -2.261491221367962 | -1.962225271530802 | -2.550597937577418 |
| Al | -0.224733469704656 | -3.788465808982154 | -2.267975366234273 |
| Al | 6.317002397396328  | -0.329222621622254 | -1.168380922823021 |
| Al | 4.410401244329744  | 1.725216624936202  | -0.823772908659905 |
| Al | -4.219057907808481 | 1.840556642147142  | -0.763135030922324 |
| Al | 2.499961494503935  | 3.611965047233388  | -0.508216319620789 |
| Al | 0.453799371751359  | 1.768540216386315  | -0.210168920682394 |
| Al | 2.389327874543662  | -0.226203499323971 | -0.384202635834299 |
| Al | 4.314080620640731  | -2.209043944007666 | -0.703033122429449 |
| Al | -1.954491946376764 | 3.323306179579049  | -0.565157740388040 |
| Al | -1.965249662521327 | 0.218059930307248  | -0.591576487300516 |
| Al | 0.205805954499132  | 4.875210255054606  | -0.174922300654581 |
| Al | -4.234540446612972 | -1.451339739262597 | -0.612927070026002 |
| Al | 0.004918099060465  | -1.802938396892698 | -0.349408389826275 |
| Al | 2.117447176799850  | -3.669567807224671 | -0.303742855195338 |
| Al | -2.399762163369214 | -3.452648745068608 | -0.443741629197913 |
| Al | -0.168071399309087 | -4.878336374290167 | 0.135639634289568  |
| Al | 4.818297036686884  | -0.168473725930774 | 1.140201356379350  |
| Al | 3.059982797671466  | 1.897039883844302  | 1.569038035353822  |
| Al | -3.730796687705934 | 3.041166370867035  | 1.608167802458341  |
| Al | 3.113092879708471  | -2.213899128596584 | 1.826108289898807  |
| Al | 0.952153227144354  | -0.139972807633670 | 1.825624396053680  |
| Al | -3.819990311047840 | 0.245018113070490  | 1.487136873446230  |
| Al | 1.124013460483939  | 3.552415554900815  | 2.001565940116627  |
| Al | -4.036547171263448 | -2.632129256421955 | 1.783267852081911  |
| Al | -1.868244046104213 | -4.226113847410716 | 2.161562878142616  |
| Al | -1.549713793674961 | -1.357571144695052 | 1.719080539729820  |
| Al | 3.461328780517144  | -0.028142944207245 | 3.479913356979997  |
| Al | -1.484954984812852 | 4.485169419139368  | 1.926235583095274  |
| Al | 1.458732888318799  | -1.624069772569163 | 4.123954109882439  |
| Al | -3.134807628279924 | 1.602983335924737  | 3.865107074478580  |

|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| Al | -0.782491494430040 | 3.005126749550653  | 4.094061017669031 |
| Al | -1.288600719706166 | 1.578605020033597  | 1.743978055512841 |
| Al | 1.526301987359375  | 1.661772069677468  | 3.915991168774630 |
| Al | -3.264201404045243 | -1.099620583511919 | 3.932561641214265 |
| Al | 0.732145574027708  | -3.227592781773637 | 2.059398525922477 |
| Al | -1.032555545113265 | -2.673573985836143 | 4.193056446730171 |
| Al | -0.717160839920327 | 0.108929806431721  | 4.043579087785263 |

## 2.32 Al\_51

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.142616012793992 | 0.574094847716967  | -5.059622238432010 |
| Al | 0.567400145539004  | -1.842765983232809 | -4.115068606654530 |
| Al | 1.050585693991319  | 2.858355506529918  | -4.165763535445381 |
| Al | 2.226787158020080  | 0.422898198423891  | -3.630860097106033 |
| Al | -2.136253590019738 | -0.560974883399636 | -3.736044699492635 |
| Al | -1.330981336772554 | 1.984921671240009  | -3.043634481473505 |
| Al | -1.370208412131422 | -2.733547932754288 | -2.372978221586701 |
| Al | 1.096466612306532  | -4.006978602790666 | -2.557025366225426 |
| Al | 3.326389001675565  | 2.876434346571797  | -2.642579855662493 |
| Al | -0.173811168353467 | 4.444847319795910  | -2.360007722371531 |
| Al | -4.118727056686962 | -1.610840964729542 | -2.288061319547653 |
| Al | 2.600720947281772  | -1.736717912057628 | -2.256313389255649 |
| Al | -0.002043570805501 | -0.292135716157929 | -1.922456318071289 |
| Al | -3.561733512464423 | 1.122272504284471  | -2.045894857908905 |
| Al | 4.101177980199044  | 0.517490947494336  | -1.638881479395758 |
| Al | 1.026490763583858  | 2.054443517552164  | -1.305202631941285 |
| Al | -2.532167798669901 | 3.667995618439372  | -1.257789358476996 |
| Al | 2.129132016855952  | 4.625339516680237  | -0.885242646379488 |
| Al | 1.621922117081112  | -6.215626375631315 | -0.832084541238711 |
| Al | -0.777291812056447 | -4.789862412521345 | -0.650516100150613 |
| Al | 3.052887495653767  | -3.885347320816062 | -0.614749268442058 |
| Al | -3.235189227644581 | -3.628329991583255 | -0.538931232750690 |
| Al | 0.581186915482121  | -2.388246500507395 | -0.274505240830106 |
| Al | -5.868385993338515 | -2.705596880211009 | -0.384711390879882 |
| Al | 4.664409705886039  | -1.743190025659496 | -0.422564148005318 |
| Al | -2.219010649178038 | -1.039632638855004 | -0.377432932128000 |
| Al | 1.981245631204173  | -0.191337507760879 | 0.087852433805637  |
| Al | -4.973563313014687 | -0.133866117017014 | -0.067024406027210 |
| Al | -1.387150167865132 | 1.443573996687300  | -0.119068444398410 |
| Al | 3.287849125633715  | 2.270452353661412  | 0.261081316008822  |
| Al | -4.349180909403045 | 2.457117516090308  | 0.291602304161284  |
| Al | -0.315176255526460 | 3.975460071820814  | 0.401332402462955  |
| Al | 1.168582283546597  | -4.570368561007476 | 1.361914473322178  |
| Al | -1.423083274908080 | -3.122143936530410 | 1.445989000144646  |
| Al | 4.308131888583599  | 0.029867591120551  | 1.545527444769897  |
| Al | 2.949575852421070  | -2.435672279738565 | 1.712615008937991  |
| Al | -0.414562914654468 | -0.549658461375284 | 1.625733837177155  |
| Al | -4.011356985079884 | -2.088966680988502 | 1.648209091749324  |
| Al | 2.069995961448821  | 4.207913161285397  | 1.795315582710911  |
| Al | 0.781529365625149  | 1.762214249634207  | 1.635120681886267  |
| Al | -3.209889761157984 | 0.516703262786407  | 1.927792540543887  |
| Al | -2.397656836931478 | 3.136343127123443  | 1.905697929798698  |
| Al | 3.288384107518617  | 2.031364419379390  | 3.021850723964256  |

|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| Al | -0.239330019619912 | 3.734351978589604  | 3.311187963123885 |
| Al | 2.028241074681356  | -0.407931850147028 | 3.049801715439978 |
| Al | 0.594262635163217  | -2.710596772949779 | 3.181861795971324 |
| Al | -2.172303756564954 | -1.469218097522228 | 3.540305372877221 |
| Al | -1.445290967579915 | 1.216160751237007  | 3.673636809061264 |
| Al | 2.203860704878890  | 4.115814009146233  | 4.599400840338486 |
| Al | 1.071340373215357  | 1.623413920111268  | 4.614444523044114 |
| Al | 0.028409745744874  | -0.810289997457886 | 4.926740738978033 |

### 2.33 AI\_52

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | 0.069398330593453  | 0.272818317057853  | -6.518979328472231 |
| AI | -1.868934680015398 | -0.289152270899992 | -4.670400059040214 |
| AI | 1.960967448860835  | 0.702375224685785  | -4.509528661778723 |
| AI | 0.561732901722699  | -1.740324345792839 | -4.649857550115422 |
| AI | -0.458355522856357 | 2.077978317874550  | -4.385358653005807 |
| AI | -3.991739043822356 | -0.841373706512696 | -3.137768832750380 |
| AI | 3.953859956155179  | 1.229842287070337  | -2.744524090963880 |
| AI | -2.339759918586731 | 1.355130746981562  | -2.543048927987261 |
| AI | -1.602424810196066 | -2.433223410538321 | -3.015938541595722 |
| AI | 1.528521550574258  | 2.574827890727144  | -2.523968052829421 |
| AI | 2.657796282030533  | -1.207242142526542 | -2.610652819614398 |
| AI | 0.185119916317047  | -0.029674817882865 | -2.508085962143586 |
| AI | 1.014016450577854  | -3.327437234370407 | -2.566636935164400 |
| AI | -0.970403243787500 | 3.762329986032391  | -2.323747620418764 |
| AI | 6.176259695646836  | 1.766710857660390  | -1.140634492075339 |
| AI | -3.575664816289660 | -2.562039940855414 | -1.063882991580519 |
| AI | -4.221773262963421 | 0.432501456976251  | -0.820087290102636 |
| AI | 3.802119003434589  | 3.166107394140420  | -0.701256109049940 |
| AI | 4.746627064156847  | -0.677170014343140 | -0.874508290318229 |
| AI | 1.341835352720805  | 4.097023155175437  | -0.322337128211549 |
| AI | -1.615259071521141 | -0.693196932099948 | -0.719107006180607 |
| AI | 2.305607054707654  | 0.843504807349614  | -0.546651890203021 |
| AI | -0.983185141596959 | -3.503600130505667 | -0.738435709659418 |
| AI | -2.955995274997784 | 2.919956998773772  | -0.571252996693175 |
| AI | 3.350712132726295  | -2.984432787554295 | -0.597018817570094 |
| AI | 1.039498560868953  | -1.552910469711692 | -0.348356212357929 |
| AI | -0.304434836760176 | 1.793221085888378  | -0.552707978649879 |
| AI | 1.433182146830822  | -4.918417111507217 | -0.483473103475793 |
| AI | -1.111328176632703 | 5.022621493831432  | 0.089101601841330  |
| AI | 4.751644793581097  | 1.230820582630157  | 1.134019748217973  |
| AI | -3.536997339957281 | -1.085909033172314 | 1.283380686251682  |
| AI | -2.834662557354036 | -3.763192705253909 | 1.251646779701950  |
| AI | 2.599774718446488  | 2.760211091117615  | 1.793696244822746  |
| AI | -4.443904478800501 | 1.738616143206283  | 1.484013232371693  |
| AI | 3.275604461327717  | -1.049711860313547 | 1.395050853938749  |
| AI | -0.715453579223338 | -1.819600034185905 | 1.583632305093662  |
| AI | -1.775694787296302 | 1.005771715997501  | 1.471970834862836  |
| AI | 1.959762857584598  | -3.246492642652202 | 1.794334584403009  |
| AI | 0.880452002870877  | 0.453459608689695  | 1.620857376398174  |
| AI | -0.277315437332133 | -4.779425049585509 | 1.569540261000794  |
| AI | -2.605783318096574 | 3.837864405218143  | 1.966388177029883  |
| AI | 0.046994543338759  | 3.197441789011279  | 1.886590198620597  |
| AI | 3.670656429570505  | 0.736433072258810  | 3.531287921253842  |

|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| Al | -2.578197355398043 | -2.268578783170662 | 3.588757790496194 |
| Al | -3.336530804440145 | 0.336690938522151  | 3.665274692652812 |
| Al | -4.234438735669542 | 2.961708801301642  | 3.983598097903920 |
| Al | 1.865984273716942  | -1.202354550967820 | 3.708408602434583 |
| Al | -0.650074686001908 | -0.281371203864863 | 3.851040444038231 |
| Al | 0.080316466422648  | -3.164090648950358 | 3.866679005598407 |
| Al | -2.010909926631186 | -5.001819234564740 | 3.675911668328705 |
| Al | 1.195044512128543  | 1.784356185207891  | 3.984087503419785 |
| Al | -1.454268100685551 | 2.362416708396418  | 4.008937441326751 |

## 2.34 AI\_53

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| AI | -0.993331550774563 | 0.816206676753513  | -4.311541231305792 |
| AI | 1.262403569022364  | 2.145994378500854  | -4.355559785184588 |
| AI | -3.012884619750364 | -0.841945740558888 | -3.999722437974528 |
| AI | 3.658488913371389  | 3.532384060582640  | -4.226521627361563 |
| AI | -0.961918711568323 | -2.558692882290209 | -3.799101555437234 |
| AI | 1.195458525058037  | -0.738097713155046 | -3.764789999425780 |
| AI | 3.519299997334505  | 0.810800201332668  | -3.462571267540489 |
| AI | 1.476972490958955  | -3.465561480567882 | -3.237520218611013 |
| AI | 3.659815351181125  | -1.808533165886656 | -2.955309034456249 |
| AI | -2.755559116088355 | 1.417076370884340  | -2.329265535747679 |
| AI | -0.731324655726473 | 2.987767720672192  | -2.613583695369335 |
| AI | 3.983872679211911  | -4.620540019401575 | -2.766122318696048 |
| AI | 1.651239487177955  | 4.215888727543661  | -2.468127933932282 |
| AI | -4.636667797078345 | -0.557177295993798 | -1.897981344081337 |
| AI | -0.800736176358701 | -0.627620517426609 | -1.846096801396394 |
| AI | -2.853349982520220 | -2.669565281154631 | -1.780851138850670 |
| AI | 1.369615886824230  | 1.115766698538549  | -1.748339945251161 |
| AI | -0.382718728280652 | -3.428408293914368 | -1.213843690977994 |
| AI | 3.769227414842875  | 2.724246215964337  | -1.524675852932948 |
| AI | 1.699056572876058  | -1.580763198186304 | -0.998047863924146 |
| AI | -0.259487954035652 | 5.155383597522619  | -0.840740398232050 |
| AI | 3.940897332834666  | 0.013158480532061  | -0.826052271044704 |
| AI | -2.505790133879720 | 3.570288043674219  | -0.604014356941967 |
| AI | 1.956946068316752  | -4.785609001619214 | -0.872046517222470 |
| AI | -4.510804243694994 | 1.749384104123576  | -0.275183002536362 |
| AI | 4.078145224375678  | -2.990258080670345 | -0.505283081227493 |
| AI | -6.450731191771673 | -0.265462962413865 | 0.105633280182371  |
| AI | -2.576044144857885 | -0.308955901104497 | 0.101369464435287  |
| AI | -0.677860846362845 | 1.543309364672654  | -0.167745344517018 |
| AI | -4.486395201900579 | -2.220763821995915 | 0.454703509642506  |
| AI | 1.569532122891244  | 3.204343635302958  | 0.058565477531026  |
| AI | -0.178159412706703 | -1.503712122713790 | 0.845169450226431  |
| AI | -2.184875523311915 | -3.586421630448618 | 0.769685971131601  |
| AI | 1.785861672281019  | 0.430051533408703  | 0.935841624058495  |
| AI | 3.955317989362866  | 2.142682067944168  | 1.048981066728793  |
| AI | -0.026124662435084 | -5.213796455406342 | 0.915697481392092  |
| AI | 1.994806365215320  | -3.268651719348917 | 1.324655088905041  |
| AI | -0.592934062577887 | 3.867601718947805  | 1.499964731701052  |
| AI | -2.625521018637788 | 2.135992620502948  | 1.757204929892611  |
| AI | 3.970041093440313  | -1.343911288468354 | 1.529169903917479  |
| AI | 1.785929976377600  | 2.772673735627546  | 2.653788326925573  |
| AI | -2.716563560368378 | 2.800339331826880  | 5.769606496052202  |
| AI | -0.425283153953654 | 0.611614438529278  | 2.438379092884007  |



|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| Al | 3.717052823438131  | 0.632053073674893  | 3.243323255866045 |
| Al | 1.769401542615549  | -1.436571361057490 | 3.123252279311380 |
| Al | -2.410711881994962 | -1.587056579239910 | 2.545738185784696 |
| Al | -0.273216064214880 | -3.331202853589402 | 2.889972910918873 |
| Al | -4.516859221757181 | 0.152862507087297  | 2.047070334855469 |
| Al | -0.661569601665757 | 2.866256112336689  | 3.990804698495655 |
| Al | 1.442975174025030  | 0.863379097359607  | 4.595156893549476 |
| Al | -2.725091688771545 | 0.664083405790363  | 4.101524366624318 |
| Al | -0.538922380300606 | -1.227339095315251 | 4.563172274078182 |
| Al | -0.740920985687914 | 1.025030542290933  | 6.082207155088685 |

## 2.35 Al\_54

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 3.846903071859114  | -1.951078618296101 | -4.378042347509639 |
| Al | 0.267990697836572  | -2.041540843353296 | -4.253141873920666 |
| Al | 2.105296323987137  | 0.122315889996429  | -4.182430735454237 |
| Al | -2.276158920584832 | -1.598089104787558 | -3.396995095066855 |
| Al | 2.251903057323627  | -4.072193509923145 | -4.036293932355752 |
| Al | -0.556414787618241 | 0.519578034393612  | -3.338582543741318 |
| Al | -3.183761736865180 | 1.116004001618348  | -2.711392012817772 |
| Al | 1.325080855170299  | 2.519656363237095  | -3.371763372604466 |
| Al | -1.455540221713832 | 3.064158075206056  | -2.709771500140242 |
| Al | 0.427854585428804  | 4.951365455770260  | -2.757611110311263 |
| Al | 0.213284688275184  | -4.083780450364181 | -2.233284880089290 |
| Al | 4.319056231066941  | -3.667924077602309 | -2.314540887160350 |
| Al | 4.149075182579766  | -0.147637975606543 | -2.200683570016963 |
| Al | 2.122431874589621  | -1.998857546383134 | -2.169183191935337 |
| Al | -2.975946778218441 | -0.886764326022591 | -0.588857316606342 |
| Al | 3.395363674270195  | 2.385188322850222  | -1.498688531682545 |
| Al | -0.408396478084299 | -1.452165206946878 | -1.344710451500788 |
| Al | -2.252354915929850 | -3.537263511675163 | -1.382234472325541 |
| Al | 1.418460958079585  | 0.500056756890809  | -1.362191823943245 |
| Al | 2.583829544675766  | 4.849975380241668  | -0.805467955550446 |
| Al | -1.244143235771880 | 1.122679402013929  | -0.548623714096410 |
| Al | 0.607686664632972  | 6.754319837613528  | -0.770723246687778 |
| Al | -6.683426549044088 | -1.147093073687916 | -0.746031085257894 |
| Al | -4.994059586941233 | 1.000391788839709  | -0.556185681844863 |
| Al | 2.274907111714644  | -4.068253981318415 | -0.290467464380734 |
| Al | 0.601795388819795  | 3.012288949957398  | -0.645997071813021 |
| Al | -3.270310970403359 | 3.002700247790987  | -0.461154473115355 |
| Al | -1.428894730122513 | 4.907637056089857  | -0.555687526255220 |
| Al | -4.827954044221902 | -1.001461782873195 | -2.716733568764012 |
| Al | 1.531020631723127  | -1.416602731655425 | 0.562978692866231  |
| Al | -0.340627241191111 | -3.515025079844722 | 0.549640209912916  |
| Al | 3.484130055083604  | 0.590556898715935  | 0.521857766542643  |
| Al | 4.190871337161992  | -2.041564012973167 | -0.111274049499833 |
| Al | -4.717044151817463 | -2.998167208650282 | -0.569892672292919 |
| Al | 2.797317920973994  | 3.125495983224162  | 1.279323232177266  |
| Al | -1.078293979669153 | -0.842235199813265 | 1.427191152148989  |
| Al | 0.775253162353586  | 1.160987638925041  | 1.396461980786512  |
| Al | -4.879922943433137 | -0.999787149029649 | 1.416597816364142  |
| Al | -2.896263789427342 | -3.012250674355288 | 1.408326912490715  |
| Al | 0.661069124686424  | 4.932609762100267  | 1.331568151623086  |
| Al | -3.241661130452658 | 1.184276635947000  | 1.596050828014512  |
| Al | -1.417071883602491 | 3.148434422545209  | 1.583794501610894  |
| Al | 3.687404278155736  | -1.336517684052303 | 2.435086699667952  |

|    |                    |                    |                   |
|----|--------------------|--------------------|-------------------|
| Al | -0.923549045494946 | -3.019681543381736 | 3.227154184375822 |
| Al | 0.735823151270944  | 3.138630781260868  | 3.313517672269978 |
| Al | -3.030095231423577 | -0.927684431389457 | 3.364583517520158 |
| Al | 0.957147295426328  | -0.822725988747338 | 3.301504349641242 |
| Al | 1.706140873519693  | -3.546986505372843 | 2.350556458590940 |
| Al | -1.264216336492762 | 1.210070745762733  | 3.510699222943860 |
| Al | -1.090097987250434 | -1.084364159178444 | 5.166510561147019 |
| Al | 2.977789421778112  | 1.252620339807279  | 3.186873086827909 |
| Al | 1.259749037422131  | -1.081030005614764 | 6.331651896411017 |
| Al | 2.702443820039777  | -2.525094163344062 | 4.598166327703350 |
| Al | 1.059126655869198  | 1.251821775444782  | 5.148542937103935 |

## 2.36 Al\_55

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|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | 3.934905779360893  | 0.597980439290851  | -1.175848085658066 |
| Al | 0.485549701030763  | 3.544033104524324  | -2.598609360316672 |
| Al | 2.984282696397269  | -1.398671094942120 | -3.013772034409740 |
| Al | -2.229205196184175 | 3.264799122894329  | -2.003944321153009 |
| Al | -3.262080084231968 | 1.258612096024649  | -0.569943366741201 |
| Al | 0.699666563993356  | 6.663705133418102  | 0.956115216168966  |
| Al | -0.716159126440626 | -2.277914513027532 | 4.103318526696153  |
| Al | 1.859585941248795  | 1.105305856342310  | -2.808562127471793 |
| Al | 4.642490424293483  | -2.082048779733134 | -1.044598187937712 |
| Al | 1.109824660180628  | -0.721328372982747 | -4.827774363209912 |
| Al | -2.444675497954455 | 3.370565211456231  | 0.960575659600122  |
| Al | -4.005064448095078 | -1.099084630908543 | -2.029680743288345 |
| Al | 2.524857101846438  | 4.592805271378740  | 1.342251322275853  |
| Al | -3.391725967034937 | -1.564371315722864 | -4.643734708725373 |
| Al | -1.535382258888053 | -0.042189556265836 | -5.951970548272983 |
| Al | 0.375391028666360  | 3.210947478862392  | 0.166879694291138  |
| Al | 3.666244945325611  | -3.915599535695276 | 1.088003590427054  |
| Al | 6.190094920721796  | -2.712132646566671 | 1.107961263917302  |
| Al | 1.655246422405781  | 0.962125157861353  | 0.459449402626355  |
| Al | 2.189588947021418  | -3.503587363894724 | -1.125096105933608 |
| Al | 5.262445756501714  | -0.043721150548812 | 1.097204031146976  |
| Al | 1.429073069745620  | 2.542605961031333  | 2.777146424827453  |
| Al | -4.597863800607556 | -0.808083227571180 | 0.610124966581102  |
| Al | 4.388948973862103  | -1.901678485447004 | 2.967905209393914  |
| Al | -0.383119123537044 | 1.731692658525375  | -4.282130284162642 |
| Al | -2.294343328367706 | 0.117779543536166  | 4.521592407359179  |
| Al | -3.624869974866860 | 1.320448332406512  | 2.256234007491615  |
| Al | -1.126140927241863 | 5.281000001501834  | -0.599805513205439 |
| Al | 0.077728236492888  | -1.464880470878926 | 1.398066077525682  |
| Al | -1.763004969262280 | -1.030898926549813 | -0.487463818405441 |
| Al | 1.718637072457675  | 5.482396589079972  | -1.252546665324566 |
| Al | -2.460445440802975 | -1.257820384780938 | 2.189505941965443  |
| Al | -4.786915995004590 | -0.776563102261122 | 3.579380778743186  |
| Al | -2.945735366491119 | 1.080051193158475  | -3.476812843614598 |
| Al | -1.327009332723417 | 2.448700672435852  | 3.557603726029349  |
| Al | 2.880692944515355  | 3.046466484035696  | -1.131104480102863 |
| Al | 0.349968758029297  | 0.223969311513212  | 3.629630646668321  |
| Al | -1.733056535270441 | -3.832124928533348 | 2.030936819300935  |
| Al | 0.999038600739809  | -4.028078699853318 | 1.571464886112764  |
| Al | -2.197372828008907 | -3.201691184041815 | -2.700190393891157 |
| Al | -1.060621700309801 | -0.805652115306437 | -3.200141273265366 |
| Al | -0.950510101996727 | -2.605464446103412 | -5.376359854925058 |
| Al | -0.513652853374305 | -3.476003564380932 | -0.579353779790752 |

|    |                    |                    |                    |
|----|--------------------|--------------------|--------------------|
| Al | -0.570757330153484 | 1.199193984519894  | -1.459091247734047 |
| Al | 4.036394012900624  | 2.349968791424930  | 1.286738501725916  |
| Al | -4.606788555316356 | -3.187583239024848 | 2.169049373975543  |
| Al | 3.032267982380112  | 0.415532974881657  | 2.967397995156974  |
| Al | -3.306688939956581 | -2.868495918060900 | 4.467187084470224  |
| Al | 0.906580582973914  | -1.038414381003273 | -1.264296779251540 |
| Al | -3.376357723384070 | -3.243234630528031 | -0.247904626729898 |
| Al | 1.823125476584949  | -2.099304590837788 | 3.403255036724147  |
| Al | 0.617918687560332  | -3.073368605422088 | -3.212091288926269 |
| Al | 2.734396430501761  | -1.383069037946578 | 0.770995886482332  |
| Al | -0.974834932559357 | 1.011985516102369  | 1.283506277437917  |
| Al | -0.390563379673910 | 4.620388012613418  | 2.343346047326131  |