

# Modeling the ternary chalcogenide $\text{Na}_2\text{MoSe}_4$ from first-principles\*

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In the ongoing pursuit of inorganic compounds suitable for solid-state devices, transition metal chalcogenides have received heightened attention due to their physical and chemical properties. Recently, alkali-ion transition metal chalcogenides have been explored as promising candidates to be applied in optoelectronics, photovoltaics and energy storage devices. In this work, we present a comprehensive theoretical study of sodium molybdenum selenide ( $\text{Na}_2\text{MoSe}_4$ ). First-principles computations were performed on a set of hypothetical crystal structures to determine the ground state and electronic properties of  $\text{Na}_2\text{MoSe}_4$ . We find that the equilibrium structure of  $\text{Na}_2\text{MoSe}_4$  is a simple orthorhombic (*oP*) lattice, with space group *Pnma*, as evidenced by thermodynamics. Electronic structure computations reveal that three phases are semiconducting, while one (*cF*) is metallic. Relativistic effects and Coulomb interaction of localized electrons were assessed for the *oP* phase, and found to have a negligible influence on the band structure. Finally, meta-GGA computations were performed to model the band structure of primitive orthorhombic  $\text{Na}_2\text{MoSe}_4$  at a predictive level. We employ the Tran-Blaha modified Becke-Johnson potential to demonstrate that *oP*  $\text{Na}_2\text{MoSe}_4$  is a semiconductor with a direct bandgap of 0.53 eV at the  $\Gamma$  point. Our results provide a foundation for future studies concerned with the modeling of inorganic and hybrid organic-inorganic materials chemically analogous to  $\text{Na}_2\text{MoSe}_4$ .

## I. INTRODUCTION

In recent years, a class of ternary chalcogenides known as alkali-metal(ion) transition-metal (and post transition metal) chalcogenides have presented a new landscape of opportunities in solid-state physics and chemistry. These materials, with general composition  $\text{AMX}_n$  where A is an alkali metal, can be obtained by facile solid-state reactions or by intercalation of a  $\text{A}^+$  in a  $\text{MX}_n$  lattice [1]. This, and the fact that they are well known ionic conductors, make them an attractive class of inorganic compounds [2, 3]. In this scene, Kanatzadis and coworkers have shone light on the fundamental chemistry of these compounds, discovering systems as different as semi- and superconductors [4, 5]. Regarding alkali metal transition selenides, particularly, we highlight the discovery of the layered metal  $\text{NaCu}_6\text{Se}_4$  with mixed valency [6], the mixed-valent two-dimensional metal  $\text{NaCu}_4\text{Se}_3$  [7] and the two-dimensional metal  $\text{NaCu}_4\text{Se}_4$ , which presents high hole mobility and giant magnetoresistance [8].

On another note,  $\text{AMX}_n$  semiconductors are also gaining traction. In 2018, Z. Xia *et al* [9] have discussed the chemistry of a new alkali-transition metal chalcogenide,  $\text{CsCu}_5\text{Se}_3$ . This caesium copper selenide, grown via the solvothermal method, is a pseudo-direct bandgap semiconductor with a bandgap of 1 eV. It crystallizes in the *oP* lattice, space group *Pnma* (No. 51). Authors also explored the complete family of materials by synthesizing and characterizing  $\text{CsCu}_5\text{S}_3$  and propos-

ing  $\text{CsCu}_5\text{Te}_3$ . Since then, an  $\alpha$  phase of  $\text{CsCu}_5\text{Se}_3$  has been achieved and proposed as a high-performance thermoelectric [10] and an *ab-initio* study treats the defect physics in  $\text{CsCu}_5\text{Se}_3$  and its potential in optoelectronics [11]. In a recent review on chalcogenides for photovoltaic applications,  $\text{CsCu}_5\text{Se}_3$  and other new chalcogenides are compared to their oxide analogs [12], to highlight opportunities for new materials.

Recent advances worth highlighting include the emergence of a new class of transition metal chalcogenide perovskites. The interplay between alkali-metal ( $\text{A}^+$ ) and MX chemistry promises solar-cell absorbers with electronic properties on a par with those of Hybrid perovskites [13–15]. One of the major advantages of TMC perovskites is high resistance to decomposition [16, 17], as well as their malleable compositional, structural and electrical and optical properties.

In this work, we aim to model  $\text{Na}_2\text{MoSe}_4$  (See Fig. S1), a molecule chemically analogous to the well known sodium molybdate ( $\text{Na}_2\text{MoO}_4$ ) [18], in the solid state. The following question arises: what will be the crystal structure of  $\text{Na}_2\text{MoSe}_4$ ? The answer is not obvious, as molecular  $\text{Na}_2\text{MoSe}_4$  is as related to  $\text{Na}_2\text{MoO}_4$  as it is to  $\text{Cs}_2\text{WS}_4$  [19]. Additionally, upon modeling  $\text{Na}_2\text{MoSe}_4$ , we will gain insights into the physics of its crystalline relatives. To this end, we employ a crystal structure prediction method combined with Kohn-Sham Density Functional Theory (DFT) to determine the ground state properties of this alkali metal transition metal chalcogenide. Our crystal structure prediction is based on the generation of hypothetical candidates through the ionic substitution of experimentally known analogs of Na-Mo-Se compounds. We demonstrate that  $\text{Na}_2\text{MoSe}_4$  is a direct bandgap semiconductor with simple orthorhombic

\* Supplementary information available

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(*oP*) structure, *Pnma* space group.

## II. METHODS

### A. Crystal Structure Prediction

To address the issue of structure determination, we followed the crystal structure prediction method of Hautier *et al* [20] based on data mined ionic substitutions. It has been demonstrated that the method of data mined ionic substitutions can generate likely crystal structures at a fraction of the computational cost of evolutionary algorithms [21], due to the fact that the substitutions are generated posterior to the analysis of existing crystal structures listed in the Inorganic Crystal Structure Database (ICSD). For the reader's convenience, we briefly outline the data mining + ionic substitution (DM+IS) methodology in the supplementary section.

Our DM+IS search generated over sixty-five hypothetical sodium molybdenum selenide structures, of which twenty-one present our hypothesized stoichiometry. The list of  $\text{Na}_2\text{MoSe}_4$  candidates is found in Table S1 along with other hypothetical Na-Mo-Se structures have been made public in the Materials Project Database [22] and can be downloaded directly from our repository [23]. Here, we focus our attention to the four candidates with highest likelihood of existing in their predicted space groups.

### B. Density Functional Theory Computations

Our first-principles computations based on Kohn-Sham Density Functional Theory (DFT) were carried out in the Quantum ESPRESSO package [24, 25]. We employ the General Gradient Approximation (GGA) functional with corrected-for-solids Perdew-Burke-Ernzerhof (PBEsol) parametrization for the exchange-correlation interactions [26]. It has been shown that the PBEsol is better suited to approximate lattice constants and surface energies when compared to PBE and LDA [27–29]. The ground state structures were determined by varying the volume isotropically, to control symmetry while fully relaxing atomic positions under a tolerance of  $13.605 \times 10^{-5}$  eV/atom for total energy and net forces of  $0.025 \text{ eV}\cdot\text{\AA}^{-1}$  per atom. These computations were performed using ultrasoft pseudopotentials (USP) developed by Dal Corso [30], with a plane-wave kinetic energy cutoff of 40 Ry (544 eV), charge density cutoff of 320 Ry (4,354 eV) and a convergence threshold of  $1 \times 10^{-8}$  eV for self-consistency. Long-range Grimme D2 Van der Waals interactions were included [31]. Monkhorst-Pack  $\Gamma$ -centered integration grids were used to sample the first Brillouin Zone [32]. For structural optimization computations of the *cF*, *oF*, *oP* and *mC* phases, integration grids of  $6 \times 6 \times 4$ ,  $8 \times 6 \times 5$ ,  $6 \times 8 \times 4$ ,  $5 \times 5 \times 4$  were used, respectively. To model the electronic structure we then

increased the density of the *k*-points integration grids to  $24 \times 28 \times 20$  (*cF*),  $32 \times 24 \times 20$  (*oF*),  $24 \times 32 \times 18$  (*oP*) and  $30 \times 30 \times 24$  (*mC*).

The General Gradient Approximation fails to predict the fundamental bandgap of semiconductors. Therefore, we then model the electronic band structure of the favored phase by performing meta-GGA (MGGA) calculations using the the semilocal Trahn-Blaha modified Becke-Johnson (TB-mBJ) exchange potential [33]. In order to obtain optimal results, our TB-mBJ computations were carried out in the framework of the Augmented Planewave plus Local Orbital (APW+lo) method as implemented in the all-electron code Wien2k[34].

## III. RESULTS

### A. Candidate structures

The structures considered in detail are: a face centered cubic (*cF*) phase with space group *Fd-3m* (227), a face centered orthorhombic (*oF*) with space group *Fddd* (70), a primitive orthorhombic (*oP*) with space group *Pnma* (62) and a base centered monoclinic (*mC*) with space group *C2/m* (12). The unit cells are sketched in Fig. 1 (a)-(d). The structural details for the evaluated systems are shown in Table I, which includes the raw (generated) DM+IS and the computed (equilibrium) parameters. From the obtained results it is clear that the equilibrium volume changes significantly with respect to the DM+IS volume, with the exception of the *oP* structure. For example,  $\Delta V^{oP} = 2\%$  versus  $\Delta V^{cF} = 26\%$ . Since we have unit cells with different number of atoms, to formally asses which of the candidates is the favored ground state phase of  $\text{Na}_2\text{MoSe}_4$ , we proceed to calculate the formation enthalpy  $\Delta H_{f(AMX)}^s$ . Thus, Eq. (1) reads

$$\Delta H_{f(AMX)}^s = U_{(AMX)}^s - \sum_i^3 N_i \mu_i^s, \quad (1)$$

where  $U_{(AMX)}^s$  is the total energy of our ternary compound,  $N_i$  is the number of atoms of a constituent *i* with chemical potential  $\mu_i$  in solid phase. The chemical potentials  $\mu$  for the constituent species were calculated based on DFT energies of their equilibrium crystal structures under the same relaxation criteria and convergence thresholds as the  $\text{Na}_2\text{MoSe}_4$  candidates. Additionally, analysis was cross-checked by fully relaxing all candidate structures at the PBE level. No significant changes on the lattice parameters were found. The values for  $\Delta H_{f(AMX)}^s$  are shown in Table II. Note that *oP* is the favored phase of the selenide with  $\Delta H_f^s$  values of -5.14 (PBEsol) and -5.11 (PBE) eV per formula unit (eV/f.u.). We find these relative formation enthalpies to be in good agreement, as the relative error between the PBE/PBEsol calculations is less than 2% for all phases with the exception of the least stable *cF*, in which a larger discrepancy is observed.

TABLE I. Predicted and optimized lattice parameters for the  $\text{Na}_2\text{MoSe}_4$  candidate structures computed at the GGA-(PBEsol) level of theory.

System	<b>a</b> (Å)	<b>b</b> (Å)	<b>c</b> (Å)	V (Å <sup>3</sup> )	$\alpha$	$\beta$	$\gamma$
<b>cF</b> : $Fd-3m$ (227)							
DM+IS	6.44	6.44	9.11	188.89	45	45	90
This work	8.14	8.14	11.51	238.76	45	45	90
<b>oF</b> : $Fddd$ (70)							
DM+IS	6.31	8.39	10.85	223.77	130	149	56
This work	7.71	10.24	13.26	273.38	130	149	56
<b>oP</b> : $Pnma$ (62)							
DM+IS	9.32	6.84	12.08	770.49	90	90	90
This work	9.12	6.69	11.81	753.92	90	90	90
<b>mC</b> : $C2/m$ (12)							
DM+IS	6.88	6.08	7.54	254.92	90	67	63
This work	7.78	6.88	8.52	368.58	90	67	63

TABLE II. Formation enthalpies  $\Delta H_f$  for  $\text{Na}_2\text{MoSe}_4$  candidate structures calculated using the GGA(PBEsol) and GGA(PBE) computed energies. Here,  $\Delta H_f$  is shown in eV per formula unit (eV/f.u.) and per atom (eV/atom).

System	$\Delta H_f$ (eV/f.u.)	$\Delta H_f$ (eV/atom)
<b>cF</b>		
PBEsol	-4.39	-0.63
PBE	-4.02	-0.57
<b>oF</b>		
PBEsol	-4.90	-0.70
PBE	-4.83	-0.69
<b>oP</b>		
PBEsol	-5.14	-0.73
PBE	-5.11	-0.73
<b>mC</b>		
PBEsol	-5.03	-0.72
PBE	-4.91	-0.70

This is reminiscent of the Cs perovskite  $\text{CsPbBr}_3$ , where through cooling, it undergoes a transition from a cubic to an orthorhombic lattice [35]. We encourage further studies to calculate the phonon band structure for the metastable phases, as we recognize the importance of these calculations to assess its dynamical stability. Here, our scope is to determine the most energetically stable structure for the already synthesized  $\text{Na}_2\text{MoSe}_4$  compound [36, 37], and calculate its electronic properties. Once the most stable phase is determined, we focus our attention only to the *oP* phase with  $Pnma$  space group. To further cross check the ground state of the  $Pnma$  candidate, we take the initial (DM+IS) structure and optimize it at the APW+lo/PBEsol level of theory.

The calculated lattice parameter is  $a = 9.21$  Å, which is in agreement with our pw(pseudo-potential) computed lattice constant  $a = 9.12$ , with an absolute error  $e_a = \leq 1\%$ . At  $V_0$ , the bulk modulus for  $\text{Na}_2\text{MoSe}_4$  is  $B_0 = 56.07$  GPa with a numerical derivative of  $B'_0 = 4.41$ .

Due to the fact that the hypothetical crystal structures were generated from chemical analogs, it is reasonable to infer that the true equilibrium structure of our material (*oP*) should result isostructural to other mate-

rials governed by same chemical principles (e.g. valency, electronegativity, position within the periodic table etc.). Furthermore, it is inferred that compounds governed by the same chemical principles as  $\text{Na}_2\text{MoSe}_4$  will also be favored in the *oP* lattice. This is in fact the reasoning and strength behind probabilistic models for crystal structure prediction. [38]. Therefore, the probability  $P$  of a material to exist in a specific crystal system (e.g. *oP* ( $Pnma$ )) will be influenced by the size of the known isostructural family[20, 38–40]. We match our evaluated structures to their analogs. First, we found our least stable candidate *cF*( $Fd - 3m$ ) to be isostructural to the stable phase of  $\text{Na}_2\text{MoO}_4$  [ICSD No. 44523]. The metastable phase of  $\text{Na}_2\text{MoO}_4$  [ICSD No. 151971] is characterized by the  $Fddd$  space group and isostructural to our *oF* candidate. The *mC* candidate is isostructural to the molecular crystal  $\text{K}_2\text{MoO}_4$  [ICSD No. 16154]. Lastly, our most stable *oP* phase is isostructural to (or based on)  $\text{K}_2\text{MoS}_4$  [ICSD No. 409563]. Notably, the following alkali-ion transition metal chalcogenides have been synthesized and are isostructural to  $\text{Na}_2\text{MoSe}_4$  with *oP*( $Pnma$ ) symmetry:  $\text{K}_2\text{MoS}_4$ ,  $\text{Cs}_2\text{MoSe}_4$ ,  $\text{Rb}_2\text{MoSe}_4$ ,  $\text{Cs}_2\text{MoS}_4$ ,  $\text{Rb}_2\text{MoS}_4$ ,  $\text{K}_2\text{WSe}_4$  and  $\text{Rb}_2\text{WS}_4$  [41–45] and could potentially be intrinsic semiconductors. Additionally, hybrid organic/inorganic members of the family  $\text{R}_2\text{MX}_4$  (e.g.  $\text{R} = \text{CH}_3\text{NH}_3$ ;  $\text{M} = \text{Mo}, \text{W}$ ;  $\text{X} = \text{S}, \text{Se}$ ) have also been reported to be stable in the *oP* phase [46, 47]. Note, throughout the  $\text{A}_2/\text{R}_2\text{MX}_4$  family,  $\text{X}$  is either S or Se. To the best of our knowledge, no theoretical studies of these materials are available in the literature. Given the electronic structure of  $\text{Na}_2\text{MoSe}_4$  and  $\tau_4$  (see section VI), it results of interest to the broader community to understand the physics and chemistry of the *oP*  $\text{A}/\text{R}_2\text{MX}_4$  compounds and their potential applications. Hence, we intend for our work to serve as a primer for future theoretical and experimental studies on the structural and electronic properties of both the inorganic and hybrid organic-inorganic analogs of  $\text{Na}_2\text{MoSe}_4$ . With respect to the question posed in the introduction; in the solid-state, the stable phase of  $\text{Na}_2\text{MoSe}_4$  is isostructural to  $\text{Cs}_2\text{WS}_4$  (*oP*) and not  $\text{Na}_2\text{MoO}_4$  (*cF*).

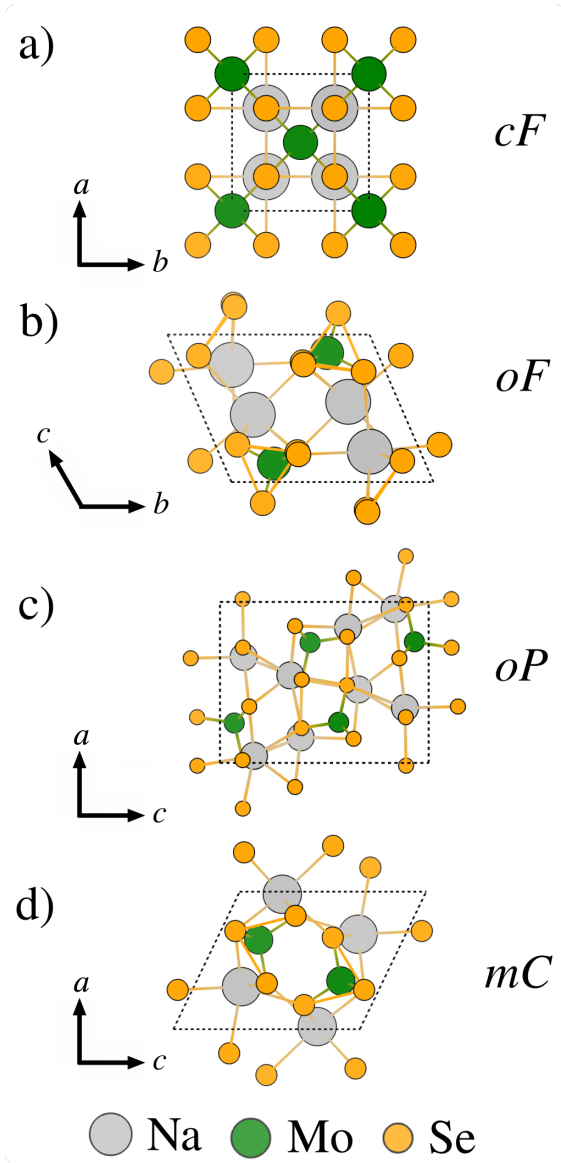


FIG. 1. Sketch of the candidate structure unit cells for  $\text{Na}_2\text{MoSe}_4$ . The candidates are labeled by their lattice type (a) face centered cubic  $cF$ , (b) face centered orthorhombic  $oF$ , (c) simple orthorhombic  $oP$  and (d) base centered monoclinic  $mC$ . Na, Mo and Se are depicted as grey, green and orange circles, respectively.

### B. Structure and symmetry of orthorhombic sodium molybdenum selenide

The disodium molybdenum tetraselenide, which can be written as  $\text{Na}_2[\text{MoSe}_4]$ , is a molecular crystal favored to grow in the simple orthorhombic ( $oP$ ) crystal system with space group  $Pnma$  (No. 62), as shown in Fig. 2. It has inversion symmetry, with eight symmetry operations. In Figure 2-a, an illustration of the frontal and isometric views are shown. The  $\text{Na}_2\text{MoSe}_4$  structure is three-dimensional, (i.e. there are no van der Waals gaps)

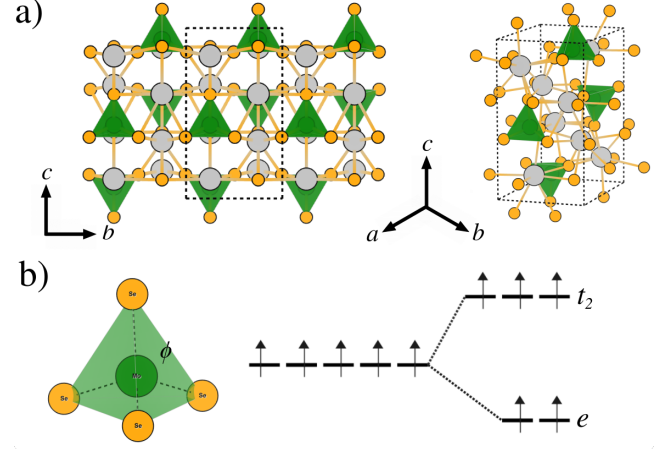


FIG. 2. Illustration of (a) isometric and front views of  $oP$  ( $Pnma$ ) phase of  $\text{Na}_2\text{MoSe}_4$  and (b) the  $[\text{MoSe}_4]^-$  tetrahedron with a schematic illustration of the tetrahedral crystal field that splits the  $\text{Mo } 4d^5$  orbitals into two groups, namely  $t_2$  and  $e$ . As seen in (a)  $\text{Na}^+$  ions are intercalated through the  $\text{MoSe}_4$  tetrahedral layers.

with two equivalent  $\text{Na}^{1+}$  sites. Note that  $\text{Mo}^{6+}$  is in tetrahedral coordination  $\tau_4$  (i.e.  $\tau_4 = 1$ ), bonded to four  $\text{Se}^{2-}$  atoms at each vertex. There is no Mo-Na bond, and the Na cations are intercalated throughout the lattice. This promotes the one-dimensional (directional) growth of the  $[\text{MoSe}_4]^-$  tetrahedrons. Hence,  $\text{MoSe}_4$ -terminated slabs can be achieved for Na conduction. Additionally, the electronic charge of Na induces a distortion in the tetrahedron (distortion index  $t'_4 = 0.006$ ) yielding a slightly elongated Mo-Se bond ( $d = 2.33 \text{ \AA}$ ) versus the other three ( $d = 2.30 \text{ \AA}$ ), and a broadening of Se-Mo-Se bond angle  $\phi$  from  $\phi = 109.55^\circ$  to  $\phi = 115^\circ$  as shown in Figure 2-b. The average Mo-Se bond length is  $\bar{l} = 2.31 \text{ \AA}$ . This distortion is caused by electronic charge effects of a  $\text{Na}^{1+}$  atom in proximity to one Se vertex ( $r = 2.98 \text{ \AA}$ ). The distortion can be also measured by the tetrahedron edges, conformed by Se-Se inter-atomic distances. The tetrahedron edge lengths are  $l = 3.778, 3.576$  and  $3.793 \text{ \AA}$  respectively. The omitted length values are redundant in nature. A spread of Na-Se bond distances can be found in the two Na sites, with values ranging between  $2.98 - 3.48 \text{ \AA}$ . In one  $\text{Na}^{1+}$  site,  $\text{Na}^{1+}$  is bonded in a 8-coordinate geometry to eight  $\text{Se}^{2-}$  atoms. In the other, our sodium cation is bonded in a 9-coordinate geometry to 9 Se anions. There are three nonequivalent  $\text{Se}^{2-}$  sites. In the first Se site,  $\text{Se}^{2-}$  is bonded in a single coordinate geometry to four  $\text{Na}^{1+}$  and one  $\text{Mo}^{6+}$  atom. In the second site,  $\text{Se}^{2-}$  is bonded in a 6-coordinate geometry to five  $\text{Na}^{1+}$  and one  $\text{Mo}^{6+}$  atom. In the third Se site,  $\text{Se}^{2-}$  is bonded in a 5-coordinate geometry to four  $\text{Na}^{1+}$  and one  $\text{Mo}^{6+}$  atom. Our structural analysis is in agreement with the available information for its reported analogs [41–45]. Note, the intercalation of Na atoms could favor an energetically low-cost substitution (or displacement) of cations, enabling the tunability of



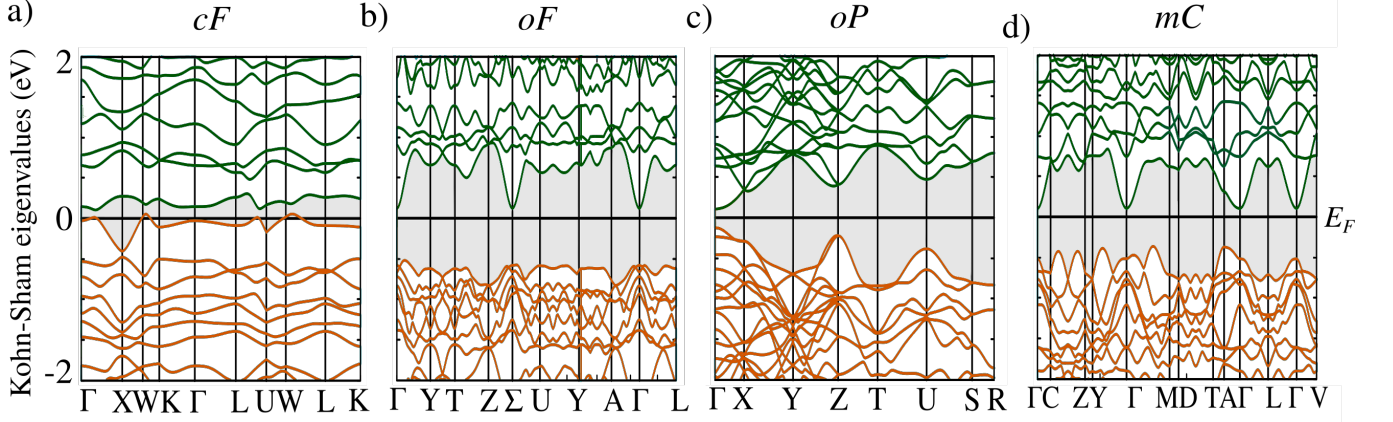


FIG. 3. The electronic band structure is shown for the (a)  $cF$ , (b)  $oF$ , (c)  $oP$  and (d)  $mC$  candidate phases of  $\text{Na}_2\text{MoSe}_4$ . The Fermi energy  $E_F$  is set to 0.

TABLE III. Band structure summary with  $E_g$  values at the GGA level of theory. To classify the phases, metals are identified by  $M$ , while  $D$  and  $I$  are employed to label direct and indirect semiconductors. The labels  $k_c$  and  $k_v$  correspond to the points at the CBM and CVM respectively.

System	$E_g^{GGA}$ (eV)	$E_g$ Type	$k_c - k_v$
$cF$	0.00	M	— — —
$oF$	0.62	D	$\Gamma - \Gamma$
$oP$	0.24	D	$\Gamma - \Gamma$
$mC$	0.45	I	$A - \Gamma$

these materials' properties. Lastly, due to the structure of  $\text{Na}^+[\text{MoSe}_4]^-$ , it can be safely inferred that the crystal is a typical ionic conductor [45].

### C. Electronic structure

The electronic structure of all candidates was investigated. Fig. 3 depicts the electronic band structure of the candidate phases of  $\text{Na}_2\text{MoSe}_4$ . Conventional  $k$  paths were used [48, 49]. In all plots, the Fermi energy ( $E_F$ ) is set to zero. From the band structure plots, it is clear that one of four phases is semi-metallic;  $cF$  (Figure 3(a)), where the valence band slightly crosses the Fermi level. The remaining three are intrinsic semiconductors. The  $oF$  phase is a direct bandgap semiconductor (Figure 3 (b)), with a bandgap  $E_g = 0.62$  eV at  $\Gamma$ , while the  $mC$  phase is an indirect bandgap ( $A - \Gamma$ ) semiconductor with  $E_g = 0.45$  eV (Figure 3(d)). Lastly, our GGA calculations show that  $oP$   $\text{Na}_2\text{MoSe}_4$ , the stable phase, has a fundamental bandgap of  $E_g^{GGA} = 0.24$  eV at  $\Gamma$  (Figure 3 (c)). A summary of the electronic properties computed at the GGA level is displayed in table III.

Since the  $oP$  structure is energetically favored, we focus only on this phase from here in and shall refer to this

phase simply as " $\text{Na}_2\text{MoSe}_4$ ".

In order to better understand the electronic band structure of  $\text{Na}_2\text{MoSe}_4$ , we adopt an alternative  $k$  path from the work of Xia *et al* [9] on  $oP$   $\text{CsCu}_5\text{Se}_3$ . Note that its band dispersion is highly isotropic with clear parabolic topology at the VBM and CBM around the  $\Gamma$  point. Moreover, as an ionic conductor [2], it can be considered as a mixed conductor.

At this point, we analyze the nature of the  $\text{Na}_2\text{MoSe}_4$  band structure. Let us recall that the Mo [ $d^5$ ] is in coordination  $\tau_4$  with Se [ $4s^24p^4$ ] atoms at each vertex. Given the electronic nature of our species, it is inferred that the valence bands that contribute to the  $E_F$  are composed of Se  $p$  orbitals, with mild hybridization with the Mo  $d$  orbitals. The  $s, p$  hybridization of the Mo-Se bond is common in high-spin  $\tau_4$  compounds (see Fig. 2). Therefore, it can be said that the electronic (semiconducting) properties of the material arise mainly from the  $[\text{MoSe}_4]$  sub-unit and that the effects of Na  $s, p$  electrons are negligible. However, although Na doesn't influence the band structure of the selenide, it is reasonable to believe that the displacement of Na throughout the lattice or its substitution by another ion (or molecule) could have an effect on the value of  $E_g$ . We further investigate the  $\tau_4$  sub-unit by treating the Se atoms as point charges around the Mo central ion, to study their bonding strength in terms of Coulombic interactions. In this sense, Crystal Field Theory (CFT) qualitatively predicts how the electrons in the Mo  $4d$  orbitals respond to the effective electrostatic potential imposed by its neighboring Se atoms. In  $\tau_4$ , the initial five-fold degeneracy of the  $4d$  orbitals will break. This will give rise to the the orbital groups  $t_2$  orbitals ( $d_{xy}, d_{yz}, d_{yz}$ ) and the  $e$  orbitals ( $d_{x^2-y^2}, d_{z^2}$ ). A schematic illustration of an idealized  $\tau_4$  CF is shown in the right hand side of Figure 2(b). The qualitative CFT analysis indicates that spin orbit coupling (SOC) effects in  $\text{Na}_2\text{MoSe}_4$  are weak [50].

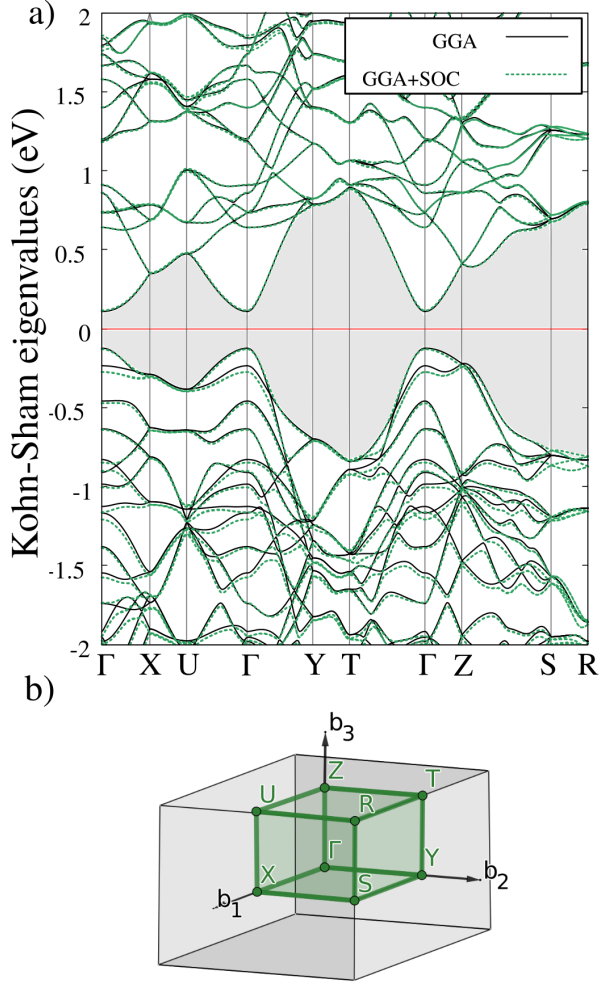


FIG. 4. Electronic band structure of *oP*  $\text{Na}_2\text{MoSe}_4$  is shown in (a) modeled at the GGA (black - solid) and GGA+SOC (green - dashed) levels. The irreducible Brillouin zone and  $k$ -path for the *oP* lattice is shown in (b).

We then perform full-relativistic calculations to model the band structure and the effects of the spin-orbit coupling term. To see effects of SOC effect on the band structure, we adopt an alternative path of integration along the FBZ. The band structure of  $\text{Na}_2\text{MoSe}_4$  with and without SOC  $\Gamma \rightarrow X \rightarrow U \rightarrow \Gamma \rightarrow Y \rightarrow T \rightarrow \Gamma \rightarrow Z \rightarrow S \rightarrow R$   $k$ -path (Figure 4). The FBZ is shown in Figure 4 (b). As expected, no change was observed in  $E_g$  upon calculating the band structure confirming the weak (essentially null) SOC effect. As we have seen,  $\text{Na}_2\text{MoSe}_4$  has relatives where the  $A^+$  cation is either Rb or Cs. Therefore, although SOC is weak in the Na member of the family, it is necessary to account for relativistic effects when modeling the band structure [35].

For completeness, we also plot the band structure by introducing an empirical Hubbard potential (GGA+ $U$ ;  $U = U_{eff} = 3.5$  eV [51]) to test for Coulomb effects in Mo  $d$  electrons (see Supplementary Information, Fig.

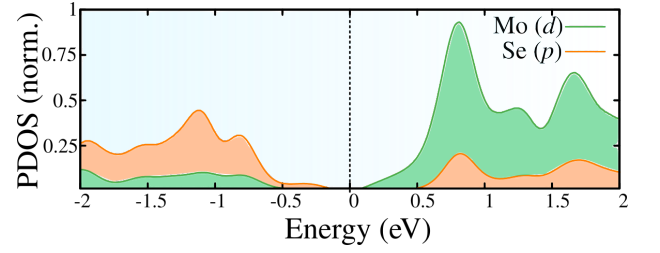


FIG. 5. Projected density of states is shown for the valence orbitals Mo and Se. Here,  $E_F$  is at 0, indicated by a dashed line.

S3.). No significant change was found as well, so there is not an apparent strong electronic correlation in the  $\text{Na}_2\text{MoSe}_4$  structure.

In the region,  $U \rightarrow \Gamma \rightarrow Y$ , the electron and hole effective masses  $m_h^*$  and  $m_e^*$  are expected to be constant, as the dispersion of  $E(k)$  near  $\Gamma$  is linear.

In Fig. 5, the projected DOS (PDOS) is shown for the Mo  $d$ , Se  $p$  and Na  $s$  electrons. Clearly the Na  $s$  states do not contribute near the Fermi level. The hybridized  $d/p$  orbitals; Mo  $d$  states (note  $0 \leq E_F \leq 2$  eV) and the Se  $p$  states are the ones dominating the contributions around the Fermi level. In other words, in the vicinity of  $E_F$ , the orbital composition of the conduction bands are Mo  $d$  while the valence bands are Se  $p$ . Note that higher PDOS is observed above the Fermi level, due to the unoccupied states in the shells. A decomposition of the summed Mo  $d$  orbitals can be found in the supplemental material.

Although ground state GGA computations can provide a reasonable first approximation to model the electronic structure of a material, they systemically fail to predict the fundamental value of  $E_g$ . Hence, a precise approach such as the use of meta-GGA, hybrid functionals or the many-body perturbation theory method  $GW$  is desired. To achieve a predictive level, we have opted for the Meta-GGA, Trahn-Blaha modified Becke-Johnson potential (TB-mBJ), designed and proven to yield robust results comparable to those obtained by other theoretical treatments and observed in experiments[33, 52–57]. In the present scenario, the use of TB-mBJ is considered adequate as no further corrections for correlation or spin-orbit coupling are needed (since the band topology and  $E_g$  do not change). Details regarding the TB-mBJ exchange potential are provided in the Supplementary Information file [33].

The band structure of *oP*  $\text{Na}_2\text{MoSe}_4$  calculated with the semilocal potential TB-mBJ is shown in Fig. 6. Note that the band topology is invariant and located at  $\Gamma$  (Figure 4), but a widening of the bandgap is observed. When modeling the electronic band structure of  $\text{Na}_2\text{MoSe}_4$  with the modified Becke-Johnson potential through an all-electron treatment,  $E_g^{TB-mBJ} = 0.53$  eV at  $\Gamma$ . The CBs are composed mainly of Mo  $d$  states whereas the Se  $p$  states, depicted as larger points, dominate the VB contributions, in agreement with the pDOS in Figure 5. It is important to point out that TB-mBJ semilocal po-

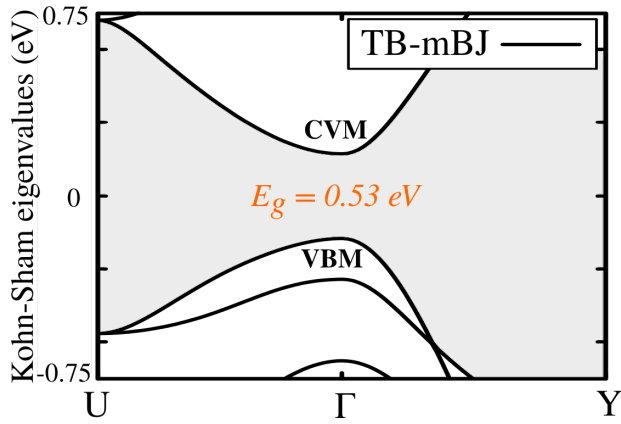


FIG. 6. Electronic band structure schematic of  $\text{Na}_2\text{MoSe}_4$  obtained by TB-mBJ, with emphasis along  $U \rightarrow \Gamma \rightarrow Y$  to better visualize the bandgap.  $E_F$  is set to zero. For the full band structure, see Fig. S4.

tential enables the modeling the electronic structures of large and chemically complex semiconducting materials as well as insulators with predictive accuracy [58, 59], and is currently considered the most accurate semilocal potential for semiconductor modeling [60, 61].

Although the bandgap isn't yet "ideal" for applications in traditional optoelectronic devices or photovoltaics,  $\text{Na}_2\text{MoSe}_4$  with an intrinsic direct bandgap of 0.53 eV holds potential for applications in infrared optoelectronics and high-speed electronic heterostructures and devices. Since the bandgap is tunable via ionic displacement and/or substitution, a natural interest in studying these effects throughout the lattice rises. More importantly, it will be interesting to build upon the work presented and investigate the interplay between chemical composition, thermodynamic stability and electronic structure in the inorganic and hybrid relatives of  $\text{Na}_2\text{MoSe}_4$ .

#### IV. CONCLUSIONS

In this study, we have presented a theoretical investigation on the alkali-ion transition metal chalcogenide  $\text{Na}_2\text{MoSe}_4$ .

First-principles computations based on periodic Density Functional Theory were systematically performed on a set of data-mined hypothetical crystal structures to determine their stability and model their electronic properties. The face centered cubic, face centered orthorhombic, simple orthorhombic and base centered monoclinic phases were studied in detail. The stability analysis reveals that the simple orthorhombic is the favored phase, with space group  $Pnma$ , therefore  $\text{Na}_2\text{MoSe}_4$  is isostructural to  $\text{Cs}_2\text{WS}_4$  and  $(\text{CH}_3\text{NH}_3)_2\text{MoS}_4$ . Additionally, the electronic band structure depicts semiconducting behaviour for three candidates, while the cubic phase is semi-metallic. The band structure of orthorhombic  $\text{Na}_2\text{MoSe}_4$  is modeled at the meta-GGA level of theory with the Trahn-Blaha modified Becke-Johnson exchange potential, yielding a direct fundamental bandgap of 0.53 eV at  $\Gamma$ . Thus, this bandgap makes  $\text{Na}_2\text{MoSe}_4$  suitable for applications in infrared optoelectronics and high speed electronics. Our analysis indicates that the qualitative physics of  $\text{Na}_2\text{MoSe}_4$  may be transferable to *oP* chemical analogs, and raises questions regarding the interplay between the composition, energetics and band structure of similar compounds. We hope our work will serve as a first step towards the understanding of the alkali-ion transition metal chalcogenides family as well as the hybrid organic-inorganic transition metal chalcogenide semiconductors.

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