Molecular Level Interpretation of Vibrational Spectra of Ordered Ice Phases

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Abstract

We build on results from our previous investigation into ice Ih using a combination of classical many-body molecular dynamics (MB-MD) and normal mode (NM) calculations to obtain molecular level information on the spectroscopic signatures in the OH stretching region for all seven of the known ordered crystalline ice phases. The classical MB-MD spectra are shown to capture the important spectral features by comparing with experimental Raman spectra. This motivates the use of the classical simulations in understanding the spectral features of the various ordered ice phases in molecular terms. This is achieved through NM analysis to first demonstrate that the MB-MD spectra can be well recovered through the transition dipole moments and polarizability tensors calculated from each NM. From the normal mode calculations, measures of the amount of symmetric and antisymmetric stretching are calculated for each ice, as well as an approximation of how localized each mode is. These metrics aid in viewing the ice phases on a continuous spectrum determined by their density. As in ice Ih, it is found that most of the other ordered ice phases have highly delocalized modes and their spectral features cannot, in general, be described in terms of molecular normal modes. The lone exception is ice VIII, the densest crystalline ice phase. Despite being found only at high pressure, the symmetry index shows a clear separation of symmetric and antisymmetric stretching modes giving rise to two distinct features.
Introduction

Ice is a common occurrence in our everyday lives, and whether found at the surface of a frozen lake, falling from the sky in the form of snow, or in an ice cube in a glass, the water molecules are always arranged in a hexagonal structure. This hexagonal ice, known as ice Ih, is the most common phase of ice found on the Earth’s surface and in its atmosphere as it is stable at ambient conditions. Under conditions of high pressure and/or low temperature, however, ice can transition to a number of other crystalline phases, each with their own unique structure. These conditions occur naturally only in extreme locations, such as inside glaciers in Antarctica, in the planet’s upper atmosphere, and on the icy moons and satellites across our solar system. Thus, having a molecular level understanding of the crystalline ice phases is important to a range of fields spanning terrestrial and oceanic, to atmospheric, planetary, and interstellar research.

Since the discovery of ice II over a century ago, when the naming scheme for future ice phases was first established, a total of 17 different crystalline ice phases have been identified. Each phase adheres to the Bernal-Fowler “ice rules” where for every oxygen atom there are two hydrogen bonds being accepted and two hydrogen bonds being donated, forming a tetrahedral hydrogen bond network. These phases can be divided into two main categories based on whether the hydrogens atoms of their water molecules exhibit long range order, referred to as either proton ordered or proton disordered phases. There are at present six known ordered/disordered ice pairs: XI/Ih, IX/III, XIII/V, XV/VI, VIII/VII, and XIV/XII. The remaining crystalline ices include the ordered ice II with no known proton disordered counterpart phase and ice Ic and ice IV as the only proton disordered phases currently missing their corresponding counterparts. Ice X is a special case and lone exception to the Bernal-Fowler rules, where increasing pressure on ice VIII or VII results in each hydrogen atom being shared evenly between two oxygen atoms. Within the past few years, two additional proton disordered ices have been formed through the process of emptying a type sII clathrate hydrate and a hydrogen-filled ice, resulting in ices XVI and XVII, respectively. Notably,
the different ice phases that meet with the phase boundary for liquid water are all proton disordered, showing a trend that with increased temperature and entropy effects, the ordered phases transform to their disordered counterparts before eventually transitioning to liquid.

Much research has been done to understand the structure of ice I\textsubscript{h} due to its common occurrence in nature and importance to many fields. The oxygen atoms of ice I\textsubscript{h} are arranged in the hexagonal ‘wurtzite’ crystal lattice\textsuperscript{14} with the hydrogen atoms bonded randomly to ensure the oxygen atoms satisfy the aforementioned Bernal-Fowler ice rules. The hydrogen atoms, therefore, have no long range order, resulting in an infinite number of possible lattice configurations for ice I\textsubscript{h}. By contrast, its proton ordered counterpart ice XI has a well-defined unit cell. While the crystal lattice structures for the oxygen atoms in both ice I\textsubscript{h} and ice XI are identical, the ice XI hydrogen atoms are in fixed positions. This proton ordering results in a finite number of phonon mode vibrations, which have been calculated for the unit cell through computational methods.\textsuperscript{15}

Vibrational spectroscopy\textsuperscript{16–31} and diffraction\textsuperscript{32–41} experiments have proven invaluable tools for understanding the complex molecular structures and hydrogen bonding networks of the various ice phases. The OH stretching region in particular, extending from 3000 cm\textsuperscript{-1} to 3800 cm\textsuperscript{-1}, provides a direct measure of the strength of the hydrogen bond network in an aqueous system. Experimental infrared\textsuperscript{16,17,19,24,29} and Raman\textsuperscript{18,20–23,25–28,30,31} spectroscopic studies have been used to characterize the ice phases for decades. As condensed phase computational methods have become more sophisticated, complimentary theoretical studies have also increasingly proven invaluable in both interpreting spectra and predicting structures and electrostatic properties.\textsuperscript{30,42–53}

Complicating matters is water being a network composed almost entirely of hydrogen bonds. This results in a general lack of damping effects on intermolecular coupling that would be induced by the presence of heavier molecules, creating difficulties in both experimental and computational structural interpretations. This delocalization and strong intermolecular coupling also suggests that a simple assignment of vibrational modes to individual spectral
peaks in water and ice is not appropriate. Recently, experimental and computational two dimensional IR (2DIR) spectroscopy studies have been used to disentangle some of the vibrational cross-couplings. The extra frequency dimension of 2DIR spectra provides information on the couplings and dynamics of different modes. However, interpretation of the spectra is far from trivial. With more sophisticated methods for modeling water, computational investigations can better aid in interpreting the one and two dimensional IR and Raman spectra beyond the limits of experiment. Recent computational investigations into the ice phases have focused on spectroscopic interpretation, energetics of phases and competition between proton order and disorder, the ferroelectricity (or lack thereof) of ice XI, phase transitions and volume isotope effects, phonon/normal mode calculations, and the pressure dependence of proton hopping.

It has recently been demonstrated that a rigorous representation of the water properties can be derived from many-body molecular dynamics (MB-MD) simulations performed with the MB-pol potential energy function. MB-pol represents a highly accurate molecular model of water and correctly predicts structural, thermodynamic, and dynamical properties as well as vibrational spectra of water from the gas to the condensed phase. Small energy differences between different ice phases demonstrate the importance of an accurate determination of the underlying potential energy surface for a quantitative assignment of the relative stability of the different ice phases. MB-pol has accurately reproduced the lattice energies for the ordered ice phases with known experimental comparisons, and provided predictions for the remaining ordered phases yet to be measured experimentally. In particular, MB-pol has shown excellent agreement with both ice Ih IR and Raman spectra and the energies of the ordered ice phases. In Ref. 52, it was argued that classical MD captures the same, though narrowed, vibrational features that quantum centroid molecular dynamics (CMD) simulations do for ice Ih. Owing to the excellent agreement between experiment and CMD simulations at 200 K, normal mode calculations were employed to further understand the spectral features.
Building off the work of Ref. 52, this study applies a similar MB-MD and normal mode treatment to the ordered ice phases. Classical MB-MD simulations in conjunction with normal mode calculations are used to gain a more complete molecular level understanding of the ordered ice phases and the observed spectral features in the OH stretching region. Ice Ih is used as a point of comparison for the disordered phases and the computational methods employed.

**Computational details**

All MD simulations were performed within the many-body molecular dynamics (MB-MD) formalism. System sizes differed between ice phases, with ices Ih, XI, and XV having 360 molecules, IX having 324 molecules, VIII having 384 molecules, ice XIV having 540 water molecules, ice II having 768 water molecules, and ice XIII having 756 water molecules. The simulations were carried out using in-house software based on the DL_POLY 2.0 MD software, modified to include the MB-pol PEF. The initial configurations were taken from previous trajectories carried out in the isobaric-isothermal (NPT) ensemble. The equations of motion were propagated using the velocity Verlet algorithm and the temperature was controlled via Nosé-Hoover chains of four thermostats coupled to each degree of freedom. A cutoff of 9 Å was used to calculate the short-range interactions, while the long-range electrostatic interactions were evaluated in reciprocal space using the Ewald summation technique. A time step of 0.2 fs was used in the MD simulations.

After 100 ps of equilibration in the canonical (NVT) ensemble, 5 independent trajectories for 100 K simulations and 1 trajectory for 10 K simulations of 100 ps each were simulated in the microcanonical (NVE) ensemble. From the classical MD NVE trajectories, classical spectra were calculated from the system dipole moment (for infrared spectra) or system polarizability tensor (for Raman spectra). The system dipole moments and polarizability tensors were calculated along the MD trajectories using the many-body MB-μ and MB-α
The vibrational spectra were calculated within the time-correlation function (TCF) formalism that relies on the following expressions. The IR activity is calculated as

$$I_{\text{IR}}(\omega) = \left[ \frac{2\omega}{3V\hbar c\epsilon_0} \right] \tanh (\beta \hbar \omega) \int_{-\infty}^{\infty} dt \: e^{-i\omega t} \langle \mu(0) \mu(t) \rangle$$  \hspace{1cm} (1)$$

with $V$ the system volume, $c$ the speed of light, $\epsilon_0$ the permittivity of free space, and $\beta = (kT)^{-1}$, with $k$ the Boltzmann constant. The unpolarized Raman activity is calculated as

$$I_{\text{Raman}}(\omega) = \frac{2\omega}{(\omega_I - \omega)^4} \tanh (\beta \hbar \omega) \int_{-\infty}^{\infty} dt \: e^{-i\omega t} \langle \alpha(0) \alpha(t) \rangle$$ \hspace{1cm} (2)$$

and can be decomposed into isotropic

$$I_{\text{iso}}(\omega) = \frac{2\omega}{(\omega_I - \omega)^4} \tanh (\beta \hbar \omega) \int_{-\infty}^{\infty} dt \: e^{-i\omega t} \langle \bar{\alpha}(0) \bar{\alpha}(t) \rangle$$ \hspace{1cm} (3)$$

and depolarized

$$I_{\text{dep}}(\omega) = \frac{2\omega}{(\omega_I - \omega)^4} \tanh (\beta \hbar \omega) \int_{-\infty}^{\infty} dt \: e^{-i\omega t} \langle \text{Tr} \left[ \beta(0) \beta(t) \right] \rangle$$ \hspace{1cm} (4)$$

contributions. Here, $\omega_I$ is the frequency of the incident laser (20,500 cm$^{-1}$, corresponding to a wavelength of 488 nm), $\bar{\alpha} = (\alpha_{xx} + \alpha_{yy} + \alpha_{zz})/3$ the isotropic component and $\beta$ the anisotropic component, respectively, of the polarizbaility tensor $\alpha = \bar{\alpha} I + \beta$, with $I$ the unit tensor.

The normal mode displacements and frequencies were obtained from the diagonalization of the Hessian matrix of the simulation box in periodic boundary conditions. For this analysis, frames were extracted from classical MD trajectories propagated at 10 K. Within the normal mode approximation, the IR and Raman spectra were then calculated from
numerical derivatives of the dipole moment and polarizability along each normal mode, respectively, as approximations to the transition dipole moment and polarizability tensor. Within the normal mode approximation, the transition dipole moment is expressed as

$$\left| \frac{\partial \mu}{\partial q} \right| \approx \frac{1}{\Delta q} \left| \mu_{\text{init}} - \mu_{\text{disp}} \right| \propto \sqrt{(\mu_{x,\text{int}} - \mu_{x,\text{disp}})^2 + (\mu_{y,\text{int}} - \mu_{y,\text{disp}})^2 + (\mu_{z,\text{int}} - \mu_{z,\text{disp}})^2}. \quad (5)$$

The transition isotropic polarizability tensor is expressed as

$$\left| \frac{\partial \bar{\alpha}}{\partial q} \right| \approx \frac{1}{\Delta q} \left| \bar{\alpha}_{\text{init}} - \bar{\alpha}_{\text{disp}} \right| \quad (6)$$

and the transition depolarized polarizability tensor is expressed as

$$\left| \frac{\partial \text{Tr}[\beta \cdot \beta]}{\partial q} \right| \approx \frac{1}{\Delta q} \left| \text{Tr}[\beta_{\text{init}} \cdot \beta_{\text{init}}] - \text{Tr}[\beta_{\text{disp}} \cdot \beta_{\text{disp}}] \right|. \quad (7)$$

These electrostatic transitions for each mode were then broadened with Gaussians with standard deviations of $10 \text{ cm}^{-1}$, providing approximate IR and Raman spectra.

In the same vein as Ref. [43] and following from Ref. [52], we define here a symmetric and antisymmetric index,

$$S_k = \frac{1}{N} \sum_{i=1}^{N} \left| \left( \Delta ||OH_{1}^{(i)}|| + \Delta ||OH_{2}^{(i)}|| \right) \right| \quad (8)$$

$$A_k = \frac{1}{N} \sum_{i=1}^{N} \left| \left( \Delta ||OH_{1}^{(i)}|| - \Delta ||OH_{2}^{(i)}|| \right) \right|. \quad (9)$$

Here $||OH_{j}^{(i)}||$ is the $j$th OH bond length ($j = 1, 2$) of the $i$th water molecule and $\Delta ||OH_{1}^{(i)}|| = ||OH_{1}^{(i)}||_{\text{eq}} - ||OH_{1}^{(i)}||_{\Delta q}$ is the change in OH bond length between the optimized equilibrium configuration and the normal mode configuration. For a mode where all molecules undergo symmetric stretching of equal displacement $d_{\text{max}}$, $S_k \to 2d_{\text{max}}$ and $A_k \to 0$. For simplicity, a single normalized stretching index $F_{k}^{S}$ will be defined as the fraction of symmetric stretching.
to the total symmetric and antisymmetric stretching,

\[ F^S_k = \frac{S_k}{S_k + A_k}. \]  

(10)

This index ranges from 0 to 1, where 1 should be interpreted as the entirety of the OH stretching is symmetric and a value of 0 indicates the OH stretching is entirely antisymmetric. Of course, a complimentary index for the antisymmetric stretching could also be defined as

\[ F^A_k = 1 - F^S_k. \]

In addition to the symmetry index, a localization index \( m_k \) is defined as

\[ m_k = \frac{\text{Max} \left( D_i \right)}{\sum_{i=1}^{N} D_i} \]  

(11)

where \( D_i = \sum_{j=x,y,z} \left| d_j^{(i)} \right|^2 \) and \( d_j^{(i)} \) is the displacement of the oxygen atom of the \( i \)th water molecule along the \( j \)th Cartesian axis in the \( k \)th normal mode. For a normal mode localized on a single water molecule, \( m_k \rightarrow 1 \), while \( m_k \rightarrow 1/N \) for a normal mode equally delocalized over all water molecules.

**Results and Discussion**

**Structure**

The proton ordered phases of ice form at different pressure regimes, with ice XI found at the lowest pressure conditions (below \( \sim 0.25 \) GPa), ices II, IX, and XIII existing between \( \sim 0.25 \) and 1 GPa, and ices VIII, XIV, and XV being found above 1 GPa.\(^{11,31}\) Their densities also increase in unison with the pressure. Oxygen-oxygen radial distribution functions (RDFs) of ice \( I_h \) and all ordered ice phases are presented in Figure 1, calculated from 100 ps NVT simulations at 100 K. Ice \( I_h \) has a characteristic hexagonal structure with water molecules hydrogen bonded to one another and arranged in six-membered rings.\(^{11}\) The basic structure
Figure 1: O-O radial distribution functions of ice phases calculated from 100 ps NVT trajectories at 100 K.

is a series of alternating layers composed of chair-form and boat-form hexamers. For comparison, cubic ice (I\textsubscript{c}) shares the same local hexagonal oxygen configuration as ice I\textsubscript{h}, but consists of entirely chair-form hexamers resulting in a different packing geometry. However, due to sharing the same local structure (their RDFs are identical up to \( \sim 4 \) Å) and having long-range proton disorder, ice I\textsubscript{h} and I\textsubscript{c} display essentially identical infrared and Raman spectra.\textsuperscript{11,16,18} While ice XI is the ordered form of ice I\textsubscript{h} and therefore displays the same underlying hexagonal oxygen configuration, its first two peaks in the O-O RDFs are located at shorter radii than that of ice I\textsubscript{h}.

The lattice configuration of ice II is comprised of two hydrogen bonded hexamers, one chair-form and the other nearly flat. These hexamers form parallel columns along the c-axis which are spanned by eight- and ten-membered rings.\textsuperscript{8,55} It is the only ice with no known disordered counterpart. Ice IX is a metastable phase and the ordered form of ice III, with an oxygen lattice structure containing five-membered rings joined together by four-membered rings.\textsuperscript{8,55} Ice II and ice XIII also show similar RDFs, despite not sharing an underlying oxy-
gen configuration directly, with a similar first nearest neighbor position (2.787 vs. 2.785 Å, respectively), as well as a more uniform structure at farther distances. The structures of ices II, IX, and XIII are closely linked, however. Rapid cooling of ice III yields ice IX, as expected, though slower cooling results in ice II. However, ice II transforms only to the disordered form of ice XIII, ice V upon heating. Furthermore, both ice II and ice XIII display unique distortions to their local tetrahedral hydrogen bonded networks with the root mean squared deviation from the ideal tetrahedral bond angle of ice II and ice XIII being 16.8° and 18.5° respectively. This similarity in distortion angles suggests that both ice II and ice XIII experience comparable local hydrogen bonding environments despite having different long range structures.

The remaining RDFs belong to the high pressure ices VIII, XIV, and XV, with densities also correspondingly higher. Ice VIII and ice XV achieve their high densities through their unique interlocking but non-interconnected lattices. Ice VIII is composed of interlaced Ic lattices, while ice XV can be thought of as consisting of interlocking hexamer units sharing the “cage-like” gas phase water cluster. Ice XIV has the highest density of ices not formed by interlocking lattices. Its structure (along with its proton disordered counterpart, ice XII) is unique among the crystalline ices, consisting of a double-helix motif resulting in seven-membered or larger rings. These three high pressure phases, particularly ice VIII, have counter-intuitively larger radii for their first nearest-neighbors than the lower pressure phases. However, the distances to the second nearest-neighbor oxygens are less than in other phases, resulting in overall denser structures.

**Vibrational spectra**

Vibrational spectroscopy has long been a useful tool to investigate the structure of water and ice. In particular, the OH stretching region, extending from roughly 3000 to 4000 cm⁻¹, provides detailed, though at times difficult to interpret, information about the hydrogen bonding environment, as the stretching modes are direct probes of the hydrogen bond strength in
Figure 2: Unpolarized Raman. Experimental data for ice I$_h$, XI, IX, XIII, II, XIV, XV, and VIII extracted from the indicated studies. The experimental ice XIII and XIV spectra were measured for systems with 9.0 mol% HOD in H$_2$O. MB-MD spectra calculated from classical MD simulations have been redshifted for comparison with experimental spectra.

In general, the MB-pol/MB-α spectra show excellent agreement with the experimental results. Note that the MB-MD spectra have been redshifted to align the most intense peaks...
to account for the lack of nuclear quantum effects in the classical dynamics. This redshift is only an approximate and ad hoc method and, for some ices such as ice IX, leads to poor alignment for the other features. Overall, Ice VIII displays the poorest agreement, mainly in relative peak intensity of the two main features. While the location of the two major peaks are shifted relative to experiment, MB-MD does capture the correct number of features. Perfectly matching experimental and simulation conditions is also difficult, and ice VIII has been shown to be strongly influenced by pressure and temperature. It should also be noted that MB-pol performs less well for ice VIII when comparing the lattice energies of the ordered ice phases with experimental measurements, largely due to neglecting the lattice zero point energy. As the simulations performed in this study were classical and therefore neglected nuclear quantum effects such as zero point energy, of particular importance for high density ices such as ice VIII, this may also explain the poorer agreement of the relative peak intensities observed in Figure 2 for ice VIII between experimental data and MB-MD simulations.

As discussed in Ref. 52, the lack of nuclear quantum effects can lead to narrower individual peaks and smaller overall spectral ranges as a result of the steeper potential energy surface experienced by the system. The majority of the MB-MD spectra displayed in Figure 2, however, appear to show minimal narrowing of spectral peaks when compared with experimental spectra. This is likely due to the low temperatures the experiments and simulations were performed at, resulting in less thermal broadening effects overall that need to be captured by the classical simulations. Another possible explanation is the proton ordering yielding less inhomogeneous broadening, though this was shown to not be a contributing factor in ice XI.

The general agreement of MB-MD with experimental Raman spectra, as well as ice Ih being shown previously to match the experimental IR spectrum, suggests MB-MD can satisfactorily capture the important spectral features of the various ordered ice phases. To better understand the molecular level details of the spectra, normal mode (NM) calculations
**Figure 3:** Shown for each ice, from top to bottom, are the MB-MD (solid) and normal mode (dashed) derived infrared, isotropic Raman, and depolarized Raman spectra.
are employed. As explored in Ref. 52, the obtained frequencies can be combined with approximations to the transition dipole moments and transition isotropic and anisotropic polarizability tensors. Together, these yield effective IR and Raman spectra after artificial broadening with Gaussian functions. The agreement between the MB-MD spectra and NM spectra provides a measure of the validity of the NM treatment, allowing for further analysis to provide physically meaningful interpretations. More details on this process are provided in the Supporting Information as well as in Ref. 52.

Normal mode calculations have been previously performed on primitive cells of some ordered ice phases containing $2^{66}$ or 4 molecules. While these phonon mode studies allow for precise assigning of peaks in terms of molecular motions, the effects of delocalization cannot be properly investigated with single unit cells. An earlier study on ice I$_h$ using a model Hamiltonian also introduced a similar symmetry index to Eq. 10 referred to therein as the fractional symmetric character. Both Refs. 43 and 52 claim the interpretation of the ice I$_h$ spectrum in terms of molecular vibrational modes is not appropriate. Instead of the spectral peaks being assignable to well-defined symmetric and antisymmetric stretch modes, at best the peaks generally have only a majority of one stretch motion in a given region, while other regions contain a mixture of symmetric and antisymmetric stretching. The method of analyzing the normal mode calculations of ice I$_h$ described in Ref. 52 is here applied to the ordered ice phases. Figure 3 shows the calculated infrared, isotropic Raman, and depolarized Raman spectra from both MB-MD and NM analysis. For every ice phase, the spectra display excellent agreement between the MB-MD and NM spectra, allowing for some small discrepancies due to the artificial line broadening through Gaussian functions. Key features to note are the large variances in frequency ranges spanned by the different phases, with ice XI/I$_h$ being the most redshifted spectra, and ice VIII the most blueshifted. Note that, unlike Figure 2 the spectra in Figure 3 have not been redshifted to account for the lack of nuclear quantum effects.

It must be noted that the isotropic Raman spectra shown in Figure 3 for ices XI, IX, XV,
and VIII were modified by removing single modes with artificially strong isotropic peaks. These strong intensities are a result of the highly symmetric nature of the ordered phases and the resulting normal mode approach overestimates the isotropic signal produced from some modes with a high degree of symmetric stretching, which the isotropic Raman signal is sensitive to. These single large intensity modes lead to poor agreement with the MB-MD spectra for the ice phases where they appear. Comparing ice Ih and XI, the disorder of ice Ih dampens the effect of the concerted stretching motion seen in the ordered ice XI. The spectra before removing the artificially large isotropic modes are shown in the Supporting Information.

Figure 4: Correlation between the peak position of the first nearest neighbors in the O-O RDF at 10 K and the median NM frequency for each ice phase. Also shown is the linear fit obtained from the data.

As discussed previously, ices VIII, XIV, and XV are high pressure and high density ices, though their first oxygen-oxygen nearest neighbors are in fact located at farther distances
than the lower pressure ices, as shown in the O-O RDFs of Figure 1. This is explained by the fact that their second nearest-neighbors are located at much closer distances than the lower density ices, yielding an overall higher density. Figure 4 shows the correlation between the position of the first nearest-neighbor peak of the O-O RDFs at 10 K (RDFs at 10 K are provided in the Supporting Information) and the median frequency obtained from the normal mode calculations for the various ice phases. Also shown is the linear fit to the data. The ice phases follow a general trend of larger spacing between neighboring water molecules leading to a higher frequency OH stretching region. This suggests the first coordination shell predominantly affects the frequency shift in the OH stretching region, with the second nearest neighbors having little influence. This follows from the stronger hydrogen bond formed as the water molecules are forced closer together. Also apparent is that of the ordered ice phases, ices XI and VIII are outliers, found at either extreme of the observed trend, with the remaining ices clustered in a more central group. The trend can be understood as, if taken to the extreme case of large $r_{O-O}$, approaching a gas phase water molecule with neighboring water molecules at large distances. The median stretching frequency would approach that of a gas phase water molecule.

**Indexes**

The normal mode analysis also allows for molecular level detail of the ice spectra with each mode examined individually. This is accomplished here through the categorization of each mode through indexes, namely the degree of symmetric/antisymmetric stretching present in a mode and the amount of displacement caused by the stretching motion as a measure of localization. Figure 5 shows the normalized symmetric stretching parameter described in Eq. 10. For each mode, the index ranges between 0 and 1, with 0 describing a system where all stretches are antisymmetric, 1 where all stretches are symmetric, and transitional values referring to a mixture of both symmetric and antisymmetric stretching. There is a clear trend of lower frequencies having mainly symmetric stretching character, logically
Figure 5: Symmetry index calculated for each mode in each ice ordered phase and ice I$_h$. Shown is the ratio of symmetric stretching to total stretching character of a given mode, $F^S_k$, as described in Eq. 10.

corresponding to regions where isotropic Raman signals tend to be strongest. Shifting to higher frequency yields a mixture of both symmetric and antisymmetric stretching for most ices, with the ratio eventually leveling off and remaining roughly equally divided between symmetric and antisymmetric.

Ice I$_h$/XI deviates somewhat at their highest frequency values, with regions of high symmetry appearing again around 3600-3650 cm$^{-1}$. The relations between the spectra and indexes for ice I$_h$ were discussed in Ref. 52, and ice XI follows similar trends. They both display weak IR signals in this region, whereas the other ice phases besides ice VIII have generally retain roughly the same IR intensity over their entire range. Experimental spectra$^{20,20}$
show little to no difference in peak width between ice $I_h$ and XI for both IR and Raman spectra in the OH stretching region. While both this study and a previous computational study of ice $I_h$ and XI show a moderate decrease in peak width due to proton ordering in the OH stretching region, it is minimal compared to what would be expected for a hydrogen ordered ice. This suggests delocalized coupling between modes, not proton disorder, is the main cause of the broad linewidth observed in both ice $I_h$ and XI.

![Localization Index](image)

**Figure 6:** Localization index, as described in Eq. 11, calculated for each mode in each ice ordered phase and ice $I_h$.

Figure 6 shows the localization index described in Eq. 11 calculated for each normal mode of each ice phase. This index quantifies the number of molecules participating in a vibrational mode by totaling their displacements from the equilibrium position in each cartesian direction. In general, the ordered phases are overall highly delocalized, and more
delocalized than the disordered ice $I_h$, though only in narrow regions does ice $I_h$ possess highly localized modes. For a direct comparison, ice $I_h$ has one highly localized region near $3490$ cm$^{-1}$, in the region of largest IR activity\cite{52}. This region contains the largest value of $m_k$ across ice $I_h$ and all the disordered phases of 0.25, which can be interpreted as $\sim 4$ water molecules participating in the mode. In the corresponding region of strongest IR activity for ice XI around $3460$ cm$^{-1}$, the largest value of $m_k$ is only 0.08, which can be interpreted as $\sim 12$ molecules participating in the mode. Elsewhere, however, ice XI shows a similar degree of localization as ice $I_h$. As discussed, Ref.\cite{90} measured the spectra of ice $I_h$ and XI to have similar lineshapes, both in terms of peak position and width, despite the ordered ice XI theoretically having less inhomogeneous broadening and, therefore, narrower spectral peaks. They proposed this to be due to delocalization and intermolecular coupling being strong in both systems. Other than the small region near $3490$ cm$^{-1}$, the localization index values in Figure 6 are similar for ice $I_h$ and ice XI, supporting the notion that both have mostly delocalized modes.

Visualizations of selected modes are provided in Figure 7. Blue spheres represent water molecules with predominantly symmetric stretching ($F^S_k > 0.5$) and red spheres represent water molecules with $F^S_k < 0.5$, or antisymmetric stretching. The opacity of a sphere is proportional to the localization index normalized to the largest value of $m_k$ across all ice phases, which as mentioned is 0.25 in the strong IR region of ice $I_h$. Shown in the left column of Figure 7 are the most symmetric modes in ice $I_h$, ice XI, and ice VIII. These all correspond to the regions with the most intense peak in the isotropic Raman spectra in Figure 3. The right column of Figure 7 showcase modes with the most intense IR signal.

The visualizations allow for comparison between the different ice phases. The strongest isotropic modes of ices $I_h$ and XI, which are also the most symmetric modes, are not equally delocalized. The $3396$ cm$^{-1}$ ice $I_h$ mode has a darker region on the right side of the lattice, signifying a slightly more localized mode than the $3388$ cm$^{-1}$ mode of ice XI. This can be attributed to the disordered ice $I_h$ lattice configuration, where variation in local environments
Figure 7: Visualizations of selected normal modes for ices $I_h$, XI, and VIII. Blue spheres represent water molecules with predominantly symmetric stretching ($F^S_k > 0.5$) and red spheres represent water molecules with $F^S_k < 0.5$, or antisymmetric stretching. The opacity of a sphere is proportional to the localization index normalized to the largest value of $m_k$ across all ice phases. Left column: the mode for each ice with the largest isotropic signal. Right column: the mode for each ice with the largest infrared signal.

due to proton disorder limits the extent of delocalization of the mode across the lattice. The strong infrared modes in the right column of Figure 7 display much wider discrepancies in localization, with a small localized region of motion in ice $I_h$ clearly visible. In contrast, ice XI, being an ordered phase, possesses a more periodic lattice allowing for greater delocalization.
of vibrational modes and intermolecular coupling of modes across the lattice. However, for most modes, this increase in delocalization is negligible, only affecting the localized modes with strong infrared intensity.

In general, a straightforward assignment of the infrared and Raman spectral peaks for the ordered ice phases based on well-defined regions of symmetry and antisymmetry proves to be impossible. The regions can rarely be described in terms of only symmetric or antisymmetric stretching due to the delocalization and intermolecular coupling of vibrational modes. The one exception to this is ice VIII, which possesses two distinct regions with predominantly symmetric (3500-3650 cm\(^{-1}\)) and antisymmetric (3700-3750 cm\(^{-1}\)) \(F^S_k\) values, examples of which are depicted in Figure 7. Both the symmetric 3538 cm\(^{-1}\) mode and antisymmetric 3740 cm\(^{-1}\) mode are almost evenly delocalized across the lattice, a result of the unique lattice configuration of ice VIII. This suggests that ice VIII alone can be reasonably described by the molecular normal modes of water. In other words, the two main spectral features, at low and high frequency, seen in the Raman and infrared spectra of Figures 2 and 3 can be confidently assigned to be symmetric stretching and antisymmetric stretching modes, respectively. This is in contrast to ice I\(_h\), as discussed in Ref. 52 as well as the other ordered ice phases. The large spacing between neighboring oxygens, as demonstrated in Figures 1 and 4, may explain this, as ice VIII was also an outlier in this regard. The water molecules in ice VIII, despite being denser overall, have more space between their closest neighbors.

**Conclusions**

Simulated IR and Raman spectra were calculated for the proton ordered ice phases through many-body molecular dynamics combined with many-body electrostatic surfaces. MB-MD Raman spectra had good agreement when compared to experimental Raman spectra in Figure 2. With these results providing credence to the use of classical mechanics with MB-pol for capturing ice spectral features, in addition to previous studies showing MB-
pol accurately capturing the lattice energies of the ordered ice phases\textsuperscript{53} and the classical vibrational spectra of ice I\textsubscript{h}\textsuperscript{52} normal mode analyses were then carried out for each ordered ice phase to gain deeper insight into the origin of the observed MB-MD spectral features. From the normal mode calculations, symmetry and localization indexes were calculated for each vibrational mode, a continuation of the method detailed in Ref. \textsuperscript{52}. These provide molecular level explanations for the vibrational spectra of the ordered ice phases and their underlying structural relationships.

There is a clear trend between the closest oxygen-oxygen distance for each ice phase and the median frequency as calculated through normal mode analysis, shown in Figure 4, with larger O-O distances yielding higher median frequencies. As the water molecules in the ices have more space in their immediate surroundings, the hydrogen bonding is weakened, leading to stronger OH bonds and larger stretching frequencies. The ice phases also can be considered occupying three distinct groups in the correlation plot, with a main group comprised of ices II, IX, XIII, XIV, and XV, and two remaining groups composed of single ices, each being outliers on either extreme of the main group. The symmetry and localization indexes of Figure 5 support previous work on ice I\textsubscript{h} and XI that their similar spectral signatures are due to high delocalization of modes in both ices. While most of the ices display a mixture of symmetric and antisymmetric stretching across all modes, preventing assignment of features in terms of the traditional molecular modes of water, ice VIII is unique, having well-defined regions of symmetry and antisymmetry.

This study investigated only the OH stretching region as a probe of the hydrogen bonding networks of the ice phases. However, the low frequency librational region provides valuable information on the lattice structure, and the same normal mode analysis detailed here could well be applied to this and other vibrational regions. Furthermore, this study used only classical molecular dynamics methods with MB-pol. Testing MB-pol with a more robust quantum dynamics method could improve the agreement with experimental spectra, particularly the high density ice VIII.
Supporting Information

Included in the Supporting Information are radial distribution functions calculated at 10 K and isotropic Raman spectra that include the unrealistically intense low frequency mode for ices XI, IX, XV, and VIII.

Acknowledgements

This research was supported by the National Science Foundation through grant no. CHE-1453204 and used resources of the Extreme Science and Engineering Discovery Environment (XSEDE)\textsuperscript{[1]} which is supported by the National Science Foundation through grant no. ACI-1053575, under allocation TG-CHE110009 on Comet at the San Diego Supercomputer Center and Stampede2 at the Texas Advanced Computing Center.
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TOC Graphics