Functional group resolved nuclear spin relaxation in porous media

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Abstract

Understanding solid-fluid interactions within porous materials is critical for their efficient utilisation across chemical reaction and separation processes. However, detailed characterisation of interfacial phenomena within such systems is hampered by their optically opaque nature. Motivated by the need to bridge this capability gap, we detail here the application of low magnetic field 2D ^{1}H nuclear spin relaxation measurements as a non-invasive probe of sorbate/sorbent interactions, exploring the relaxation characteristics exhibited by liquid adsorbates confined to a model mesoporous silica. For the first time, we demonstrate the capacity of such measurements to distinguish functional group-specific relaxation phenomena across a diverse range of protic adsorbates of wide importance as solvents, reagents, and hydrogen carriers, with distinct relaxation environments assigned to the alkyl and hydroxyl moieties of the confined liquids. Uniquely, this relaxation behaviour is shown to correlate with adsorbate acidity, with the observed relationship rationalised on the basis of surface-adsorbate proton exchange dynamics.

Introduction

Characterisation of the solid-liquid interface is critical for the rational development of technologies across the medical, corrosion, environmental and energy sciences.[1] However, spectroscopic interrogation of the chemical and physical nature of such interfaces remains severely impeded by the need to differentiate the species of interest from the surrounding solid and liquid components. In the case of porous solids – essential to diverse industrial processes such as separations, catalysis, and energy storage – straightforward characterisation of interfacial phenomena is further hindered by the heterogeneous and optically opaque nature of the materials employed, which in turn preclude spectroscopic approaches which necessitate unimpeded access to well-defined surfaces, or require thin adsorbate films.[2]

Nuclear spin relaxation measurements have emerged as a versatile, chemically selective, and non-invasive route for the characterisation of such systems.[3,4] Utilising appropriate nuclear magnetic resonance (NMR) pulse sequences and hardware this experimental approach measures the longitudinal and/or transverse relaxation behaviour of the spin system under study, as characterised by the time constants T_1 and T_2, respectively. While these time constants conform to well-defined relationships with the translational and rotational dynamics of spin-bearing molecules in the unrestricted bulk liquid phase,[5] the restricted dynamics of fluids within porous media often results in complex relaxation behaviour associated with surface interactions and confinement effects, providing potential insight into both material characteristics and interfacial phenomena.[6] Such measurements have been employed by the rock physics and petrochemical exploration communities for over five decades,[7,8] providing estimates of oil and gas reservoir quality indicators including pore size distributions, porosity, permeability, and producible hydrocarbon content.[9] Further well-established applications include the observation of cement and plaster hydration kinetics,[10,11] while emergent fields within the materials chemistry research space now include characterisation of bespoke porous architectures including heterogeneous catalysts,[12–14] zeolites[15] and metal-organic frameworks.[16]

The most data-rich methods for elucidating spin relaxation processes in porous media are multidimensional relaxation correlation measurements.[17–19] Indeed, two-dimensional (2D) relaxation-relaxation ($T_1 - T_2$[20–22] and $T_2 - T_2$[23,24]) and diffusion-relaxation ($D - T_2$[25–28]) measurements are widely applied to characterise the dynamics of confined fluids, while higher-dimensional experiments,[29–32] including variants exhibiting spatial resolution,[33,34] have also been reported. Such measurements are of particular utility in the study of fluids confined to heterogeneous solids as they are essentially independent of the detailed chemical shift phenomena associated with traditional frequency-based NMR methods, wherein spectral peaks are likely to be (i) dominated by species away from the interface of interest, and (ii) unfavourably broadened due to the effects of magnetic susceptibility differences at the solid-liquid interface,[35] often requiring exotic approaches (see e.g. ref [36]) to avoid an intractable loss of spectral resolution.

Here we focus on the interpretation of 2D ^{1}H (proton) $T_1 - T_2$ relaxation correlation data as a probe of adsorbate identity and interfacial dynamics. Such measurements are particularly advantageous in the presence of multiple fluids, with differences in observed relaxation times – often by an order of magnitude or more – facilitating distinction between components. An established example of this phenomenon is the characterisation of hydrocarbon-bearing shale rocks,[37,38] wherein the contributions of oil, gas and bituminous hydrocarbons are often readily identified. As first demonstrated by Weber *et al*.,[39] this concept is also extensible to the observation of competitive displacement processes, wherein the relative populations of correlation peaks associated with different liquids may be followed simultaneously to provide insight into displacement kinetics.

In this work, the concept that multiple fluids within the same porous material can present distinct relaxation correlation populations is extended to the realisation and interpretation of relaxation data associated with a single probe fluid exhibiting multiple observable nuclear spin relaxation environments. Specifically, through the exploitation of a series of short-chain alcohols and carboxylic acids imbibed within a model mesoporous silica, we demonstrate for the first time that a wide range of polar-protic adsorbates exhibit discrete proton relaxation characteristics associated with their different hydrogen-bearing moieties. We find that such observations facilitate clear spectroscopic distinction between alkyl and hydroxyl groups without the need for chemical shift analysis. Further motivated by the discovery of a correlation between relaxation characteristics and adsorbate acidity, we provide a clear rationalisation for the observed functional group-specific surface relaxation phenomena in terms of interfacial proton exchange dynamics.

Results

Nuclear spin relaxation of confined liquids

Figure 1 summarises our experimental approach: Measurements utilised a 2D $T_1 - T_2$ correlation pulse sequence (**Figure 1a**) implemented on a 12.7 MHz benchtop NMR spectrometer (**Figure 1b**). Samples comprised a range of short chain liquid hydrocarbons imbibed within a particulate silica material (4 mm diameter particles) exhibiting 15 nm pores (**Figure 1c**); mesoporous silica was selected as a model porous solid in this work due to its wide-ranging application as a support material in heterogeneous catalysis, and hence relevance to a plethora of interfacial processes.[40] Further experimental details are provided in **Methods**.

The $T_1 - T_2$ correlation experiment employed here measures a series of T_2 relaxation decay processes exhibiting differing degrees of T_1 encoding (**Figure 1d**). The resulting 2D relaxation data may then be inverted[41] to provide a 2D $T_1 - T_2$ population distribution, wherein correlation peaks provide insight into the relative probability of the system exhibiting a given combination of T_1 and T_2 relaxation times. **Figures 1e and 1f** show example correlation data for imbibed cyclohexane and acetone, explored here as archetypal apolar and polar-aprotic adsorbates, respectively. Any liquid outside of the porous material was removed before analysis (see **Methods**) such that all observed relaxation characteristics are associated with confined species. A single correlation peak is observed in each case, characteristic of imbibed species exhibiting a single effective ^{1}H relaxation environment as a result of fast-exchange between adsorbed species and bulk-like molecules away from the solid-liquid interface.[42,43] Under such conditions, and assuming surface-limited relaxation,[6] a well-established general expression for the observed relaxation rates $T_{i,obs}^{-1}$ (with $i \in \{1,2\}$) is known:[17]

$$\frac{1}{T_{i,obs}} \approx \frac{1}{T_{i,bulk}} + \frac{S}{V}\frac{\delta}{T_{i,surf}}. \qquad (1)$$

Here S/V is the surface-to-volume ratio of the confining pore structure, δ the length scale of the adsorbed surface layer, $T_{i,surf}$ the relaxation time constants of the adsorbed population, and $T_{i,bulk}$ the time constants of the unrestricted fluid (see **Figure 1c**). This expression exemplifies the inherent sensitivity of nuclear spin relaxation measurements to both material properties (S/V; providing sensitivity to pore size) and interfacial chemistry ($\delta/T_{i,surf}$; commonly termed the surface relaxivity). Moreover, given the general case that $T_{i,surf} \ll T_{i,bulk}$, the dimensionless ratio of observed relaxation time constants can be reduced to to $T_{1,obs}/T_{2,obs} \sim T_{1,surf}/T_{2,surf}$.[44] This surface ratio $T_{1,surf}/T_{2,surf}$ is sensitive to the activated translational dynamics of molecules at the solid-liquid interface,[45] and is considered a non-invasive probe of surface affinity[21,46] (additional surface relaxation theory is detailed in **Supplementary Note 1**). As the experimentally accessible ratio $T_{1,obs}/T_{2,obs}$ is largely unaffected by the terms δ and S/V in **Equation (1)**, acquisition of this metric provides a novel route for the comparison of interfacial phenomena either between different porous

media imbibed with the same liquid, or as demonstrated here, multiple liquids within the same porous material.

As the ill-posed nature of the inversion process required to obtain the 2D relaxation distributions (an inverse Laplace transform[47]) is highly susceptible to the influence of experimental noise,[41] we restrict our analysis of such distributions to the modal relaxation characteristics of each correlation peak observed, making no attempt to infer insight from correlation peak shapes. The modal $T_{1,obs}/T_{2,obs}$ ratios of the correlation peak within **Figures 1e and 1f** (from here on in referred to as $\langle T_1/T_2 \rangle$) are indicated by a dashed diagonal lines, with the larger $\langle T_1/T_2 \rangle$ value of acetone ($\langle T_1/T_2 \rangle = 1.47 \pm 0.01$), compared to cyclohexane ($\langle T_1/T_2 \rangle = 1.13 \pm 0.02$) providing a clear demonstration of the increased affinity of polar adsorbates for oxide pore surfaces, relative to apolar alkanes.[48]

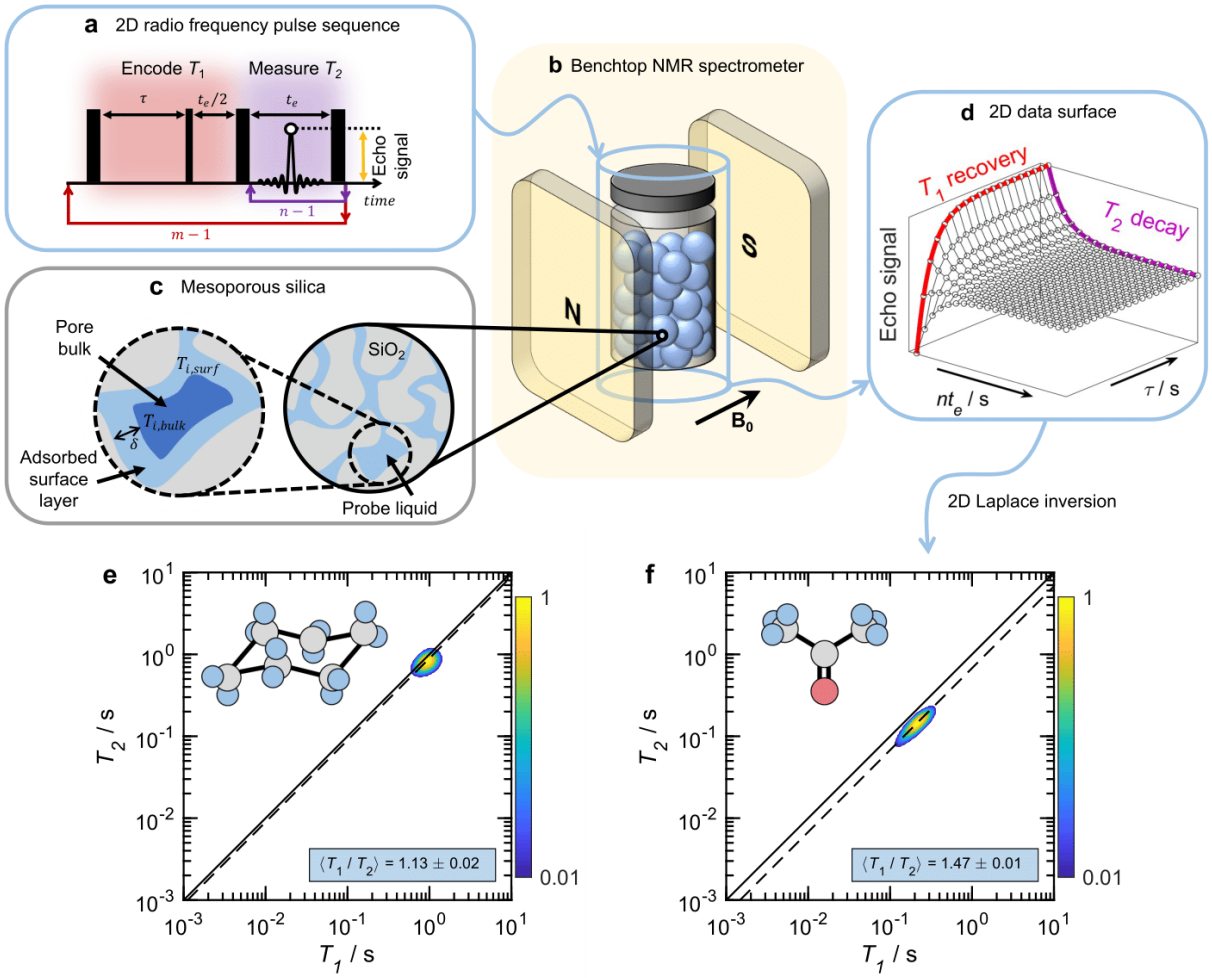

Figure 1. Experimental approach. Measurements employed (**a**) a 2D $T_1 - T_2$ pulse sequence and (**b**) a low-field benchtop NMR spectrometer with a B_0 = 0.3 T ($\nu_0\{^1H\}$ = 12.7 MHz) parallel plate magnet array. The porous silica (SiO_2) material comprised 4 mm diameter spheres with 15 nm pores (**b** and **c**), and was soaked in each probe liquid for 48 hours, with the inter-particle liquid then removed. By cycling the 2D pulse sequence in (**a**) through m different τ values and recording the intensity of the resulting n echoes a ($nt_e \times m\tau$) data surface is generated (**d**), which may be inverted to generate a 2D probability distribution of T_1 and T_2 times (see **Methods**). Example ^{1}H $T_1 - T_2$ correlation data for cyclohexane and acetone within this silica material are shown in **e** and **f**, respectively. The magnitude of each correlation peak indicates the relative probability of each system exhibiting a particular combination of T_1 and T_2 relaxation times, as indicated by the colour bars. Solid diagonal lines indicate the parity ratio T_1/T_2 = 1, while the modal relaxation time ratio $\langle T_1/T_2 \rangle$ of each correlation peak is indicated by dashed diagonal lines; $\langle T_1/T_2 \rangle$ values are specified in each case. The molecular structure of each adsorbate is also given: C, O and H atoms are coloured grey, red and blue, respectively.

Relaxation of polar-protic adsorbates

Figure 2 shows ^{1}H $T_1 - T_2$ distributions for a series of small polar-protic hydrocarbons within the same silica material; the example adsorbates studied here include short-chain primary alcohols (methanol, ethanol, 1-propanol and 1-butanol), secondary alcohols (2-propanol and 2-butanol) and simple carboxylic acids (acetic acid, propanoic acid and butanoic acid). In contrast to **Figure 1,** two distinct correlation peaks are clearly apparent within each data set, exhibiting significantly different modal T_1 and T_2 times ($\langle T_1 \rangle$ and $\langle T_2 \rangle$, respectively; values are provided in **Supplementary Table 1**). Any inter-particle liquid was again removed from these samples, such that all observed relaxation populations must arise from the confined fluids within the silica pore network. We note these peaks cannot be assigned to the presence of multiple pore environments with varying surface-to-volume ratios (see **Equation (1)**), which, given the similar self-diffusion properties of short-chain alcohols and acids to our aprotic adsorbates (demonstrated elsewhere[49–51]) would also be apparent within the cyclohexane and acetone data in **Figure 1**. Proton-proton scalar coupling artefacts[52] may also be discounted due to the very short echo time employed in these measurements (see **Methods**). Rather, these populations are assigned to the distinct relaxation characteristics of the alkyl ($C_x{}^1H_y$) and hydroxyl (-O^1H) proton environments of each adsorbate. Our assignments are supported by previous spectrally-resolved (high-field) T_1 measurements of methanol within a range of porous oxide materials,[53] which revealed the hydroxyl proton to exhibit distinctly more rapid rates of T_1 relaxation than those within the alkyl environment. Correlation peaks at short $\langle T_1 \rangle$ and $\langle T_2 \rangle$ values are therefore assigned to the hydroxyl ^{1}H environments of each adsorbate, while peaks at longer relaxation times are assigned to the corresponding alkyl environments. These assignments are further consistent with the integrated peak ratios obtained from each correlation plot, which we expect to reflect the ratio of protons within each chemical environment; these ratios are correlated against the expected alkyl/hydroxyl ratio of each adsorbate in **Figure 3**, demonstrating a strong, positive correlation. Given the complex surface exchange phenomena described and discussed below, however, we note that a 1:1 parity between observed and expected alkyl/hydroxyl ratios is not expected for these data.

While we recognise previous analyses of confined methanol suggest the existence of two distinct relaxation populations,[21,54–56] the correlation data in **Figure 2** exemplifies the first observation of such phenomena in longer chain primary alcohols, as well as the first such data for both secondary alcohols and carboxylic acids. While high and intermediate field measurements have failed to evidence the existence of multiple relaxation environments in the presence of longer chain (>C_1) liquid hydrocarbons,[21,56] we attribute the clear and persistent observation of hydroxyl group relaxation phenomena across our range of polar-protic adsorbates to the combination of short experimental echo times and low magnetic field strength (see **Methods**); these experimental conditions significantly reduce the impact of magnetic susceptibility contrast effects on the measurement of short T_2 times,[35] facilitating the accurate measurement of relaxation data associated with rapidly relaxing hydroxyl protons. Collectively, the correlation data detailed within **Figure 2** confirms, for the

first time, the widespread existence of functional group-specific relaxation phenomena across a broad range of organic molecules of importance as solvents, reagents, and hydrogen carriers. In the remainder of this work we identify and examine distinct trends in the observable relaxation characteristics presented by these adsorbates.

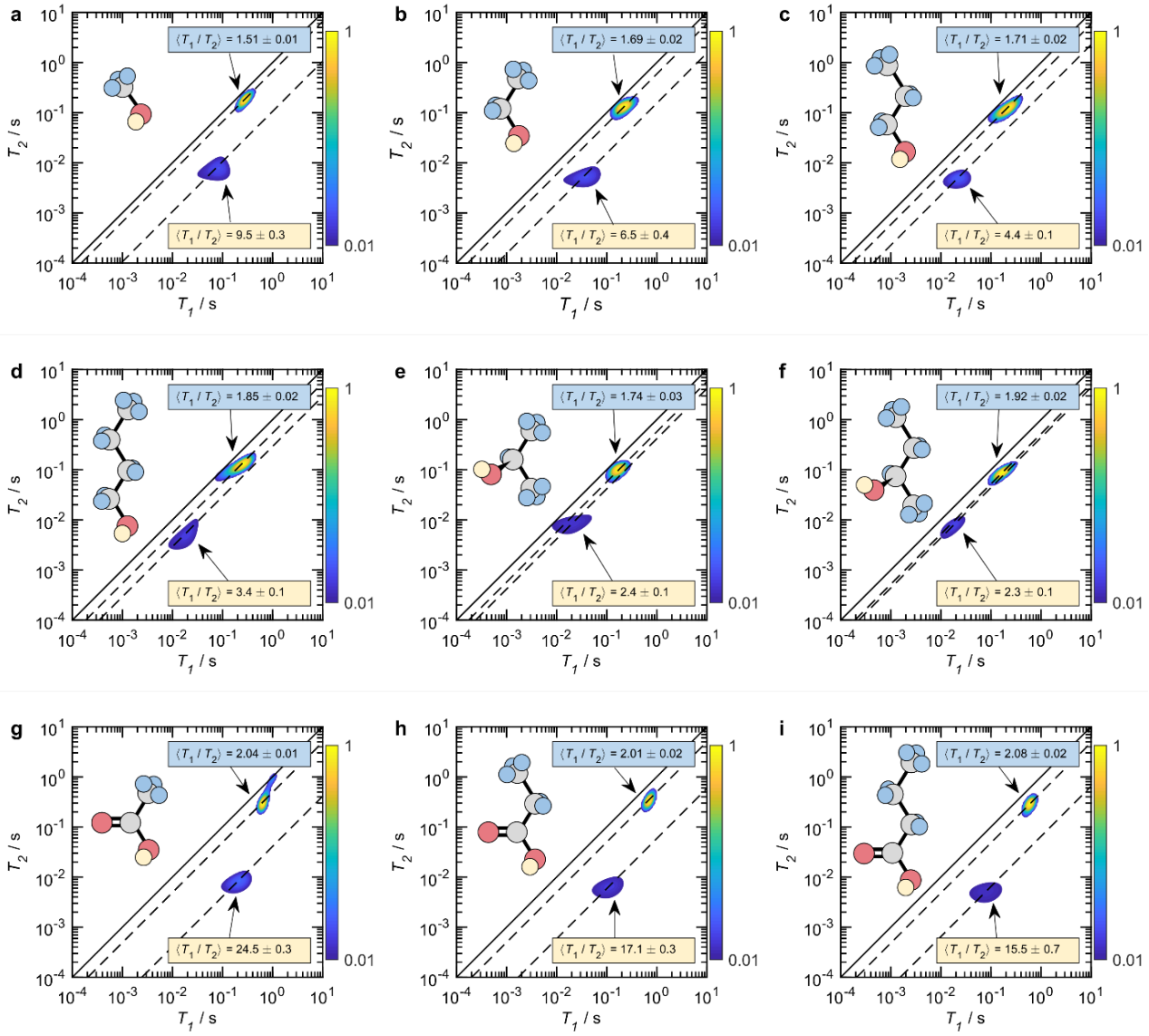

Figure 2. Relaxation characteristics of protic adsorbates. ^{1}H $T_1 - T_2$ correlation data for **a – d** primary alcohols (**a** methanol, **b** ethanol, **c** 1-propanol and **d** 1-butanol), **e – f** secondary alcohols (**e** 2-propanol and **f** 2-butanol), and **g – i** carboxylic acids (**g** acetic acid, **h** propanoic acid and **i** butanoic acid) in mesoporous silica (exhibiting 15 nm pores) at 12.7 MHz. The magnitude of each correlation peak indicates the relative probability of each system exhibiting a particular combination of T_1 and T_2 relaxation times, as indicated by the colour bars. Solid diagonal lines indicate the parity ratio T_1/T_2 = 1, while the modal relaxation time ratio $\langle T_1/T_2 \rangle$ of each correlation peak is indicated by dashed diagonal lines; $\langle T_1/T_2 \rangle$ values are specified in each figure. The molecular structure of each adsorbate is also given: C and O atoms are coloured grey and red, respectively. Aprotic H are coloured blue while protic H are shown in yellow. Correlation peaks at long and short T_2 are assigned to aprotic and protic ^{1}H-containing moieties, respectively.

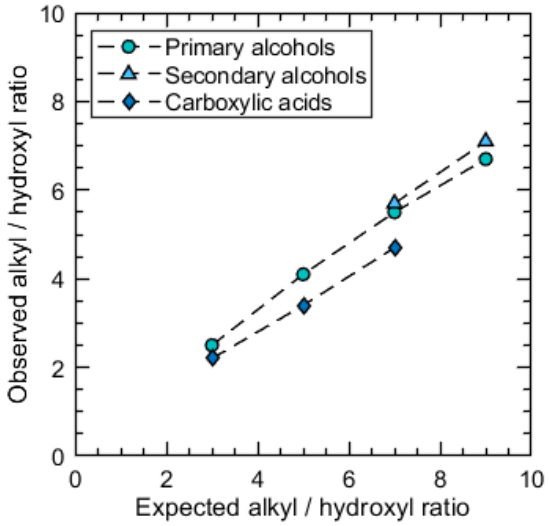

Figure 3. Correlation peak integrals. Integrated peak ratios from the correlation data presented in Figure 2 as a function of the expected alkyl/hydroxyl ratio of the same adsorbates.

Relaxation, acidity and surface-adsorbate proton exchange

Figure 4a provides a summary of the $\langle T_1/T_2 \rangle$ values obtained from the correlation data presented in **Figure 2**. A general increase in the alkyl $\langle T_1/T_2 \rangle$ of alcohols is evident with increasing carbon chain length, from $\langle T_1/T_2 \rangle = 1.51 \pm 0.01$ for methanol to $\langle T_1/T_2 \rangle = 1.85 \pm 0.02$ for 1-butanol, and from $\langle T_1/T_2 \rangle = 1.74 \pm 0.08$ for 2-propanol to $\langle T_1/T_2 \rangle = 1.92 \pm 0.02$ for 2-butanol. These observations regards the alkyl $\langle T_1/T_2 \rangle$ ratio are consistent with previous investigations of short-chain primary alcohol behaviour in porous silica[21] and alumina,[56] with the measured relaxation time ratios found to correlate with adsorbate surface affinity.[21] Conversely, our hydroxyl $\langle T_1/T_2 \rangle$ values demonstrate a clear decrease with increasing carbon chain length, with this decrease noticeably evident across both the primary alcohol and carboxylic acid data sets. To enable further discussion of this discrepancy we introduce a new metric of the form $\Delta\langle T_1/T_2 \rangle = \langle T_1/T_2 \rangle_{hydroxyl} - \langle T_1/T_2 \rangle_{alkyl}$; this metric quantifies the difference in $\langle T_1/T_2 \rangle$ between the alkyl and hydroxyl relaxation populations of each adsorbate and is independent of their absolute values (see **Figure 4a**), which we expect to scale with molecular surface affinity.[21] The overall progression of $\Delta\langle T_1/T_2 \rangle$ is: carboxylic acids $\gg$ primary alcohols $>$ secondary alcohols (see **Supplementary Table 1** for values), with values further decreasing as a function of increased carbon chain length in each case. Given these behaviours mirror the well-known trends in the acidity of these adsorbates, we conjecture that a comprehensive understanding of our observed relaxation data likely requires consideration of hydroxyl proton dissociation. This interpretation is supported in **Figure 4b** which details a plot of $\Delta\langle T_1/T_2 \rangle$ against adsorbate pK_a, employed here as a convenient metric of liquid-phase acidity. A strong correlation is clearly apparent between increasing $\Delta\langle T_1/T_2 \rangle$ and decreasing pK_a (indicative of increased acidity).

We rationalise the above trend through recognition of the dominant surface interaction mechanism between (hydroxylated) silica surfaces and polar protic molecules. Such interactions are governed by surface-adsorbate hydrogen bonding interactions, which provide not only a favourable bonding mode between protic adsorbates and surface-bound hydroxyl groups,[57,58] but also a well-established means for proton exchange.[59–61] Exchange interactions during the encoding periods of $T_1 - T_2$ correlation measurements have significant potential to bias the resulting relaxation characteristics. This concept is elucidated by considering two coupled magnetisation reservoirs, M_{sol} and M_{ads}, which we associate with solid-bound and adsorbate-bound hydroxyl protons, respectively. Relaxation during the T_1 and T_2 encoding periods of the pulse sequence in **Figure 1a** is given by:[62]

$$\frac{d}{dt}\begin{bmatrix} M_{sol} \\ M_{ads} \end{bmatrix} = \begin{bmatrix} -T_{1,sol}^{-1} - k_1 & k_2 \\ k_1 & -T_{1,ads}^{-1} - k_2 \end{bmatrix} \begin{bmatrix} M_{sol} - M_{sol}^0 \\ M_{ads} - M_{ads}^0 \end{bmatrix} \quad (2)$$

and

$$\frac{d}{dt}\begin{bmatrix} M_{sol} \\ M_{ads} \end{bmatrix} = \begin{bmatrix} -T_{2,sol}^{-1} - k_1 & k_2 \\ k_1 & -T_{2,ads}^{-1} - k_2 \end{bmatrix}\begin{bmatrix} M_{sol} \\ M_{ads} \end{bmatrix}, \quad (3)$$

respectively, where $T_{i,sol}$ and $T_{i,ads}$ (with $i \in \{1,2\}$) are the relaxation time constants of the two reservoirs. The terms M_{ads}^0 and M_{sol}^0 are the equilibrium magnetisations of the two reservoirs, which are directly proportional to the intrinsic number of protons within each environment. The exchange rate constants k_1 and k_2 then quantify the rates of proton migration between surface and adsorbate hydroxyl groups, with the overall exchange rate $k = k_1 + k_2$. Simple solutions to these expressions emerge in the limits of and slow ($k \ll \left|T_{2,sol}^{-1} - T_{2,ads}^{-1}\right|$) and fast ($k \gg \left|T_{1,sol}^{-1} - T_{1,ads}^{-1}\right|$) exchange, giving rise to either two distinctly separate relaxation populations or a single, highly averaged population, respectively.[63] In the intermediate regime between these limiting cases the observed relaxation characteristics are sensitive to the relative magnitudes of the inherent relaxation, exchange rates and equilibrium magnetisations; upon approaching the fast exchange limit a single population is expected, exhibiting relaxation times sensitive to k.[64]

Simple detection of the exchange regime relevant to our data is obviated by the lack of observable surface hydroxyl populations,[37,65] with the true relaxation behaviour of our experimental systems further complicated by adsorption/desorption processes. However, given the high sensitivity of the ratio T_1/T_2 to the adsorbed surfaced layer (see **Supplementary Note 1**), we conjecture that rapid exchange of this form provides a significant contribution to the hydroxyl group $\langle T_1/T_2 \rangle$ behaviour observed in **Figure 2**. As solid-state structures exhibit long T_1 and short T_2 values,[5] surface-adsorbate proton exchange dynamics occurring near the fast-exchange limit are expected to decrease the observed hydroxyl T_2 times, while propagating a corresponding increase in T_1. An increase in exchange rate is therefore expected to increase $\Delta\langle T_1/T_2 \rangle$ as hydroxyl group exchange dynamics progressively increase the associated $\langle T_1/T_2 \rangle$ values away from that of the non-exchanging alkyl environment.

Considering the trend demonstrated in **Figure 4b**, the increase in $\Delta\langle T_1/T_2 \rangle$ anticipated to arise from such dynamics as a function of increased exchange rate clearly suggests that more acidic adsorbates undergo more rapid rates of proton dissociation at the solid/liquid interface. Despite the typical disparity between the thermodynamic and kinetic contributions to exchange phenomena, an approximate relationship between hydroxyl exchange rates and adsorbate acidity is obtained by considering the double-well potential energy surface in **Figure 4c**. Assuming classical dynamics (i.e. neglecting tunnelling) the exchange rates between free energy minima is given by the Eyring equation: $k_i = (k_B T/h)\exp\left(-\Delta^{\ddagger}G_i/RT\right)$ (with $i \in \{1,2\}$), where k_B, h and R are the Boltzmann constant, Planck constant and gas constant, respectively, T is the absolute temperature, and $\Delta^{\ddagger}G_i$ are the activation free energy barrier heights. Thermodynamically, the free energy change for a given proton dissociation process is $\Delta G \approx 2.303RTpK_a$. As proton exchange between potential wells is considered a pair of coupled pseudo-bimolecular reactions,[66] the overall difference in hydrogen bond

potential well depth is determined by the difference in pK_a values between the two protonated states, ΔpK_a.[67] These concepts are readily translatable to the consideration of proton exchange dynamics at solid-liquid interfaces. Crucially, for the purpose of rationalising the observed trend between relaxation characteristics and adsorbate pK_a detailed in **Figure 4b**, we recognise that for the comparison of exchange processes between a single silica material and multiple distinct adsorbates, changes in ΔpK_a may be assumed dependent on only the adsorbate pK_a. With the free energy minimum associated with the solid surface essentially fixed across the range of experiments performed, adsorbate pK_a will therefore indirectly dictate the free energy barrier height, in turn influencing observable proton exchange rates $k = k_1 + k_2$. We therefore consider adsorbate pK_a an approximate and indirect indicator of overall hydroxyl group exchange rates, in turn providing a generalised rationale for the correlation observed in **Figure 4b**. This reasoning is further consistent with the peak integral ratio data presented in **Figure 3**, wherein the observed alkyl/hydroxyl ratios of our adsorbates are clearly reduced from their expected values in order of increasing acidity; adsorbates undergoing significant surface-adsorbate proton exchange will present larger hydroxyl relaxation populations than expected, hence reducing the experimentally observed alkyl/hydroxyl ratio.

To support the above interpretation, the ability of our model silica material to undergo proton exchange interactions with adsorbed hydrocarbons was confirmed via deuterium exchange experiments, with methanol-d4 (CD_3OD; D ≡ ^2H) utilised as a probe fluid. The salient features of the expected solid-liquid interactions are shown in **Figure 5a**. In the absence of interfacial proton exchange the methanol-d4 probe will remain invisible to ^1H NMR analysis; if exchange occurs, however, we expect a growth in detectable ^1H NMR signal as surface proton scrambling facilitates the formation of CD_3OH. **Figure 5b** compares ^1H T_2 decay data from methanol-d4 (nominal purity $= 99.8\ \%$) with that from a sample of mesoporous silica exposed to excess methanol-d4 for 24 hours; unlike the samples prepared for 2D $T_1 - T_2$ analysis, this system therefore comprises both free and confined liquid. While signal arising from the neat sample (bulk methanol-d4) lies within the spectrometer noise floor, the system comprising mesoporous silica in excess methanol-d4 clearly exhibits an observable T_2 decay signal, confirming the presence of NMR active CD_3OH, and hence the occurrence of surface-adsorbate proton exchange interactions. **Supplementary Note 2** further explores the consistent temporal increase of this detectable signal over the first 24 hours.

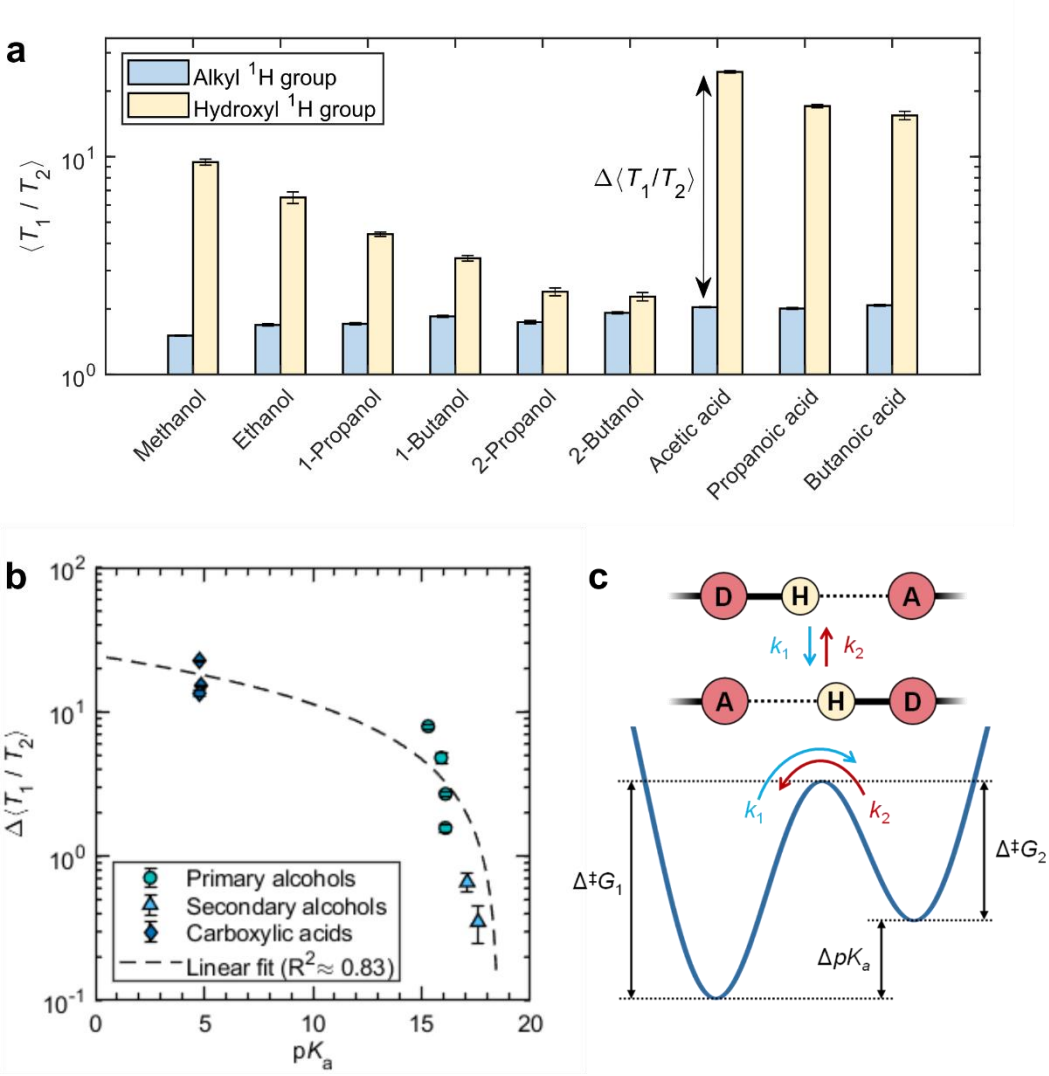

Figure 4. Relaxation trends of protic adsorbates. a Summary of modal alky and hydroxyl ^{1}H relaxation time ratios $\langle T_1/T_2 \rangle$; values correspond with the dashed diagonal lines in **Figure 2**. **b** Correlation between the difference in alkyl and hydroxyl relaxation time ratios $\Delta\langle T_1/T_2 \rangle$ and adsorbate pK_a, including a linear fit. **c** Double-well potential energy surface for proton exchange between hydrogen bond donors (D) and acceptors (A). The forward and back exchange rates k_1 and k_2 are hence determined by the activation free energies $\Delta^{\ddagger}G_1$ and $\Delta^{\ddagger}G_2$, respectively, while the difference in potential well depth is determined by the difference in pK_a of the two protonated states, ΔpK_a.

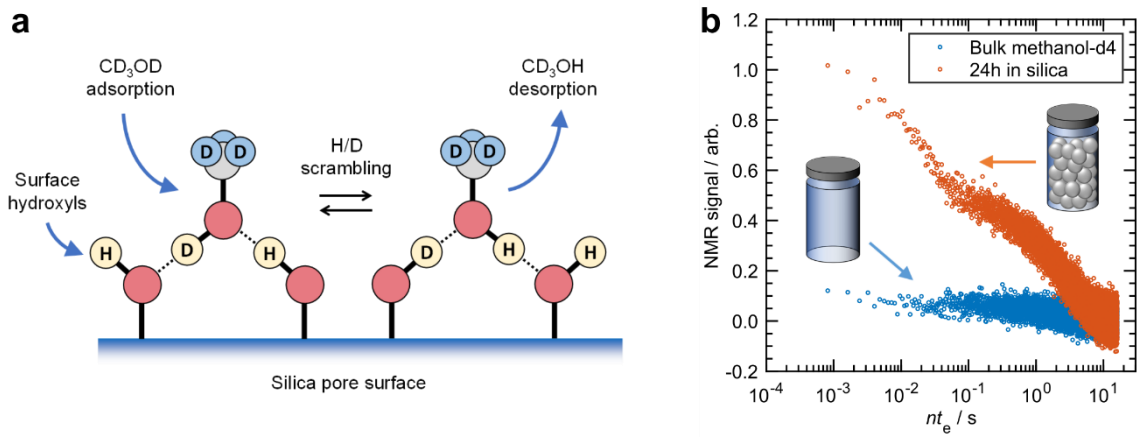

Figure 5. Proton scrambling at the methanol/silica interface. a Expected interactions between methanol-d4 (CD_3OD) and silica pore surfaces. ^1H NMR-invisible CD_3OD molecules adsorb via hydrogen bonding interactions with surface hydroxyl groups, facilitating H/D scrambling. Desorbing CD_3OH will then be visible to ^1H NMR analysis. **b** CPMG T_2 decay data for bulk CD_3OD (blue) and CD_3OD combined in excess with dry mesoporous silica for 24 hours (orange). The clear ^1H T_2 decay signal in the case of CD_3OD imbibition within silica confirms surface H/D scrambling and the formation of ^1H NMR-active CD_3OH.

Conclusions

The identification of functional group-specific nuclear spin relaxation across the diverse range of adsorbates examined here is of clear significance for the accurate interpretation of NMR relaxation data acquired from systems involving polar-protic probe fluids and hydroxylated pore surfaces. The example solid/liquid systems explored here are of particular relevance to the interrogation of solvent effects in liquid-phase heterogeneous catalysis, wherein the utilisation of nuclear spin relaxation measurements to infer interfacial phenomena is now a rapidly evolving field. Furthermore, the unambiguous assignment of multiple relaxation populations to individual adsorbates undergoing surface-adsorbate proton exchange is expected to better inform the application and interpretation of multidimensional relaxation measurements when applied to a range of complex porous systems, averting the erroneous interpretation of correlation data in terms of material structures or the presence of multiple adsorbates. Although not explored here, we note that the clear distinction between alkyl and hydroxyl relaxation populations along the T_2 dimension of our 2D correlation data suggests functional group-specific insight is also readily accessible via rapid one-dimensional T_2 measurements, facilitating the extension of such analyses to temporally resolved experimental systems; this concept is of particular relevance to the study of competitive displacement dynamics, which we aim to pursue in future work. Finally, we note that the clear correlation between nuclear spin relaxation characteristics and adsorbate acidity strongly motivates further investigation into how relaxation phenomena may be exploited as a robust and non-invasive probe of adsorbate identity and/or material surface chemistry within a diverse range of porous architectures.

Methods

Materials and sample preparation

A commercial Q15 mesoporous silica gel material (4 mm diameter spherical particles; nominal mean pore size: 15 nm) was obtained from Fuji Silysia Chemical Ltd. (Japan). The material was first refluxed in deionised water (obtained onsite at the Australian Resources Research Centre, Perth, Australia) at 120 °C for 4 hours to hydroxylate the pore surfaces. The Q15 was then dried in air at 110 °C for 12 hours, and for an additional 3 hours at 110 °C under vacuum (10^{-3} bar).

Cyclohexane (Thermofisher Scientific, >99%), acetone (ChemSupply Australia, >98%), methanol (ChemSupply Australia, >99.9%), ethanol (ChemSupply Australia, >99.9%), 1-propanol (ChemSupply Australia, >99.8%), 2-propanol (ChemSupply Australia, >99.5%), 1-butanol (ChemSupply Australia, >99%), 2-butanol (ChemSupply Australia, >99%), acetic acid (Merk, >99%), propanoic acid (ChemSupply Australia, >99%), butanoic acid (ChemSupply Australia, >99%), and methanol-d4 (Cambridge Isotope Laboratories, 99.8%) were used as received.

Imbibed silica samples for $T_1 - T_2$ analysis were prepared by soaking in excess liquid for at least 48 hours under ambient conditions. Silica particles were then separated from each liquid and rolled over a pre-soaked filter paper to remove any inter-particle liquid, with the imbibed silica spheres transferred to sealed 7 ml glass vials for analysis. Each sample consisted of approximately 2.5 g of Q15, corresponding with around 160 particles; as such, each sample provided a well-averaged measurement of the surface-adsorbate interactions present between probe liquid and the model silica material employed. Sample preparation for methanol-d4 exchange analysis is described in **Supplementary Note 2**.

NMR hardware

^{1}H nuclear spin relaxation measurements were performed using an Oxford Instruments Geospec NMR spectrometer equipped with a $B_0 = 0.3$ T parallel plate permanent magnet array (providing a ^{1}H frequency of $\nu_0 = 12.7$ MHz) and a 53 mm Q-sense probe. All measurements were performed at room temperature (25 ± 1 °C) and ambient pressure.

2D relaxation analysis

$T_1 - T_2$ relaxation correlation data was acquired by applying the 2D pulse sequence in **Figure 1a**,[20] wherein 90° and 180° radio frequency (RF) pulses are represented by think and thick vertical bars, respectively. Following initial polarisation of the spin system along the direction of the static magnetic field B_0 (conventionally termed the z-axis), a 180° RF pulse inverts the sample magnetisation onto the -z axis. Here, longitudinal T_1 relaxation processes drive the spin system back towards thermal equilibrium; this recovery is

characterised by the relaxation time constant T_1, and was encoded within the indirect dimension of our 2D relaxation data through the application of a variable recovery time τ. The spin system is then rotated into the transverse plane via a 90° RF pulse, inducing transverse relaxation processes. A train of n 180° RF pulses induces n spin echoes, which decay in magnitude due to T_2; each echo magnitude $S(nt_e, \tau)$ was recorded as a single data point (white data point in **Figure 1a**), defining the direct dimension of our relaxation data. The pulse sequence was then cycled to encode for m different τ recovery times, forming an $(m \times n)$ data surface (**Figure 1d**). The indirect dimension of our measurements was encoded by employing 16 logarithmically spaced τ values between 1 ms and 12 s, while the direct dimension was encoded using $n = 40{,}000$ echoes separated by an echo time of $t_e = 100$ µs. This short echo time, together with the low static magnetic field strength used here, was employed to mitigate undesired transverse relaxation phenomena resulting from magnetic susceptibility contrast effects at the solid-liquid interface.[35] Measurements included 8 repeat scans separated by a recycle delay of 12 s ($\gg 5 \times T_1$), taking approximately 40 minutes per correlation measurement and resulting in signal-to-noise ratios of approximately 400.

$T_1 - T_2$ relaxation correlation data may be described by a 2D Fredholm integral equation of the first kind:[47]

$$\frac{S(nt_e, \tau)}{S(0, \infty)} = \iint K_{12}(nt_e, T_2, \tau, T_1)\, F(T_1, T_2)\, d\log_{10}(T_1)\, d\log_{10}(T_2) + \varepsilon(nt_e, \tau). \quad (4)$$

Here $S(nt_e)/S(0, \infty)$ is the normalised spin echo signal magnitude, while $\varepsilon(nt_e, \tau)$ represents the experimental noise, assumed Gaussian with zero mean. The kernel function $K_{12}(nt_e, T_2, \tau, T_1)$ describes the predicted form of T_1 and T_2 relaxation, which for the pulse sequence in **Figure 1a** is given by:[20]

$$K_{12}(nt_e, T_2, \tau, T_1) = \left[1 - 2\exp\left(\frac{-\tau}{T_1}\right)\right] \exp\left(\frac{-nt_e}{T_2}\right). \quad (5)$$

Finally, the term $F(T_1, T_2)$ represents the targeted 2D distribution of T_1 and T_2 relaxation time constants. Distributions were obtained via a numerical inversion of each acquired relaxation data surface according to Equations (4) and (5). As this is an ill-posed problem in the presence of experimental noise,[41] stability of the inverted distributions was achieved through the application of Tikhonov regularisation,[68] with the degree of smoothing determined via the generalised cross-validation (GCV) method.[69] Distributions were limited to (200 × 200) values, with each dimension bound within the range $\{10^{-4}, 10^{1}\}$ s (note that correlation plots in **Figure 1** are shown with reduced bounds to increase correlation peak clarity). Inversions were performed using a 2D fast Laplace inversion algorithm written in Matlab, as first implemented by Mitchell *et al.*[70]

H/D exchange

For the analysis of H/D exchange at the methanol-d4/silica interface, T_2 decay data was acquired by applying the standard Carr-Purcell Meiboom-Gill (CPMG) sequence. The magnitude of $n = 40{,}000$ spin echoes were acquired separated by an echo time of $t_e = 400$ µs. Further details are provided in **Supplementary Note 2**.

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Additional information

Author Contributions

N.R. and M.L.J. designed the research. N.R. performed the experiments and analysed the data. E.F.M and M.L.J. supervised the research. N.R. and M.L.J wrote the manuscript. All authors discussed the results and commented on the manuscript.

Supplementary Information Details

The Supplementary Information file contains:

- Supplementary Note 1: Surface Relaxation Theory
- Supplementary Note 2: Methanol-d4 Exchange Experiments
- Supplementary Table 1
- Supplementary Figures 1 – 3

Data Availability

Associated data is available in the Supplementary Information file or from the corresponding author upon reasonable request.

Competing Interests

The authors declare no competing interests.

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